SOUND

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A PHYSICAL TEXT-BOOK

BY

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PREFACE TO FIFTH EDITION

At the time of the first edition of this book, there was no text-book in English which gave the reader an account of the considerable advances in the subject which had taken place in and following the First World War. Now after a Second World War the position is very different. There are a number of books dealing with special aspects of applied acoustics and there are two periodicals dealing solely with the subject, while in industry the penetration of applied acoustics has been far-reaching.

While it is hoped that sufficient mathematical theory has been given to enable the physical conceptions to be followed, this book does not pretend to take the place of the mathematical treatises of Rayleigh and Lamb, to which the reader is referred for a more rigorous and complete analysis when necessary. A knowledge of physics and mathematics to Intermediate Science standard and of calculus is assumed. The book covers all that a candidate for the pass and honours examinations of British and American Universities should require, and at the same time the needs of the research worker and technician have been met by describing the important work which has been done in the subject up to the present, with copious references to original papers where further details may be discovered.

It has proved impossible to adhere to the original intention to print all references, but all important recent ones are quoted as well as those of historical importance and the reader will find bibliographies in a number of these references and in the *Journal of the Acoustical Society of America.* The reader may add to those given by referring to *Science Abstracts* or the *Annual Reports on the Progress of Physics* which the Physical Society publishes.

In this, the fifth edition, a considerable re-arrangement of the text has been made and a considerable quantity of new material—particularly in the latter half of the book—has been added. The references are now grouped at the ends of chapters. I am indebted to many teachers, students and research workers for their continued support of the book, particularly to those who have written proffering suggestions, many of which I have incorporated in later editions.

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CHAPTER ONE

THE VELOCITY OF SOUND

First Principles. In seeking the origin of the phenomena which we classify under " sound " we have to deal with a sudden compression at a point in a medium, and the *raison d'être* of the phenomena we are about to study lies in the elasticity of the medium. If the latter were perfectly compressible, our interest in the matter would cease at its inception, but the particles which are pressed together tend to resume their original positions, and, in doing so, react on the surrounding particles, which are in turn compressed ; so that the original sudden change in the state of the medium travels out from the point at which it arose in the form of a wave.

Again owing to inertia, the return of the disturbed portions of the medium is never " dead beat " ; inertia causes the particles of the medium to overshoot their position of rest, so that in the track of the compression there follows a rarefaction, and another (but less intense) compression, as the oscillations of the particles about their former position are more or less rapidly damped. A single wave of compression is thus only an ideal conception, but is approximately realized by a short, sharp noise called a " pulse." The state of things at our point of origin may be made to pass through a recurring series of changes by artificial means, resulting in the emission of a train of compressions and rarefactions at regular intervals : this we term a musical note. The type of wave-motion (transverse, longitudinal, torsional) may be changed during propagation as it passes from one medium to another, but finally becomes apparent to us by an excitement of the auditory nerves ; or, if a registering apparatus is used instead of the ear, of the optical nerves, since a recording apparatus usually functions by moving a ray of light. The velocity with which the disturbance travels will obviously depend on the medium for the type of deformation involved, and on the closeness of packing of the material of the medium (its density).

The study of this subject as a branch of physics has had a peculiar history. It lay at first entirely in the hands of musicians who accumulated a great deal of more or less empirical data. It was known to the ancient Greeks that rapid vibration produced a tone of " high pitch " ;

and slow vibration, one of "low pitch." The pioneer physicists in modern times who endeavoured to set the subject on a scientific basis were: Mersenne,[1] Newton,[2] Young,[3] Chladni,[4] the brothers Weber,[5] and Savart.[6]

Velocity of Longitudinal Waves in Gases, especially Air. The sounds which affect a normal ear come through the atmosphere, so that the propagation of sound in the air is of prime importance. When the air follows a simple pendular motion under the maintaining action of the source of sound, the ear hears a single tone of which the number of complete oscillations per second is called the "frequency" of the source; and if an instantaneous picture of the medium be imagined, the length of the unit of the "pattern," or the distance between successive compressions, is known as the "wave-length."

The fractional decrease in volume, or the decrease in volume per unit of original volume, which a compressed body of gas may undergo, is known as the "condensation" s; in symbols, $s = -\frac{\delta v}{v}$. The compressibility may be defined as the fractional decrease of volume produced by unit change of pressure. The elasticity E is the inverse of this; the more compressible the gas the less elastic it is; in symbols

$$E = -\delta p \frac{v}{\delta v} \quad . \quad . \quad . \quad . \quad (1)$$

(the negative sign shows that increase of pressure produces decrease of volume).

Consider a tube of gas (Fig. 1) of unit cross section, and two plane

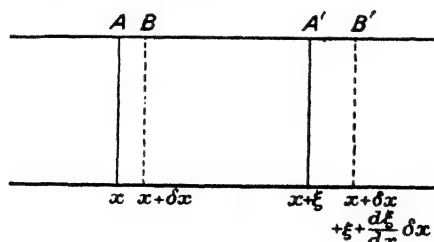



Fig. 1.—Velocity of Longitudinal Waves.

sections A, B, of this tube, whose co-ordinates measured along the tube are, before the passage of the wave, x and $x + \delta x$ respectively; so that the initial volume of the slice so cut off is δx.

At an instant of time δt later, let the arrival of the disturbance have

displaced the end A of our slice by an amount ξ to A', so that its co-ordinate is now $x + \xi$. Other sections of the slice will have a different displacement, for the displacement will vary with x at a rate $\frac{\partial \xi}{\partial x}$, so that, e.g., the displacement of the end B will be $\xi + \frac{\partial \xi}{\partial x}\delta x$, and this plane will now occupy a position B' given by $x + \delta x + \xi + \frac{\partial \xi}{\partial x}\delta x$, and the new volume of the slice will be $\delta x + \frac{\partial \xi}{\partial x}\delta x$, representing an increase of volume of $\frac{\partial \xi}{\partial x}\delta x$; this, divided by the original δx, is therefore, by definition, the condensation $s = -\frac{\partial \xi}{\partial x}$. Now the pressure on a section differs from the normal by an amount which varies with its position; so that, if that on the plane through A' is δp, that on B' is :—

$$\delta p + \frac{\partial(\delta p)}{\partial x}.\delta x = Es + E\frac{\partial s}{\partial x}.\delta x, \text{ in virtue of (1).}$$

The total force on the slice = the difference of pressure on the two ends :—

$$-E\frac{\partial s}{\partial x}.\delta x = E\frac{\partial^2 \xi}{\partial x^2}.\delta x.$$

Equating this to the mass $\times$ acceleration of the slice, the mass being $\rho\delta x$ where ρ is the density of the gas, we have :—

$$E\frac{\partial^2 \xi}{\partial x^2}.\delta x = \rho\delta x\frac{\partial^2 \xi}{\partial t^2};$$

or

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho}.\frac{\partial^2 \xi}{\partial x^2} \quad . \quad . \quad . \quad . \quad (2)$$

Putting

$$\frac{E}{\rho} = c^2 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

we find that $\sqrt{\frac{E}{\rho}}$, i.e., c, has the dimensions of a velocity. In the succeeding chapters it will be shown that the solution of (2) represents longitudinal waves travelling with this velocity.

This formula is only true for plane waves, and in the case where the original disturbance is of infinitesimal amplitude, but is approximately fulfilled for sounds of ordinary strength. Having regard to the correct value of the elasticity E for the type of wave in question, this formula

also applies to longitudinal waves in liquids and solids, and to torsional waves in solids, and to the transverse vibrations of strings (but not, without modification, to the transverse vibrations of bars and plates).

Value of E in Free Air—Newton's Formula and Laplace's Correction. From the formula (2) we can calculate the velocity (c) of sound in terms of the pressure and density, if we assume some relation between the pressure and volume (or density) of a gas. Newton [2] obtained, by other reasoning, a similar formula, and adopted the isothermal relation (pv = a constant) implying that the heat produced in the compression was rapidly conducted away from the layer of air affected, so as not to raise its temperature appreciably. Differentiating,

$$\frac{\delta p}{p} + \frac{\delta v}{v} = 0,$$

$$E = -\delta p \frac{v}{\delta v} = p$$

From (3) $c = \sqrt{\frac{p}{\rho}}$ . . . . (4)

At 0° C., $p = 76 \times 13{\cdot}6 \times 981 = 10^6$ dynes per sq. cm. (approx.).
$\rho = {\cdot}0013$ gms. per c.c.
$\therefore\ c = 280$ metres per second.

Now the earliest experimental determinations had shown a value considerably in excess of this. Newton ascribed this discrepancy to the fact that he had neglected to allow for the space occupied by the molecules in the actual atmosphere. This would imply that if one used a formula taking account of this space, there would be agreement between the experimental and theoretical results. Newton endeavoured to make some such correction by assuming that the molecules of air were incompressible, so that the disturbance passed instantaneously from one side of a molecule to the other, but he could not by this assumption raise his value to meet the experimental results.

It was left to Laplace [7] some 120 years later to provide the explanation which has borne the test of 100 years' subsequent experiment. He held that the compressions and rarefactions succeed each other so rapidly that the adiabatic relation is pertinent. The heat produced in compressing each stratum of the gas has not time to be dissipated in the body of the gas, but is expended in warming the compressed rticles themselves. There are, therefore, no exchanges of heat

between neighbouring strata, and the appropriate equation is

$$pv^{\gamma} = \text{const.}$$

$$\log p + \gamma \log v = \log \text{const.}$$

$$\frac{\delta p}{p} + \gamma \frac{\delta v}{v} = 0$$

$$E = -v\frac{\delta p}{\delta v} = \gamma p.$$

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which gives a value for c of 331·5 m. per sec. at 0° C.

Signal Method. What may be termed the large-scale method of finding the velocity of sound in air has remained the same in principle, though improved in technique, until the present day. A gun is fired and at a distant point observations are made of the instants (1) at which the gun is fired to produce the sound, and (2) at which the sound arrives at the observation post.

In the first experiments of Mersenne,[8] and of Gassendi,[9] the first instant was noted by looking for the flash accompanying the report, light being propagated over a few miles instantaneously in comparison with sound. The instant of arrival was determined by ear. The time for the sound to travel from the source to the observer was taken as the time on a clock between the incidence of the flash and the report. By employing distances greater than 3,000 metres, and allowing errors of starting and stopping the clock not exceeding $\frac{1}{2}$ sec., an accuracy of 5 per cent. was possible. On the appearance of Laplace's Memoir, the Bureau des Longitudes [10] organized extensive researches between five stations round Paris, with eminent observers, over distances of about 18,000 metres, cannon being fired consecutively from alternate ends of each line in an endeavour to eliminate the effect of the wind. The mean result gave 333·2 m./sec., when reduced to 0° C. The fault of their method lay in the "personal equations" of the observers, whereby the recorded times depended on the quickness of the perception and response of the experimenter. Stone,[11] in an endeavour to estimate this "psychological time (t)," fired small guns near the observers, so that sound of the same intensity was heard by them, as from the large guns far away.

Assuming an approximate value c_1 for the velocity of sound over this small distance l, the time "clocked" by an observer for the small

gun was $\frac{l}{c_1} + t$, from which t was found and substituted in the formula of the actual velocity c over the larger distance L, $c = \frac{L}{T - t}$, T being the observed time over this range.

Regnault[12] first endeavoured to get rid of the human element, by using electrical registration apparatus. A drum D (Fig. 2) is revolved by clockwork at constant speed, on which a style S, normally attracted by an electromagnet M, marks a straight line. At the sending station, a wire W in the circuit is broken by the firing of the

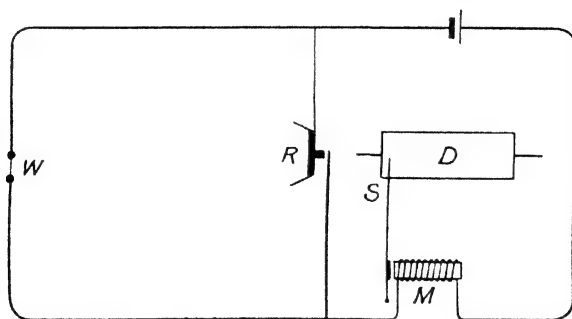



FIG. 2.—Regnault's Apparatus.

gun, releasing S which now makes a mark parallel to the former but a little to the left. The sound travels out till it reaches the receiving station, and pushes in the membrane R, so that it momentarily re-establishes the circuit, and pulls the style suddenly to the right. The time of travel of the sound is thus found by measuring the distance between two kinks in the mark on the drum, the speed of rotation of the drum being known. Regnault found, however, that the time lag in the instrument was, at least, equal to that of a trained experimenter.

Another method which, although subjective, does not involve human error in the estimation of time, is that of Bosscha,[13] which depends on the observation of coincidences. Two small electromagnetic hammers are arranged so that periodic interruption of the current which excites them causes them to produce simultaneously a loud tap at equal intervals of time. The periodic " make and break " usually consists of a vibrating reed having a short wire attached to its free end, alternately entering and leaving a cup of mercury placed in the primary circuit of an induction coil, of which the secondary circuit

contains the tappers. The observer takes one of these tappers with him, and walks away from the other stationary one. Owing to the time taken by the sound to cross the intervening space, the sound from the stationary tapper lags behind that from the one with the observer. If he goes farther away the ticks will again be in step; but this time the tick produced nearby will coincide with the previous tick of the distant instrument. If n ticks are being made per second, and l is the distance of separation, this time is obviously $\frac{1}{n} = \frac{l}{c}$, whence c is determined.

If a reflecting wall is used only one ticker is necessary; the experimenter moves away from the wall until the tap and the echo from the preceding one are in phase. This reduces the distance required, but some intensity is lost in reflection. Another modification in the

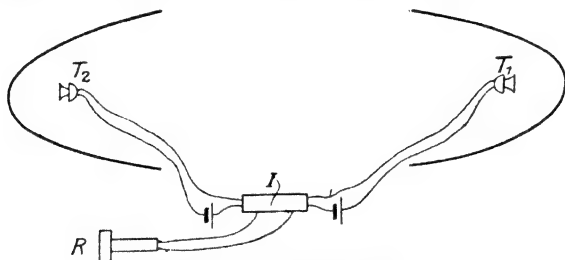



Fig. 3.—Hebb's Apparatus.

coincidence method consists in fixing the distance between the tappers, and varying the number of ticks per second, until coincidence is obtained. Obviously, the greater the distance, the greater the accuracy of the results, but then the intensity of the sound from the distant source is overpowered by that from the one close at hand. Probably, if the relative intensity of emission were made adjustable so as to produce equal intensity at the observer's ears, the technique of the method would be improved.

An objective method on " coincidence " lines is that of Hebb.[14] He placed a telephone transmitter T_1 in the focus of a paraboloid of plaster of Paris, with a corresponding receiver T_2 in another paraboloid (Fig. 3). These were set facing each other, so that plane waves were sent out from one mirror, and were brought by the other to a focus on the receiver. Both transmitter and receiver were connected in two primary circuits wound on the same secondary coil, I. The receiving apparatus could be moved away from the transmitter until the two

primary currents round the coil were "dead out of phase" and so produced no effect in the secondary; a telephone R connected across the latter would then remain silent. Knowing the frequency of the note maintained in the transmitter, the velocity of sound was calculated precisely as in Bosscha's experiment.

The latest series of observations reverts to the objective method of Regnault, and was made on the one hand by Angerer and Ladenburg [15] in the Bavarian Mountains, during the First World War. Instead of a style and a revolving drum, they used an Einthoven string galvanometer, whose fluctuations were recorded by the movement of a spot of light on sensitized paper moving with constant speed. As before, the explosion (in this case of powder alone) broke a wire in the primary of a circuit, the galvanometer being in the secondary circuit. The arrival of the sound-pulse at a distant microphone caused oscillations in another primary on the same coil. A helium tube controlled by a tuning-fork gave time intervals, and in this way they claimed to measure lapses of time as short as two-thousandths of a second. A number of stations 6 or 10 km. apart, over mountainous country round a rough triangle, were chosen, and their positions carefully surveyed. The anomalies observed will be mentioned in a later section, but the reduced result is 330·8 ± 0·1 m. per sec., comprising many observations.

During the course of researches on gunfire, Esclangon [16] obtained 330·9 m. per sec. (reduced to 0° C.) as the mean of a large number of experiments by a similar method over a fixed range of 14 km. in all kinds of weather. He found that, over this distance, the time of propagation was independent of the calibre of the gun, i.e., of the amplitude of the initial pulse.

Other methods of measuring the velocity of sound will be given later (pp. 204 and 252).

Atmospheric Influences--Temperature. It appears from the formula (3) that any cause that will alter the density of the medium, supposed gaseous, will alter c. The most frequent source of change in the velocity will be the temperature. According to the well-known law of Gay-Lussac, the density of a gas varies inversely as the absolute temperature. Equation (3) then shows that the velocity of sound will vary directly as the square root of the absolute temperature, or if one wishes to reduce the velocity c—observed at θ° Cent.—to the velocity c_0 at 0° Cent., one can use the approximate formula

$$c_0 = \frac{c_\theta}{\sqrt{1 + \alpha\theta}} = c_\theta(1 - \tfrac{1}{2}\alpha\theta) \quad . \quad . \quad . \quad (6)$$

where α is the coefficient of expansion on the Centigrade scale. Data to test this formula have rested mainly on small-scale researches by the methods described on p. 27 ; allowing for rather uncertain corrections, the agreement with theory is good. Open-air observations by Greeley [17] during a Polar expedition add confirmation down to $-45°$ C.

Influence of Pressure. According to Boyle's Law, the relation between pressure and density is constant, so that change of pressure, *per se*, can have no influence on the velocity. There is no reason to believe that this is not the case, though the secular range of pressure in the atmosphere is hardly sufficient to test the deduction.

Influence of Humidity, and Fog. Like the temperature effect, the effect of humidity is mainly an alteration of density, but admixture of the foreign vapour requires the use of a new value of γ for the ratio of the specific heats. Fog itself—as distinct from humidity—has no influence on the propagation of a pulse, but the accompanying temperature-changes in the air strata may cause anomalous propagation.

Experimental study of the effect of fog on surface propagation through the air was begun by Tyndall [18] in 1870. The sirens at the South Foreland were chosen for the experiment. He reached the conclusion, surprising at that time, that an overall fog accompanied a favourable reception of the signals at a distance. This is not due to the atmospheric moisture itself, but to the fact that fog is an indicator of stable and homogeneous atmospheric conditions. It is when the atmosphere is " patchy," i.e., having a diversity of density at different points, with unequal temperature and humidity distributions, that fluctuations occur in the audibility of the sirens, both in time and in space. Sometimes these patches in the form of acoustic clouds caused continuing echoes of the blast as long as 15 secs. after the siren had been turned off. Owing to the presence of such reflecting patches in a fog a siren giving a continuous note is sometimes heard better, and sometimes worse, than the crack of a signal gun giving a short pulse.

More recently Tucker [19] and Eagleson [20] have made measurements with sensitive detectors of the sound in the open air received at a distance from a continuous source, the former over the sea and the latter on land. Both found irregularities of the signal strength up to several hundred per cent. on days when the air was very turbulent or broken by fog banks and clouds. On the other hand low frequency tones were propagated in good agreement with theory under good homogeneous conditions (see also p. 321).

Influence of Wind. When a steady air stream is imposed on

the motion of the sound waves, its velocity is to be added to or subtracted from c, if it blows directly in the line of observation ; in this case the arithmetical mean of the times of travel of sound in the two opposite directions will give the true value of c with sufficient accuracy. If, however, there is a side wind, the state of affairs is quite complex, and not then merely the resolved sum ; while, with a wind at right angles to the direction of propagation of the sound, the observed velocity in both directions is the same, i.e. the wind has no effect. The actual conditions in the atmosphere involve, however, so many different effects, that it is generally impossible to pick out those due to the wind alone, nor is there any advantage in choosing a still day for velocity determinations. We must assume that the mean of a large number of observations under all possible atmospheric conditions will give the best results.

Fall of Intensity in Propagation. It is a matter of common observation, that under normal conditions, the intensity of the sound rapidly diminishes as one recedes from the source. This is to be explained (quite naturally) if one remembers that energy must be conserved. It can be shown that, following the principle of energy conservation, the intensity falls off as the square of the distance from the source. For if O be the source (Fig. 4) which has just sent out a sound impulse, then the wave-front of the emitted sound, travelling out as an expanding sphere, has now covered a sphere of radius r_1, of which $A_1B_1C_1$ is a section. A receiver, such as the ear placed at A_1 on the surface of the sphere, gains energy I_1 from the unit area of the surface, so that the total energy over the surface $= I_1 4\pi r_1^2$. An instant later the wave-front covers a larger sphere of radius r_2, and the receiver at A_2 collects energy I_2 from unit area of the new wave-front, on which the total energy crossing is $I_2 4\pi r_2^2$. Then

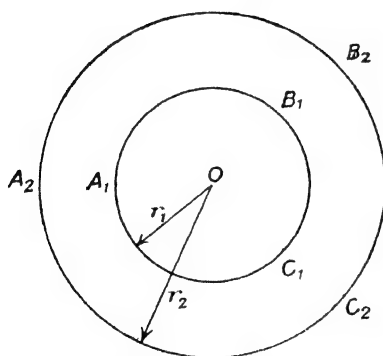



Fig. 4.—Inverse Square Law.

$$I_1 4\pi r_1^2 = I_2 4\pi r_2^2$$

and

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

There are three remarks which should be made with regard to the application of equation (7). Firstly, we have assumed a point-origin of sound. In practice, the source of the vibrations is of considerable extent, and the equation will not be true for distances of the same order as the linear dimensions of the vibrating body; at large distances the conditions of the equation will be satisfactorily realized. Secondly, the energy has been assumed to travel out from the origin in straight lines. Lastly, no account has been taken of the loss of energy dissipated in friction, and so converted into heat in the medium.

On account of these diverse influences in the actual atmosphere, the fall of intensity is much more rapid than that given by the inverse square law, especially of sound waves passing over the surface of the earth.[21] Other factors are discussed in Chapter XIII.

Reflection and Refraction of Sound. The phenomena of reflection and refraction of sound which occur when the sound reaches the boundary of a medium can be demonstrated as in the parallel case of light; except that, as sound is propagated (except in bodies of limited width) by longitudinal waves, no question of polarization can arise. The apparent differences between light and sound in reflection are merely questions of scale. The average wave-length of sound being 100,000 times the average wave-length of light-radiation, it requires rugosities of the surface 100,000 times the corresponding ones in light to produce diffuse reflection or scattering. A mirror or lens to produce concentration of sound must be enormous compared with the mirrors and lenses used in optical work. The same remark applies to gratings for producing diffraction. This difficulty of scale troubled Hertz in his work on the analogous electro-magnetic radiation which bears his name.

In place of the optical " screen," any of the sound detectors described later, microphones, sensitive flames, resonators, etc., may be used.

The law of reflection, that the angle which the incident ray makes with the normal to the surface is equal to that made by the reflected ray, can be proved by taking a directional source of sound, such as a watch at the end of a cardboard tube which is pointed at a large plane wall. A tubular resonator or a detector at the end of a tube must be placed so that the tube points to the same spot on the wall as the sending tube, and makes an equal angle with the normal (Fig. 5) in order that the detector may respond to the source. With curved surfaces the existence of foci can be demonstrated in a similar manner. In particular a small source of sound at the focus of a mirror of parabolic section made of iron or cardboard sends out a train of plane waves,

which can be brought to a focus at another similar paraboloid, and there detected by a microphone or a sensitive flame, as in Hebb's experiment (p. 7).

If the source gives a pulse instead of a continuous note, and if its

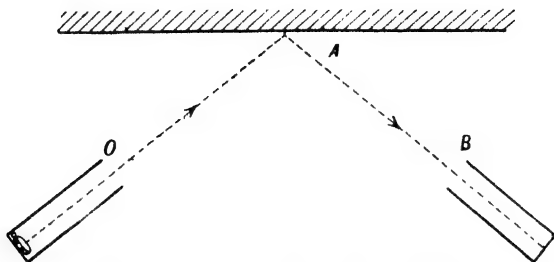



FIG. 5.—Reflection from Plane Surface.

distance from the wall is considerable, at a point such as B (Fig. 5) sound coming direct from O may be heard considerably in advance of the reflected sound travelling along a path such as OAB. When there is more than one-fifth of a second (corresponding to a distance of 70 m.) between the two times, an observer at B gets the impression of a distinct " echo."

Harmonic Echo. Reflection plays an important part in Building Acoustics, which, because of its technical importance, will be dealt with in Chapter XII There are several interesting cases, however, which are best dealt with now. The echo of a complex note returned by a reflecting surface may not necessarily be a faithful reproduction of the original. If the reflector has the property of reflecting certain wave-lengths better than others, then these components, if present in the source, will predominate over the others in the echo. Cases of this kind, in which the upper components of a complex note predominated in the echo, were noticed by Rayleigh,[22] who gave them the name of " harmonic echoes," as the pitch of the returned note was apparently raised ; the edge of a wood, for example, seemed to return a note as its octave, owing to the predominance of the octave in the reflected sound.

Musical Echo. The conversion of a single pulse into a succession of reflected pulses (or a musical note) is accomplished by wooden fences, the palings of which overlap in echelon formation. In consequence of this, the portion of the wave-front striking each paling and being there reflected, has to travel a little farther, and consequently arrives a little later at the listener's ear, than that portion which struck

the preceding paling in the row. A clap of the hand is thus reflected as a succession of pulses, at regular intervals depending on the spacing of the palings, and is perceived as a short musical note.

Whispering Gallery. Another interesting case of reflection is that which may be observed in the Whispering Gallery of St. Paul's Cathedral in London. There the slightest sound, made at one point near the wall beneath the dome, may be heard at the opposite point, while it is less audible at intervening points round the gallery. This phenomenon, according to Rayleigh,[23] is not due to reflection, but due to the waves hugging the walls of the dome as they travel outwards, and being gradually guided round to the conjugate point of the hemisphere.

Raman and Sutherland [24] have conducted experiments in St. Paul's and found Rayleigh's theory so far confirmed in that the effect was most marked when the source of sound was directed tangentially along the surface of the gallery wall, instead of directly at the opposite side. They found, however, certain fluctuations of intensity both radially and tangentially to the gallery, which the theory does not explain. Sabine[25] ascribes a great deal of the efficacy of this and other Whispering Galleries to the inward slope of the wall, which keeps a good deal of the sound down to the level of the gallery, which would otherwise find its way up to the roof of the building, and never reach the listener.

Refraction may be shown to follow the same laws as light, i.e., if i is the incident angle in a medium where the velocity of sound is c, and r is the refracted angle in the second medium where the velocity is c', then $\frac{\sin i}{\sin r} = \frac{c}{c'} =$ a constant. Sondhauss [26] constructed an acoustic bi-convex lens of a large balloon containing carbon dioxide. Lenses made of pitch, and rubber vessels containing water may also be used. Reflection and refraction may take place at any surface where there is a change of density or of velocity, *inter alia* at surfaces between layers of gas at different temperatures (cf. pp. 19–22).

Should the longitudinal waves be incident at a large angle—nearly "grazing" incidence—on the surface of a medium in which c' is greater than c of the first medium, it may suffer "total internal reflection." Under such circumstances none of the energy penetrates the second medium; all the sound is reflected by the surface. This will arise if $\sin r$ is greater than 1, i.e., if r is imaginary, the critical angle being given by $\sin r = \frac{c'}{c} \sin i = 1$. At an air-water surface, $c = 340$ and

$c' = 1440$, therefore the critical value of $i = 13{\cdot}5°$. This explains why the voices of bathers whose heads are naturally close to the surface of the water can be heard so plainly on the shore. The phenomenon is also of technical importance (cf. p. 330).

Diffraction of Sound. The elementary phenomena of light and sound find their readiest explanation in the assumption that light and sound travel in straight lines. There are some phenomena in light, and more particularly in sound, which will not bear this assumption. Every one knows that sound can be heard round corners. In order to explain such facts, we avail ourselves of the fruitful Principle of Huygens,[27] according to which every point P_1 P_2 P_3 (Fig. 6*a*) on a wave-front, as it vibrates, becomes the origin of secondary waves, which diverge in spheres; so that the wave-front

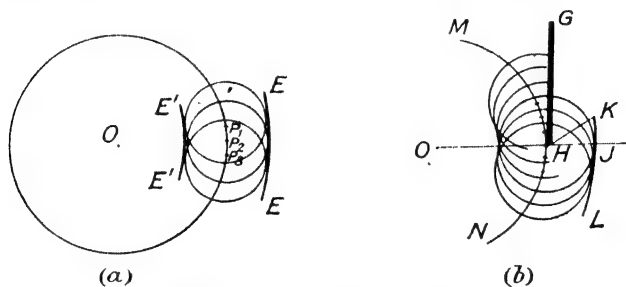



Fig. 6.—Huygens' Construction.

at a succeeding instant is the envelope E of these secondary waves. Why the disturbance is not, *ipso facto*, propagated back to the origin O by an enveloping wave such as E', required the Principle of Interference for its explanation. This was added by Fresnel [28] and Kirchhoff,[29] and from the calculation of the latter it appears that the effects of the secondary waves mutually destroy each other round the envelope E', so that the wave is propagated only in the direction away from the origin. When now an obstacle (GH, Fig. 6*b*) which will not allow the sound to pass, is placed across the path of the waves, secondaries arising from the wave-front HN teach us to expect, not only a consecutive half wave-front JL, but also a spreading of the waves into the geometrical shadow at JK; only, since half of the wave-front MH is now cut off from this side, the intensity of the sound received at a point in the shadow is less than it would have been, if the obstacle had been absent. Here again the question of wave-length comes in, and Rayleigh [30] has shown that, e.g., in the case of a spherical obstacle

whose circumference is twice the wave-length, the secondary waves conspire in the centre of the shadow, and make a concentration of sound along the axis through the origin and the centre of the obstacle.

Acoustic Shadow of a Sphere. The human head is a type of "spherical obstacle" in relation to the propagation of the sounds of the human voice to points behind the head. This case has been idealized for theoretical discussion by Rayleigh [30] in the form of a sphere having a source of sound very close to its surface. By mathematical analysis too complex for reproduction here, he, and later, Stewart [31] traced the "acoustic shadow" of such a sphere, by determining the relative intensities produced by diffraction at points behind the sphere. The theory has been checked by Stewart and Stiles [32] using for the sphere a large ball on the end of a narrow stem, which served as a conduit for the sound from a tuning fork in a box below, the sound finally debouching upon the atmosphere at a point on the surface of the sphere. Naturally in testing the effect of such an obstacle on the sound received in its shadow, reflections from neighbouring surfaces are to be excluded. Stewart placed his sphere so that it projected from the edge of a roof, covering the latter with felt to obviate reflection of the sound.

Acoustic Gratings. Other local concentrations of sound produced by diffraction are formed by acoustic gratings in the same way as in optics. Altberg [33] constructed a grating of glass rods, about 1 cm. apart, using the powerful spark discharge of a condenser as origin of sound. The sound waves were made plane by a wooden concave mirror and impinged on the grating; after being diffracted by this on to another concave mirror, they were finally detected at the focus of the latter. By moving the grating on an axis, one can trace out a sound spectrum just as with a spectrometer. Sparks have the property of originating very short waves a few millimetres only in length, and this reduces the size of the apparatus to workable proportions. Latterly continuous high frequency sources have been constructed for such demonstrations (cf. Chap. X).

Spark Photography. A method which has been of such use in studying the progression of sound waves as to deserve detailed treatment, was developed by the Toeplers (father [34] and son [35]) and called by them the Schlieren method. The sound is made by the discharge of a large condenser across a spark gap; this produces a very intense pulse of sound, sufficient to cause considerable changes of density in its passage through the air. If the field over which the wave travels

is illuminated instantaneously the position of the wave will be visible as a slight shadow on the background, owing to the somewhat different optical properties of the compressed air. In the modern method an instantaneous photograph of the wave is usually made. It may readily be understood, that if waves of considerable curvature are required, as they must be to demonstrate the reflection, etc., of sound waves, the time that elapses between the emission of the sound by the spark and the moment at which the wave or waves are photographed must be a very small fraction of a second. The photography is performed by the light from a discharge across an auxiliary spark gap.

The condenser C_1 (Fig. 7) is charged to a high potential by a large influence machine or induction coil, high enough to allow it to jump the gaps S and L, when the larger gaps G_1 and G_2 are short-circuited.

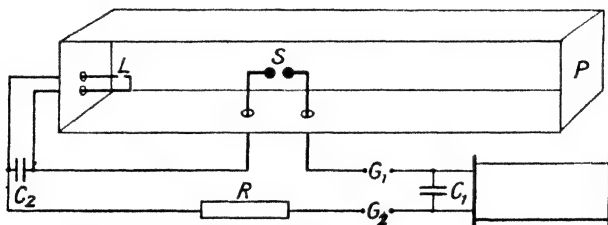



Fig. 7.—Photography of Sound Wave by Spark.

The two spark gaps, S the sound producer, and L the light producer are placed in parallel across the terminals of the condenser. As soon as G_1 and G_2 are closed, a discharge takes place almost simultaneously across the gaps L and S. Under such conditions no sound wave would be registered on the screen, as it would not have had time to leave the shadow of the knobs of the impulse gap. In order to make the illuminating spark occur a fraction of a second later, when the wave shall have attained a radius of several centimetres, a condenser C_2 is added to the branch of the circuit containing the illuminating spark. Then, if a discharge is made to pass across the gaps, it jumps the impulse gap a little before the other; the condenser causes the necessary lag in the passage of the illuminating spark, and the extent of the lag depends on the capacity of the interposed condenser. The light spark may take place along the surface of a piece of wet chalk, or between magnesium wires, in order to produce a brilliant light casting on the photographic plate P a shadow of the impulse which left S earlier.

In the arrangement used at the National Physical Laboratory, C_1 is a battery of 10 Leyden jars and C_2 of 5 jars ; a high liquid resistance R of about 10,000 ohms steadies the action. The sparks are set off by lowering glass plates into the "trigger" gaps G_1 and G_2 ; L consists of two pieces of No. 24 S.W.G. magnesium wire, with tips 3 cm. apart, enclosed in a capillary tube pointing in the direction SP ; S has platinum points 1·5 cm. apart. The total length LP is about 7 ft., and P is a "whole plate." [36]

It may be remarked, in parenthesis, that there is nothing essentially different between the two sparks, although S is intended to cause compression, and L is primarily light-producing. Both discharges are both sound and light-producing, but the light from the first, S, casts a shadow of the knobs alone on the plate, whereas L casts an additional shadow of the sound wave which left S earlier ; again, the sound wave which L starts is never subsequently registered by light on the plate. Thus, as far as the photograph is concerned, S produces a sound wave and L makes it visible.

When it is desired to study reflection problems, a piece of bent wood or card is placed below S and parallel to the plane of the screen. For refraction, a lens or small tank is similarly placed. By enclosing S with a model section of a building, the acoustical properties of it can be studied [37] (see Chap. XII).

The radius of the unreflected wave in these photographs represents the distance travelled by the sound in the time which has elapsed between the impulse and the illuminating spark ; if this time is known we have a means of determining the velocity of sound. To this end Foley [38] found the value of this short interval of time, by making the light from each spark cast a shadow of a revolving cog-wheel on another sensitized plate. The speed of revolution being known, the required time could be calculated from the angular separation of the two shadows.

Ripple Photography. Most of the problems that can be studied by the Schlieren method can be examined by following the motion of small ripples on shallow water or mercury. This method was first used by Vincent [39] for demonstrating interference of waves (see Chap. II), but, by putting obstacles in the tank in which ripples are formed to represent mirrors or diffracting edges, the method may be extended to problems in the propagation of sound ; it has been shown mathematically by Lamb [40] that, although the motion of such "capillary waves" is in two dimensions only, it is analogous to the three-

dimensional motion of actual sound waves, if we picture a plane section of the latter; and such a section is, of course, what we get in our spark-pulse photographs. To start a pulse it is sufficient to direct a puff of air on to, or to dip a rod smartly into the water; the pulse is, however, always followed by a number of lesser waves, more so in the water than in the mercury. To imitate a musical note, a prong is placed on the end of a vibrating reed or tuning-fork, so that the prong dips into the water periodically. In order to produce an acoustically denser medium, part of the tank is made more shallow than the rest; over this portion the waves travel with slower speed, and so on reaching it, are bent towards the normal of the line of separation. When the pulse is formed the motion is slow enough to be followed by the eye; and when a series of waves is being sent out, it is usually arranged that the vibrator periodically lights a helium discharge tube, which illuminates the tank from below (water) or by reflected light (mercury), and so renders the motion stationary to the eye. The ripple tank is less irksome to set up than the spark-pulse apparatus; on the other hand its indications are not so well defined.

Anomalous Propagation of Explosive Sounds. The study of the velocity of sounds from explosions has been facilitated in recent years by the need of disposing of the large dumps of explosives left over from recent wars. The earlier experiments of Mach and his pupils [41] were confined to the propagation of the explosion itself in vessels and tubes of explosive gas. As this is rather a problem in combustion for the chemist, it will not be considered further in this treatise. It is sufficient to point out that these experimenters recognized that the abnormal velocity of the waves of compression so generated was due to the intensity of the explosion, which brought it outside the pale of the small amplitudes, to which the theory leading to the velocity-equation applies. The anomalies to be noticed in the propagation of sounds from large explosions capable of penetrating many miles from the source, are: (1) abnormally large velocities in the neighbourhood of the explosion, with considerable mechanical movement of the air; (2) a second zone of normal velocity; (3) a third zone of complete silence; (4) a fourth zone where the sound reappears with unusual intensity but taking an exceptionally long time to arrive.

Of these, the occurrence of a region where the sound is inaudible, in spite of the fact that it is distinctly audible at a greater distance from the source, is the most striking. This was noticed early in the present century, and two rival theories were put forward to account

for it. Both agree in stating that the sound heard in the first and second zones travels directly over the surface of the earth. Apart from the diminution of intensity accounted for by equation (6), there is a much more powerful source of damping in the friction which the waves experience in passing over the broken surface of the earth, the tops of forests, etc., so that after an average course of 60 miles, the sound becomes inaudible to a listener. They also accede that the sound heard with renewed intensity beyond the silent zone has reached its objective by travelling up into the higher parts of the atmosphere, and being gradually bent down again to reach the earth's surface by a devious route; which accounts for the abnormal lapse of time between the explosion and the arrival of the sound in the fourth zone. They differ, however, in assigning the cause of this curved trajectory. We have noted that a bending of the direction of the sound will occur when the waves pass into a medium of different density, and wherein, therefore, their velocity is different. Such a change may occur in the atmosphere at regions where the temperature is different, or where the composition of the gas is different, and may be a sudden or a gradual change. At a sudden discontinuity in the medium reflection may be produced. Bending of the wave-front can also be produced by a gradient of wind speed, resulting in parts of the wave-front gaining on the rest. All three of these possibilities have been invoked to explain the curved trajectory of waves leaving the origin at an inclination to the earth's surface. The meteorological theory postulates the existence of regions of decreasing temperature as one ascends from the earth, whereby these sounds pursue a path which is concave upwards. At a height of about 10 miles, however, there is believed to be a temperature-inversion layer, wherein warmer regions are rapidly entered. This layer causes, either reflection of the waves, or refraction in the opposite sense (the path convex upwards) or indeed both, until the sound ultimately reaches the ground again, its renewed intensity being accounted for by the concurrence of waves having traversed slightly different paths. The physical theory put forward by von den Borne,[42] allows the upward bending in the lower regions, but ascribes the bending back to the fall of velocity in the rarefied upper regions of the atmosphere (which Borne describes as mainly helium and hydrogen), occasioned by the low density there. This theory requires that the sound should penetrate into more rarefied regions, where the sound would have difficulty in progressing. One is reminded of the well-known experiment in which an electric bell is

placed under a bell-jar, which is then exhausted ; with a good vacuum the sound fails to penetrate the few inches between the bell and the open air.

It is then a matter of putting forward hypotheses for the velocity of sound above 17 km. and testing these against the known time of propagation of the sound through this region into the abnormal zone of the earth, and such hypotheses must make the summit-velocity high, otherwise these high-penetrating rays would not get back to earth within the prescribed surface distance (180 to 300 km.). The simplest hypothesis seems to be to suppose that from 17 km. the velocity is equalled and exceeded. This fits the observed time of passage of sound to the abnormal zone, giving the rays which penetrate to the outer edge of the abnormal zone an original inclination

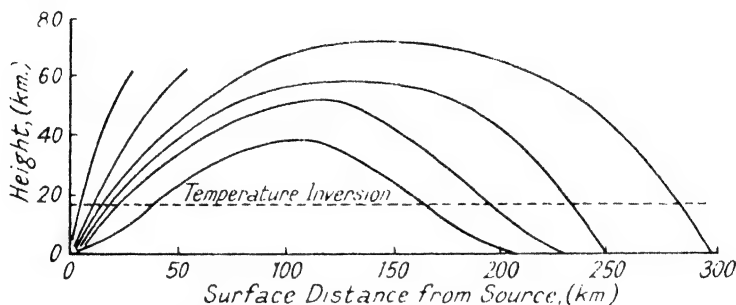



FIG. 8.—Sound Rays from Explosive Source.

of about 40° at the source, and a " summit level " of 70 km. (The assumed paths of such sound waves are shown in Fig. 8.)

Measurements of the propagation of sound from these explosions enable us to get an approximate estimate of the velocity of sound in the stratosphere, and from this we can obtain valuable information on the physics of this upper atmosphere, a subject on which little information is available. Pressure *per se* can have no effect on the velocity, but any other factor which alters the density will *ipso facto* alter the velocity of sound. The increase in the velocity in passing up through the stratosphere is therefore ascribed to a reduction in density. Granted that the density of the air progressively decreases in the stratosphere, there are two main factors which may produce this effect ; either (1) the constitution of the stratosphere becomes richer in the lighter gases—this would also affect γ—or (2) the temperature progressively increases upwards.

The first hypothesis is the older and was put forward by von den Borne. This theory considers that the temperature does not increase in the stratosphere, but that the necessary decrease in density is to be credited to a decrease in molecular weight due to the presence of lighter gases—e.g. helium and hydrogen—in these regions. Such an equilibrium would not arise if diffusion could freely take place, and it is difficult to account for the congregation of these gases in the upper region, except by postulating a vertical streaming which would upset diffusion equilibrium. It has also been pointed out that the hydrogen molecule would be more or less dissociated at such heights, and must have been dispersed in space long ago. A second objection lies in the fact that, so far, examination of the spectrum of the *aurora borealis* (a light phenomenon having its origin at great heights in the atmosphere) has failed to show any lines which can be ascribed to either of the elements named.

We are therefore led to adopt the temperature theory, which leads to the surprising result that, from the temperature inversion at 17 km., the temperature rises with height, reaches the surface temperature again at 35 km., and at 60 km. has passed 70° C! This conclusion is completely against the older ideas on the atmosphere but is supported by certain observations on meteors. Attempts which are made from time to time to penetrate the stratosphere with sealed balloons carrying observers as well as instruments may bring evidence to support the acoustical data.

Data from Explosions. The first statistical records of the audibility of an explosion were collected by Davison [43] after the accidental blowing-up of a munition factory at Silvertown, London, in 1917. In order that more accurate data might be obtained by persons prepared for the explosion, a larger quantity of explosives has been blown up on previously announced occasions, at Oldebroek in Holland, 1923, at La Courtine, France, in 1924, and at Kummendorf, Germany, in 1925. In the first there was a definite silent zone extending from 60 to 100 miles from the source. There was no trustworthy return from this region recording that the sound had been heard. It is curious that in the inaudible zone of the Silvertown explosion, windows were shaken, and pheasants, which seem to be sensitive to low frequency sounds, became restive; all of which indicated a pulse of very low pitch, not audible to the human ear (cf. p. 275). Van Everdingen,[44] who has studied results of several explosions, favours the meteorological theory, as Borne's physical theory would require the zones

to be symmetrically distributed with respect to the origin; whereas the confines of the zones are irregular, and there are in addition isolated patches of audibility in the silent zone, and *vice versa*. These would be accounted for by irregular conditions of wind and temperature in parts of the atmosphere. On the other hand, the outer limit of the silent zone, where the sound reappears from the upper path, seems to be quite a definite line, unchanged by considerable irregularities in wind and surface temperature.

The results of the La Courtine explosion in France are shown on Fig. 9 (after Maurain [45]). In the dotted regions, the sound was regis-

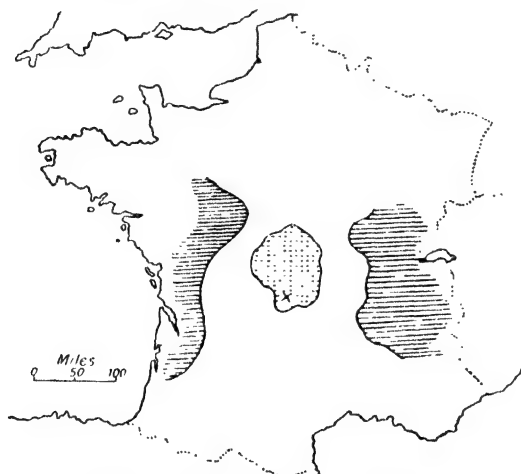



Fig. 9.—Explosion at La Courtine.

tered at normal velocity; in the hachured regions its tardy arrival indicated a long trajectory. The large areas of silence are obvious. Esclangon [46] calculates that they correspond roughly to reflection at about 17 km. in the inversion layer, according to the meteorological theory. There seems to be some evidence also for reflection by cloud masses, but wind velocity-gradients near the ground seem to play quite a secondary part. That the surface winds have little influence on the velocity higher up is proved by observations at Paris in 1914–18 on the audibility of gunfire from the Western Front. It was noticed on some days that the gunfire was better heard when the wind blew in a contrary direction to the sound, than when the wind was blowing from the front to Paris.

We have not yet discussed the region in the immediate neighbour-

hood of a large explosion. Angerer and Ladenburg noticed that the velocity of the sound as measured by their instruments was abnormally high at first, but reached its normal value after about 200 m. This is evidently due to the abnormal intensity in this region. Riemann [47] has proposed a formula for such a case—

$$c' = \frac{dr}{dt} = c\sqrt{1 + \frac{b^2}{r^2}}$$

giving the change of velocity with distance r from the source; b is a quantity dependent on the width of the layer of air compressed at the instant of explosion. After some distance has been traversed the formula evidently gives the normal velocity c. A pulse from such an explosion is very strongly damped, meaning that it is followed perhaps by only two or three waves of very low frequency. According to Villard,[48] the explosion at La Courtine produced in Paris (230 miles away), a train of waves of 1 sec. period, lasting for 3 secs. only. The most striking effect was an actual forward motion of the air, and a compression causing serious physiological derangements to those persons close to and unprotected from the explosion. On another occasion the same writer noticed an air movement of about 1 cm., at a point 1 mile away from an explosion. The mere noise is quite insignificant in comparison with these very slow pulsations, whose period apparently grows with the quantity of material exploded.

Sounds from Bodies Travelling Faster than Sound. Some very interesting facts come to light when one examines the propagation of sound from moving bodies whose speed exceeds the normal value of c. In earlier times quite normal values had been obtained from gunfire, but latterly it has been observed that the sound, produced when bullets were shot from powerful guns, travelled faster than the normal. It was at first thought that this was an effect of abnormal intensity, but attempts to collate the intensity and the velocity were unproductive. Researches of Journée [49] led him to think that as long as the bullet travelled faster than sound, the sound travelled with it; as the bullet lost speed the sound forged ahead with the normal velocity. It will readily appear that, knowing the muzzle velocity of the projectile and its deceleration, the time for the sound to travel in the direction of the line of fire is readily calculable. At points to one side, however, the calculation becomes very complicated. There is also, in practice, the complication introduced by the curved path of the projectile. Actually a double sound may be heard at a point

to one side, one direct from the muzzle, and the other which has travelled part of the way with the projectile ; but, more often, a continuous roll of sound is heard, showing that the bullet during its course is a continuous centre of emitted sound waves, which reach the hearer by paths of continuously increasing or decreasing length. Mach, on the other hand, ascribed the " roll " to reflection from the earth, buildings, etc.—the cause of the rolling of thunder, too—as at such high speeds the bullet is incapable of producing a continuous series of compressions. He and Salcher [50] obtained photographs of these waves by spark photography. The missile is made to break a wire in its flight, which sets off a spark and projects a shadow of the missile and its accompanying waves on to a sensitized plate. When the missile is travelling faster than sound, the waves sent out in its

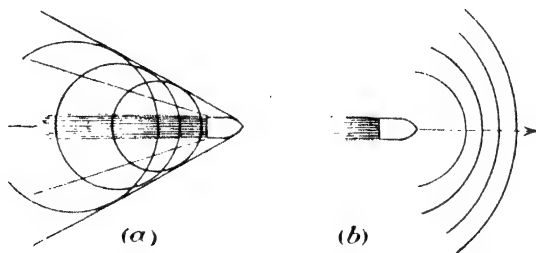



Fig. 10.—Sound Waves from Projectile.

path are enveloped by a conical wave having the bullet at its vertex (Fig. 10*a*).

When the bullet's velocity is below that of sound, the sound waves gain upon it, as in Fig. 10*b*. There is always a surface of discontinuity spreading out in cone fashion from the rear of the projectile. The conical waves appear as two pairs of straight lines on the photographic plate (10*a*) because along the surface of this cone we have a number of wave-fronts originating from different points in the missile's path, combining to produce a sufficient compression of the air to be manifest on the plate. In the absence of reflecting surfaces a listener in the neighbourhood of the trajectory of a missile travelling faster than sound, hears first this head-wave, or *onde de choc* and after an interval the muzzle-wave, which has travelled from the gun at the speed of sound throughout, accompanied by a continuous whistling due to the turbulence behind the bullet, often apparent in the photographs. We shall revert to these two sounds in connection with sound ranging.

Velocity of Sound in Tubes. A number of direct investigations were made by Regnault [51] on the velocity of sound in air in pipes up to 3 miles in length. These pipes were intended for a water system in Paris. The method employed was to fire a pistol at one end, and to receive the sound on a membrane at the other end, in conjunction with the electro-magnetic apparatus described above (p. 6). By allowing the sound to be reflected at each end, so that it passed a number of times up and down the tube, the precision of the method was improved. The now well-established fact that the velocity decreases with the width of the tube, emerged from the results. When the tube was 1 foot wide the same value was found as in the open air. The method has been repeated without considerable modification, but has been superseded by the methods described below, which are workable on quite a small scale with short lengths of tube. The tubes we are concerned with here are all at least an inch wide. Very narrow tubes, where friction is paramount, will be considered in Chap. VII.

Kundt's Dust Figures. The method of all small-scale methods which has been most fertile in development was devised by Kundt in 1866,[52] and has provided, in addition to measurements of the velocity of sound in gases under all conditions, valuable information on the molecular aggregation of their constituents. The air is contained in a glass tube closed at one end B by an adjustable stopper (Fig. 11*a*). The tube is about 150 cm. long and 3 cm. wide. Projecting into the other end is a metal or glass rod, terminating in a cork or wooden stopper at A, which just clears the tube inside. The rod is usually clamped at the centre, and is stroked with a resined cloth (if of wood) or a damp cloth (if of glass or metal) in order to produce longitudinal vibrations in the rod. This tone will have a definite frequency n and therefore a definite wave-length λ in the rod, determined by the fundamental relation $V = n\lambda$, where V is the velocity of sound in the rod. In addition, if the column of air AB is of the correct length, it will be able to vibrate with the same frequency n, and with a wave-length λ_0, where $c = n\lambda_0$.

One possible mode of vibration of the air column is that corresponding to the half-length of the vibrating bar, that is with the stopped end B fixed, and with maximum amplitude of vibration at the open end D. When the adjustment is made so that both metal and air-column are of such length (dependent on λ and λ_0) that they can both vibrate in this fashion to the same frequency, on exciting the tone in the rod the air column will also vibrate in sympathy with

the rod. This is, in fact, an example of the universal principle of Resonance. As the velocities of these longitudinal sound waves will not be the same in the metal as in the air, the wave-lengths and hence the lengths of the vibrating portions are not the same. It will be seen in the next chapter that these lengths are directly proportional to the respective wave-lengths. From the ratio of these wave-lengths the ratio $\frac{V}{c}$ can be found, since $\frac{V}{c} = \frac{n\lambda}{n\lambda_0} = \frac{\lambda}{\lambda_0}$; and the value of c found if that of V is known.

Now equation (3) applies to all longitudinal vibrations, in solid, liquid, or gas, so that if we know the value of the appropriate elasticity for the rod (and this is readily found by experiment) V is simply determined. We have only then to adjust the stopper B, and measure a few distances in air and metal to be able to calculate the velocity

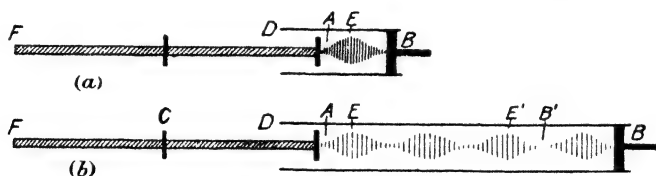



Fig. 11.—Kundt's Tube.

in the air in the tube. It is in the detection of the resounding air-column and its measurement that the speciality of Kundt's method lies; another method will be described in the next section. Instead of having the air in the tube vibrating in the manner described above, Kundt extended the tube until, in order to respond to the tone in the rod, the air-column was obliged to break up into segments, having at E' maximum vibration, at B' (Fig. 11*b*) no vibration (corresponding to the distance EB under former conditions), and then beyond this point a series of segments extending with alternate positions of maximum and no vibration down to the end of the tube B, which, by reason of the unyielding stopper, remains a point where longitudinal motion of the air is impossible.

In order to make these points visible, lycopodium or cork or pith dust is strewn lightly along the tube. At points like B', it remains undisturbed even when the air is responding to the rod, but at points such as E, and to a less extent at points in between, it is thrown into violent motion, ultimately coming to rest in peculiar bands across the tube—the significance of these bands will appear later (p. 214). The

lengths AC in the rod and EB in the air are vibrating under corresponding conditions, and their ratio represents the ratio of the wave-lengths of the respective media. As Regnault's work showed, the velocity by this method will in general be less than that in free air. In order to reduce these velocities to "absolute" values, Helmholtz [53] employed an empirical formula, which fitted Kundt's and Regnault's results. It is

$$c' = c\left(1 - \frac{k}{r}\right),$$

where r = radius of tube, and k is a constant as long as the gas, tone, and material of the tube are unchanged. If we obtain a second value from another tube of different radius, $c'' = c\left(1 - \frac{k}{r_1}\right)$ we can eliminate k, for

$$r_1\left(1 - \frac{c''}{c}\right) = k = r\left(1 - \frac{c'}{c}\right) \quad \therefore c = \frac{r_1c'' - rc'}{r_1 - r}$$

giving a value for the velocity in the open air. Another way of detecting when the tube responds to the exciter is to have a narrow side-tube leading off near A; the stopper is then adjusted until the sound transmitted down this tube to the ear by the vibrating air is a maximum. Quincke [54] and others used this method, with either a telephone transmitter or tuning-fork as exciter in place of the rod. A tuning fork was used by Quincke, who applied the listening-tube as a search-tube to find the actual positions of the maximum and minimum of vibration of the air in the tube.

Velocity in Other Gases.* By introducing other gases into the tube it is possible to obtain values of the velocity in these gases. An objection to doing this is that the free passage which must be left near A for the rod to vibrate in, will permit the outside air to diffuse into the tube. Another arrangement due to Kundt [55] is to place a second tube at F containing the gas, and some dust. When the note in the rod is excited, both tubes are adjusted to resonance, and the ratio of the wave-lengths in the air and in the gas, measured from the dust figures, gives the ratio of the speeds of sound. The latter ratio gives important information on the molecular aggregation in the gas, as the ratio of the specific heat of the gas at constant pressure to that at constant volume is a definite function of the number of atoms which go to make up the molecule.

* *See also* Ch. X.

In order to enable a chemically purified gas to be kept from contamination by the outer air, Behn and Geiger [56] introduced the ingenious improvement of enclosing the gas itself in the rod which is rubbed to produce the note. To this end the rod is made hollow and of glass, and contains dust. The gas is introduced at the mid-point which is clamped. The column of *gas* in the rod is of invariable length, and may therefore not resound to the note in its *glass* container, but the effective length of the rod is increased by screwing on metal washers to a threaded metal extension-piece at each end of the rod, until the dust inside shows that resonance has been reached. The adjustment is thus the reverse of what it was in the original apparatus, since here the rod is adjusted to resonance with the gas. It is usual to retain the air-tube, and, after the first adjustment, to adjust this

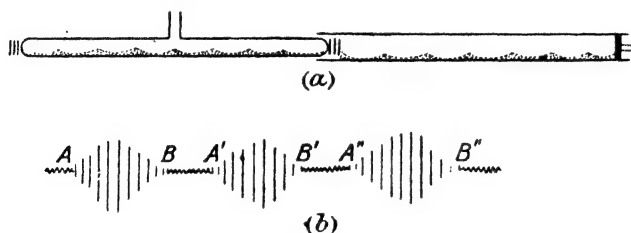



FIG. 12.—Modified Kundt's Tube (*Behn and Geiger*).

for resonance. Wave-lengths and velocities in the two gases can then be compared. The complete arrangement and method of measuring the figures is shown in Fig. 12. Corresponding points in the pattern near where the disturbance of the dust is first apparent, like A, A', A'', and B, B', B'', are chosen and distances measured across them. The Behn and Geiger apparatus has been much used by other chemists and physicists, notably by Partington.[57]

The gas tube should be about 2·5 cm. in diameter, and from 40 to 80 cm. long. The discs were cut from lead, 1 mm. thick and of diameter a trifle less than 2·5 cm., and stuck on by wax. The air tube should have a slightly larger internal diameter. In place of the discs, caps of metal foil hammered to fit the rounded ends of the gas tube may be used.[57] The gas and dust must be scrupulously dried before being admitted into the tube.

Velocity in Gases at Different Temperatures. Kundt's method provides the possibility of measuring the velocity of sound in gases at different temperatures, if the tube be surrounded by a

thermostat. At low temperatures it would be inconvenient to surround a long tube by liquid air, but Himstedt and Wedder [58] have been able to measure velocities at low temperatures, by using a very high-pitched source of sound (a Galton whistle, see p. 276) in place of the glass rod, requiring only a short length of the gas-tube accommodated in a Dewar vacuum vessel. In this case resonance in the column of gas was detected by a microphone at the bottom of the gas-tube, which was of invariable length. The frequency of the note given by the whistle, when the gas was thrown into maximum vibration, was determined by a subsidiary Kundt's tube containing air.

Velocity of Sound in Liquids. Obviously, water is the only liquid with which large-scale methods are practicable. Colladon and Sturm [59] measured the velocity of sound in the Lake of Geneva in 1827, a bell being struck under water at one side of the lake, and the sound received through the water at the other. A flash of powder made by the hammer which struck the bell took the place of the muzzle flash in the overland gunfire experiments. This classic work gave 1,435 metres per second at 8° C., against 1,441, calculated from (2), where the inverse of the compressibility of water was substituted for E.

The great strides which the science of submarine signalling has made in recent years have drawn the attention of physicists to the velocity of sound in the sea. A series of experiments in the roadstead of Cherbourg at a depth of 13 m. was made by Marti,[60] during the First World War. Small submarine charges were exploded, and the propagation of the pulse registered by a number of submarine microphones 900 m. apart. The explosion and the response of each microphone were registered on the same chronograph. The average result was high in comparison with those of other investigators (1,503 m. per sec. at 14·5° C., density 1·0245).

Stephenson [61] made careful measurements on the American coast, in which a half-kilogram bomb of T.N.T. was detonated at a depth of 10 m., and simultaneously a radio signal, instead of a light signal, was sent out. The sound signals were received on five microphones at an average distance of 10 miles from the source. The response of these microphones and the receipt of the radio signal were registered by a six-string Einthoven galvanometer recording on a moving sensitized film. The average result was 1453·5 m. per sec., at − 0·3 deg. C.; salinity 3·35 per cent.; estimated error less than 0·1 per cent.

The estimated velocity in the neighbourhood of the source rises with the quantity of explosive. This perhaps explains Marti's high

value. The changes produced by the possible variations in salinity S and temperature θ are reckoned to range over one or two metres per second only.[62] The following formula contains the results of the experiments of Wood and Browne[63] off St. Margaret's, Kent:

$$V = 4{,}756 + 13{\cdot}8\theta - 0{\cdot}12\theta^2 + 3{\cdot}73S$$

(where V is in ft./sec.; θ in degs. C.; S in parts per thousand).

Kundt's tube has been applied to the determination of the velocity in water, but it is extremely difficult to get wet dust to form the figures. Using oils, one achieves a greater measure of success, but the movements of the dust are still strongly damped. Using a resined wheel continually rubbing against a glass rod to maintain the note, Dorsing[64] was able to get values of the velocity in alcohol and in ether from a Kundt's tube, containing carefully dried powdered pumice-stone. He found that the vibrating liquid tended to force the tube containing it into sympathetic vibration, so that it is advantageous to get rod, liquid column and tube all vibrating to the same note.

In recent years most data on the velocity of sound in fluids have been obtained by using ultrasonic sources (see Chapter X).

References

1. *Harmonicarum*, 1636.
2. *Principia*, 1687.
3. *Principal Phenomena of Sounds*, 1784.
4. *Theorie des Klanges*, 1787.
5. *Wellenlehre*, 1825.
6. *Ann. Chemie*, 1820, etc.
7. *Ann. Physik*, **57**, 234, 1817.
8. *Harmonicarum*, 12, 1636.
9. *Opera omnia*, **3**, 1658.
10. *Mém. de l'Académie*, 1738.
11. *Phil. Mag.*, **43**, 153, 1872.
12. *Comptes Rendus*, **66**, 209, 1868.
13. *Ann. Physik*, **92**, 485, 1854.
14. *Phys. Rev.*, **20**, 89, 1905, and **14**, 74, 1919.
15. *Ann. Physik*, **66**, 293, 1921.
16. *Comptes Rendus*, **168**, 165, 1919.
17. *Phil. Mag.*, **30**, 507, 1890.
18. *Sound*, Lecture 7, 1870.
19. *J. Roy. Soc. Arts.*, **71**, 132, 1923.
20. *J. Acoust. Soc.*, **12**, 427, 1941.
21. Rayleigh, *Advis. Comm. Aero.*, 1916. *See also* Rocard, *J. Physique*, **1**, 426, 1930, and **4**, 118, 1933.
22. *Nature*, **8**, 319, 1873.
23. *Phil. Mag.*, **20**, 1001, 1910, and **27**, 100, 1914.
24. *Proc. Roy. Soc., A*, **100**, 424, 1922.
25. *Coll. Papers*, p. 255. *See also* Raman, *Proc. Ind. Assoc.*, **7**, 159, 1922.

26. *Ann. Physik*, **85**, 378, 1852.
27. *Traité de la Lumière*, 1690.
28. *Mém. de l'Académie*, **5**, 339, 1826.
29. *Ann. Physik*, **18**, 663, 1883.
30. *Sound*, **2**, 254, 1897.
31. *Phys. Rev.*, **33**, 467, 1911, and **7**, 442, 1916.
32. *Ibid.*, **1**, 309, 1913.
33. *Ann. Physik*, **23**, 267, 1907. *See also* MEYER, *J. Acoust. Soc.*, **7**, 88, 1935.
34. *Ann. Physik*, **131**, 180, 1867.
35. *Ibid.*, **27**, 1043, 1908.
36. DAVIS and FLEMING, *J. Sci. Instrum.*, **3**, 393, 1926.
37. SABINE, *Ann. Arch.*, **104**, 257, 1913.
38. *Phys. Rev.*, **16**, 449, 1920.
39. *Phil. Mag.*, **43**, 17, 1897.
40. *Hydrodynamics*, §§ 189 and 172.
41. *Akad. Wiss. Wien, Ber.*, **92**, 225, 1885.
42. *Phys. Zeits.*, **11**, 483, 1910.
43. *Quart. Rev.*, **452**, 51, 1917.
44. *Proc. K. Akad. Amsterdam.*, **18**, 933, 1916.
45. *Comptes Rendus*, **179**, 284, 1924.
46. *L'acoustique des canons*, 369, 1925.
47. *Gött. Nach.*, 19, 1859.
48. *Comptes Rendus*, **179**, 617, 1924.
49. *Ibid.*, **106**, 244, 1888.
50. *Akad. Wiss. Wien. Ber.*, **95**, 764, 1887.
51. *Comptes Rendus*, **66**, 209, 1868.
52. *Ann. Physik*, **127**, 497, 1866.
53. *Wiss. Abhand.*, **1**, 338, 1882.
54. *Ann. Physik*, **128**, 190, 1866.
55. *Ibid.*, **135**, 337 and 527, 1868.
56. *Ber. deutsch. phys. Ges.*, **5**, 657, 1907.
57. *Phys. Zeits.*, **15**, 601, 1914; PARTINGTON and SHILLING, *Trans. Faraday Soc.*, **18**, 386, 1923, and *Phil. Mag.*, **45**, 416, 1923; PARTINGTON and CANT, *ibid.*, **43**, 369, 1922; SHILLING, *ibid.*, **3**, 177, 1927.
58. *Zeits. Physik*, **4**, 355, 1921.
59. *Ann. Chim. Phys.*, **36**, 113 and 225, 1827.
60. *Comptes Rendus*, **169**, 281, 1919.
61. *Phys. Rev.*, **21**, 181, 1923. *See also* ECKHARDT, *ibid.*, **24**, 452, 1924; SERVICE, *J. Frank. Inst.*, **206**, 779, 1928; POOLER, *Phys. Rev.*, **35**, 832, 1930; DORSAY, *J. Acoust. Soc.*, **3**, 428, 1932.
62. LICHTE, *Phys. Zeits.* **20**, 385, 1919.
63. *Proc. Phys. Soc.*, **35**, 183, 1923.
64. *Ann. Physik*, **25**, 227, 1908.

CHAPTER TWO

VIBRATING SYSTEMS

Simple Harmonic Motion in a Straight Line. In most natural phenomena which we meet, the motion must be treated as two or three dimensional. For simplicity of mathematical treatment we shall consider first the periodic motion of a particle moving along a straight line between the points A and A' (Fig. 13a). One dimension will then suffice to describe the position of the particle at any instant. The motion is periodic if, after an interval T, the motion repeats itself; T is then called the "period" of the motion. Moreover, the

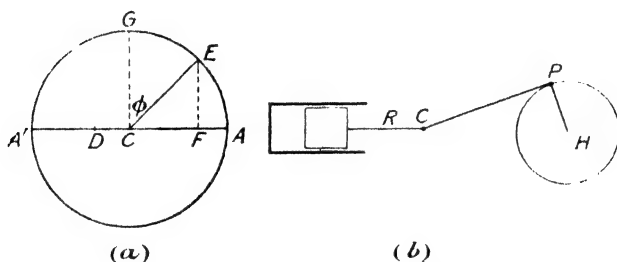



FIG. 13.—Simple Harmonic Motion.

movements of the particle are identical from period to period, so that if, after a certain fraction of the period, or at a certain "phase," the particle has moved from A to D, and we wait for its return to A, we shall find in the next period and after the same fraction of the period has elapsed, the particle will be at D again. In sound we deal almost entirely with vibrations symmetrical with regard to the mid-point of the motion; the particle goes through evolutions on the right-hand side of C similar to those on the left.

Mathematically the simplest type of vibration that we can imagine happens to be that obtained by projecting circular motion on to a straight line. We imagine a circle drawn on AA' as diameter, and a point to move round it with angular velocity ω, and suppose the particle to lie always on the projection of the point E on AA', i.e., at F (Fig. 13a). At the instant pictured let the radius vector CE make an angle ϕ with the normal, and put $CF = y$, the particle's displace-

ment from its central position. Then, if t is the time that has elapsed since E was at G and the particle was at C, $\phi = \omega t$.

Also
$$y = a \sin \phi = a \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad (8)$$
where a = the radius of the circle = the "amplitude" CA of the vibration. Also the "period" T of the motion $= \frac{2\pi}{\omega}$, and the "frequency" or number of revolutions n per second $= \frac{\omega}{2\pi}$.

Equation (8) can also be written :—

$$y = a \sin \frac{2\pi t}{T} = a \sin 2\pi n t.$$

The velocity of the particle at any instant :—

$$\frac{dy}{dt} = 2\pi n a \cos 2\pi n t = \omega a \cos \omega t \quad . \quad . \quad . \quad . \quad (9)$$

When $\phi = 0$, the particle is at C, $\cos \omega t = 1$, and the velocity is

$$\omega a = 2\pi n a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This being the maximum velocity in the motion of the particle, the velocity decreases on each side of C until, when the particle is at A or A', the displacement is a maximum a, and the velocity is 0, because $\phi = \pm 90°$, $\cos \omega t = 0$.

Finally the acceleration $= \frac{d^2y}{dt^2}$

$$\begin{aligned} &= -4\pi^2 n^2 a \sin 2\pi n t \\ &= -\omega^2 a \sin \omega t \\ &= -\omega^2 y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11) \end{aligned}$$

Since ω^2 is a constant, we have demonstrated the very important property of this so-called Simple Harmonic Motion (S.H.M.), that the acceleration is always proportional to the displacement. The minus sign indicates that the acceleration opposes the displacement, the particle being accelerated when its displacement (from C) is decreasing, and decelerated when this distance is increasing. At C itself the acceleration is nothing. If we multiply both sides of equation (11) by m the mass of the particle, the left-hand side (mass $\times$ acceleration) represents the restoring force acting on the particle, tending to bring it back to C. We may state then, alternatively, that the restoring force is proportional to the displacement.

It should be noticed that the character of the motion, as borne out by the above differential analysis, is unchanged by adding a constant under the trigonometric functions, thus :—

$$y = a \sin \left[\frac{2\pi}{T}(t + t_0)\right]$$

describes the same motion as (8) but reckons the time from another zero point. Another mode of expressing this is to say that this equation represents the motion of another particle on the same line AA' following the same motion as the one delineated by (8) but always lagging behind or leading the latter by a time t_0 according as t_0 is negative or positive. If this second particle be supposed to follow the projection of another point E' moving round the circle (Fig. 13*a*) at the same speed as E, but at a constant angle $ECE' = \delta$ behind or in front of E, then we may describe the motion of the second particle by the equation :—

$$y = a \sin \left[\frac{2\pi t}{T} + \delta\right] \quad . \quad . \quad . \quad . \quad (12)$$

This is the most common way of representing the "phase lag" or "phase lead" of one S.H.M. in reference to another of the same period, i.e., by the angle δ.

Demonstration of S.H.M. Owing to that physical property known as elasticity, systems which, when displaced from their position of rest, experience a force tending to restore them to that position and proportional to the displacement, are quite common in nature ; hence the motion discussed above is of universal importance, and forms the basis of those motions which produce sound. The sound-producing motions being too rapid to be followed by the unassisted eye, slower systems have to be called upon for demonstration purposes. The simplest of these is the conical pendulum. A weight hung on a string is made to rotate in a circle corresponding to the circular motion of Fig. 13*a*. If the observer stands at a distance from the pendulum, with his eye on a level with the weight, so as to view the motion edgewise, the circular motion appears projected on to an imaginary horizontal straight line producing simple harmonic motion, as it were along AA'. The pendulum bob itself can be made to reproduce this S.H.M., if pulled slightly aside and let go, so that it is constrained to move in a vertical plane and on a small arc of a circle. An approximate S.H.M. may also be seen on all types of reciprocating engines

(Fig. 13*b*). In such, the circular motion of the crank-pin P round the axle H is converted into the to and fro motion of the piston and piston rod R. If any point on this or on the cross-head C be watched, it will be seen to follow S.H.M. approximately. The maximum velocity of the piston occurs when the crank and connecting rod are at right angles to each other as shown. Note that the motion is symmetrical only when the connecting rod is of infinite length; but with a long rod and a short crank the motion is to all appearance S.H.M.

If, while the particle is vibrating along AA' (Fig. 13*a*), it be given an additional movement at constant speed in the direction at right angles to AA', it will trace out a sinuous curve which will be, from the nature of the motion, a sine curve representing displacement in the AA' direction, and time in the direction of the imposed motion. This provides a common way of representing vibratory motion graphically; displacements are plotted vertically as ordinates, against the time plotted horizontally.

Superposition of two Simple Harmonic Motions of equal Periods in the same Straight Line. Let the position of the particle under one periodic force be given by

$$y_1 = a_1 \sin (2\pi nt + \delta_1)$$

and, under another periodic force, by

$$y_2 = a_2 \sin (2\pi nt + \delta_1).$$

Under the action of the two forces acting simultaneously, the particle will execute a vibration such that its position at any instant will be equal to the sum of those which it would have under either force acting alone. This is the Principle of Superposition, and as we shall see, it is, in practice, applicable only when the amplitudes a_1 and a_2 are small. With this proviso, the resultant displacement:—

$$y = y_1 + y_2 = a_1 \sin (2\pi nt + \delta_1) + a_2 \sin (2\pi nt + \delta_2)$$

can be written for example:—

$$y = A \sin (2\pi nt + \Delta),$$

provided the coefficients of $\sin 2\pi nt$ and $\cos 2\pi nt$ in each expression are the same. This will be so if:—

$$a_1 \cos \delta_1 + a_2 \cos \delta_2 = A \cos \Delta,$$

and

$$a_1 \sin \delta_1 + a_2 \sin \delta_2 = A \sin \Delta.$$

Squaring and adding the last two equations, we find the amplitude of the resultant is given by :—

$$A^2 = a_1^2 \cos^2\delta_1 + a_2^2 \cos^2\delta_2 + 2a_1a_2 \cos\delta_1 \cos\delta_2 + a_1^2 \sin^2\delta_1 + a_2^2 \sin^2\delta_2 + 2a_1a_2 \sin\delta_1 \sin\delta_2$$
$$= a_1^2 + a_2^2 + 2a_1a_2 \cos(\delta_1 - \delta_2),$$

and the phase Δ of the resultant is given by :—

$$\tan\Delta = \frac{a_1 \sin\delta_1 + a_2 \sin\delta_2}{a_1 \cos\delta_1 + a_2 \cos\delta_2}.$$

The resultant amplitude is less than the sum of the component amplitudes, except when $\delta_1 = \delta_2 +$ a multiple of 2π.

An interesting case occurs when $a_1 = a_2$, and $(\delta_1 - \delta_2) = \pi$, or any *odd* multiple of π, so that $\cos(\delta_1 - \delta_2) = -1$, and therefore $A = 0$, and also $y = y_1 + y_2 = 0$. In fact, the particle is being given displacements simultaneously in two opposite directions and so remains at rest. This "wiping-out" of one vibration by a superposed vibration of opposite phase is of wide occurrence in all forms of wave motion, and is known as "interference." This term is often applied to the superposition of vibrations in general, but should strictly be limited to the mutual cancellation of motions.

Incidentally, our analysis shows that two S.H.M.'s of equal period acting in the same direction, add up to produce a vibration of equal period, but of different phase and amplitude. Analysis similar to the above can be applied to any number of vibrations superposed in the same straight line on the same particle.

Fourier Analysis. A very important theorem, or rather the physical interpretation thereof, enables us to resolve any periodic vibration into simple components. The theorem turns on the possibility (first recognized by Bernoulli [1] for the problem of the vibrating string) of representing a complex periodic function as a series of sines and cosines of continually increasing order, thus :—

$$f(x) = \tfrac{1}{2}a_0 + a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + b_1 \cos x + b_2 \cos 2x + b_3 \cos 3x, \text{ etc.} \quad . \quad . \quad . \quad . \quad (13)$$

Each of these terms will be recognized as a Simple Harmonic Motion in itself if ωt is put in place of x, so that the theorem states the possibility of representing any motion of a point along a line (as long as the motion is periodic) as the superposition of a series of S.H.M.'s whose frequencies gradually increase in the ratio of the natural numbers.

It may be noted in passing that such a series is called a "harmonic series," and the separate motions $a_1 \sin \omega t$, $a_2 \sin 2\omega t$, $a_3 \sin 3\omega t$, etc., are called "harmonics" of which the first or "prime" or "fundamental" motion is represented by $a_1 \sin \omega t$. The test of the feasibility of such a separation into component vibrations lies in the possibility of determining the values of the amplitudes a_1, b_1, etc., of the components making up a given periodic function. This was accomplished by Fourier, who thus gave us a series of far-reaching importance in physics. For Fourier's [2] method of calculating the coefficients mathematical text-books must be consulted; suffice it to say that the general coefficients are given by other series:—

$$a_m = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \sin mx dx. \qquad a_0 = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) dx.$$

$$b_m = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x) \cos mx dx.$$

It appears that the coefficients a_m and b_m are not calculable if the functions $f(x) \sin mx$ and $f(x) \cos mx$ are not directly integrable. With many of the functions occurring in physics this is not possible, but it usually is practicable to determine a_m and b_m by approximate integration.

The Fourier series can also be written as a series of sines alone, but representing component S.H.M.'s in different phases; indeed, the presence of the cosines in the above form (13) is merely a method of indicating these phase differences. Thus if in (13) we put $c_1 = \sqrt{a_1^2 + b_1^2}$; $\tan \delta_1 = \frac{b_1}{a_1}$, etc., we get the series in the form:—

$$f(x) = \tfrac{1}{2}a_0 + c_1 \sin (x + \delta_1) + c_2 \sin (2x + \delta_2) + \text{etc.}$$

representing a series of harmonics, all having different phase-angles of lead.

To obviate the tedious labour of calculating the coefficients required to complete the Fourier analysis of a given curve, a number of instruments have been devised to perform this operation in respect of the first few (and usually the most important) coefficients.

Progressive Undulation. If we imagine our particle which has been executing S.H.M. to be connected by elastic springs to a series of similar particles, then the movement of the original particle will

set in motion each succeeding particle but with a progressive phase-lag (owing to inertia) of each behind the one previously set in motion. The disturbance originally confined to one particle will be seen to progress along the row from end to end in the form of a wave, known as a progressive wave. If the particles and springs lie along a continuation of AA' (Fig. 13), so that the springs are compressed or extended by the motion, the wave is called longitudinal; this is the type of wave dealt with in the first chapter, but in one dimension instead of three. If the line of particles is at right angles to AA' and to the direction of motion of the particles, the vibration is called transverse. Finally, if instead of moving along AA', the first particle is given a rotation about its axis, causing a twist of the springs and subsequent rotation of the other particles, the vibration is torsional.

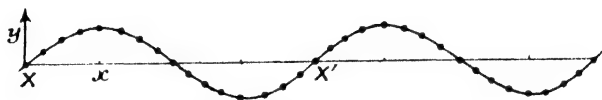



Fig. 14.—Progressive Undulation.

In accordance with the convention previously mentioned, the displacement is plotted vertically, in whatever direction it actually lies. If the distances along the row of particles be represented horizontally, Fig. 14 represents the relative displacements of the particles at any instant, though it is only for the one case of transverse vibration that Fig. 14 represents the actual relative positions of the particles. The distance XX' represents the wave-length λ, and the motion is given by :—

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right),$$

or, remembering that $n\lambda = V$, and $n = \frac{1}{T}$,

$$y = a \sin 2\pi n\left(t - \frac{x}{V}\right) \quad . \quad . \quad . \quad (14)$$

for the particle at x goes through the same evolutions as that at X, but at a time $\frac{x}{V}$ later. This lag progressively increases with x until at X' where $x = \lambda$ it has become $\frac{1}{n} = T$, so that the particles at X and at X' are in step. They are in step again at $x = 2\lambda$,

etc. Equation (14) accordingly represents a wave progressing in the positive direction of x, the second term under the sine representing the gradual taking up of the motion by subsequent particles.

It is to be noted that we have taken no account of possible loss of amplitude in transmission. The velocity of a particle at any instant $= \frac{dy}{dt} = 2\pi an \cos 2\pi n\left(t - \frac{x}{V}\right)$, and therefore lags a quarter-period behind the displacement.

Stationary Waves. In this type of motion the successive particles, over a section at least, of the vibrating series all have the same phase but with amplitude continuously diminishing from a maximum at one place (called a loop or antinode) down to zero at the end of the section (called a node). In the simplest type of stationary vibration the decrease of amplitude with distance x from antinode to node follows a sine-law, thus :—

$$y = a_0 \sin \frac{\pi}{2l} x \sin 2\pi nt \quad . \quad . \quad . \quad (15)$$

where a_0 = amplitude at antinode, and l = distance from node to antinode. This formula gives :—

$$y = a_0 \sin 2\pi nt$$

for the vibration of a particle at the antinode, falling to $y = 0$ at the node, the motion at all intervening points being in the same phase.

Stationary waves may also be produced in longitudinal, transverse or torsional form. The rod used in the Kundt's experiment (Chap. I) executed stationary longitudinal vibration, having a node at the clamped centre, and an antinode at each free end. A spiral spring clamped at the top and suspended with axis vertical will exhibit this motion if the lower end be slightly extended and let go. Stationary transverse vibration may be produced in the cord of a bow by twanging it at the centre.

A body may vibrate in a number of segments containing alternate nodes or antinodes by suitably arranging the fixed points in it (cf. succeeding chapter). The distance between two successive antinodes represents the unit-pattern of the vibration, and may be termed, in accordance with a preceding definition, the wave-length of the stationary motion, but as in fact two successive loops have opposite phases (see Fig. 15), it is more usual to call the length of

two loops the wave-length, and this falls into line with the concept of stationary motion demonstrated in the next section.

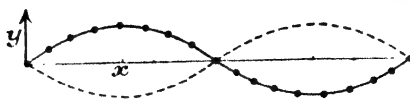



Fig. 15.—Stationary Vibration.

Reflection of Progressive Waves. By means of our formula (14) we can show that a stationary wave can be regarded as the superposition of two progressive waves of equal amplitudes, frequencies and velocities, but travelling in opposite directions. Thus, as

$$y = a \sin 2\pi n\left(t - \frac{x}{V}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

represents a wave travelling in the positive direction of x from left to right,

$$y = a \sin 2\pi n\left(t + \frac{x}{V}\right)$$

represents a wave travelling from right to left. The resulting motion is obtained by adding these two equations :—

$$y = a \sin 2\pi n\left(t + \frac{x}{V}\right) + a \sin 2\pi n\left(t - \frac{x}{V}\right)$$
$$= 2a \sin 2\pi nt \cos \frac{2\pi x}{\lambda}.$$

If this be compared with the formula (15) it will be seen to represent stationary vibration in which the amplitude of particles at the antinodes $\left(\text{at } x = \text{a multiple of } \frac{\lambda}{2}\right)$ is $2a$, or twice the amplitude of either progressive wave. Also the distance l from node to antinode is a quarter of the wave-length of the progressive undulation. By similar reasoning a contrary transformation may be demonstrated, i.e., that two superposed stationary vibrations, one advanced a half period in front of the other, make up a progressive wave advancing in the direction of the stationary vibration with the leading phase, and having amplitude equal to the antinodal displacement of either of the stationary waves.

Both the propositions in this section may be proved by graphical means, by drawing the two sets of waves after the style of Fig. 14, but at a number of succeeding instants in the period ; a process more tedious than the analytical demonstration given here.

Phase Change on Reflection. If we regard stationary vibration as made up of progressive waves, it is of interest to examine the conditions at the end of the series of particles, or of the medium in vibration. When this end is free the maximum vibration is possible, and we find at such an end an antinode of stationary vibration; when the end is fixed, or, in the case of a fluid column is limited by a rigid wall, no motion at any instant is possible, and so this point becomes a node. At the antinode, the motion being given by $2a \sin 2\pi nt$, the actual displacement at any instant can be regarded as made up of two superposed displacements, each equal to $a \sin 2\pi nt$ and of the same phase. To get this displacement out of the two progressive waves we may imagine the incident wave to come up to the wall, and to recede without change of phase; this will make the total displacement equal to the arithmetical sum of the two separate displacements at every instant. In order to get no motion at the nodal end at any instant we must postulate a complete reversal of phase (π) on reflection, in order that the separate amplitudes may mutually destroy each other, thus:—

$$y = a \sin 2\pi nt + a \sin (2\pi nt + \pi) = 0.$$

The case of imperfect reflection is worthy of consideration. Suppose that owing to transmission of part of the vibration at an " end " into the neighbouring medium, there is a loss of amplitude in the reflected wave; then the superposed motions of the direct and retrograde waves will be represented by:—

$$y = a \sin 2\pi\left(nt - \frac{x}{\lambda}\right) + b \sin 2\pi\left(nt + \frac{x}{\lambda}\right)$$

where b is less than a. Expanding:—

$$y = (a+b) \sin 2\pi nt \cos 2\pi\frac{x}{\lambda} - (a-b) \cos 2\pi nt \sin 2\pi\frac{x}{\lambda}$$

This represents a set of imperfect stationary vibrations with pseudo-antinodes where the vibration is $(a + b) \sin 2\pi nt$, and pseudo-nodes where $y = (a - b) \cos 2\pi nt$. The motion in the former points is less than before, while there are no real nodes, their place being taken by points of minimum vibration.

This type of motion will be met with in Melde's experiment (Chap. IV) and in the measurements of absorption coefficients (Chap. IX).

Superposition of Vibrations in Directions at Right Angles. This type of superposition is purely artificial, that is to say, no natural

sounds involve this type of vibration, but it leads to an important precision method for comparing the relative frequencies and phases of the vibrations executed by two bodies. Let a particle be acted on by a periodic force tending to displace it in the x direction according to the law :—

$$x = a \sin 2\pi nt \quad . \quad . \quad . \quad . \quad (16)$$

and at the same time by another force which, acting alone would make it execute vibrations in the y direction at right angles to the former ;

$$y = b \sin (2\pi nt + \delta) \quad . \quad . \quad . \quad \text{as} \quad (17)$$

Then, under the combined forces, the particle will trace out a two dimensional figure in the plane of xy. The *form* of this figure, but not the rate at which it is traced, will be obtained by eliminating t from (16) and (17). Expanding (17) and substituting from (16) :—

$$\frac{y}{b} = \frac{x}{a} \cos \delta + \sqrt{1 - \frac{x^2}{a^2}} \sin \delta.$$

Squaring :
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \delta = \sin^2 \delta \quad . \quad . \quad . \quad (18)$$

This represents an ellipse, contained within a rectangle of sides a and b, but of an eccentricity and at an inclination depending on the phase difference, and the individual amplitudes.

A number of special cases will be noticed :—

(1) If $\delta = 0$ or π, or any multiple of π, the equation gives one of two straight lines, diagonals of the rectangle, $y = \pm \frac{b}{a} x$

(2) $\delta = \frac{\pi}{2}$ or any odd multiple of $\frac{\pi}{2}$ gives the ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

symmetrically situated in the rectangle, and traced out in one or other direction. If in addition $a = b$, the ellipse becomes a circle.

When the components are vibrations of different frequency, the method of elimination of t follows the same lines, but the resulting curves are too complex to be treated mathematically in this treatise. The curve is in general bent on itself, and shows " double points," e.g., when the ratio is 1 to 2, one double point ; when 1 to 3, two double points ; in fact, the number of these points is a function of the period ratio. In these cases, though the phase for each vibration

remains fixed, yet the relative phase (if one can speak of relative phase when the periods are different) is constantly changing, owing to dissimilarity of periods. For example, confining one's attention to successive passages of the faster vibration past its null point, the first time the slow vibration may have zero displacement; the second time the fast vibration reaches the null phase, it may have a small positive displacement; the third time, a larger positive displacement, etc., until they will reach the null-displacement point once more together, provided the periods are commensurate.

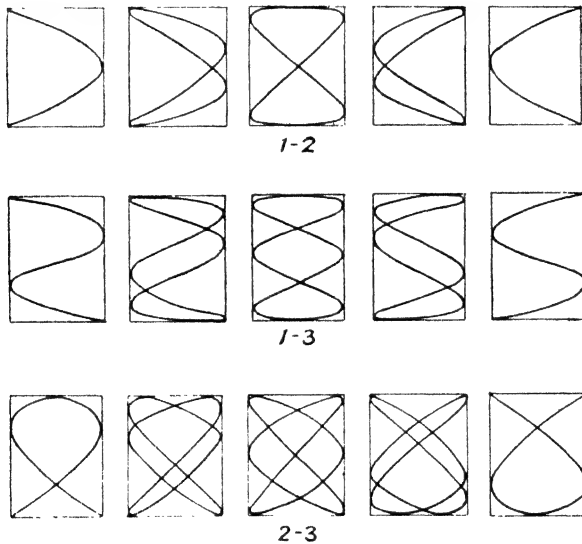



FIG. 16.—Lissajous' Curves.

For pure demonstration purposes, one or other of the "harmonographs" based on the principle of Blackburne's Pendulum may be used, by which a pencil or ink-marker is given perpendicular motions, an adjustment being provided for varying the periods of the two pendulums by which this motion is caused. In sound, however, superposition of rectangular vibrations is of importance because it serves as a test for the equality of the periods of two vibrating bodies, and was first so used by Lissajous. The figures in Fig. 16 are usually known as Lissajous' Curves.[3]

The method was incorporated by Lissajous into a "vibroscope" consisting of a microscope, the object-glass of which was detached

from the rest of the system and vibrated to and fro on one of the prongs of a tuning fork. The vibroscope is placed with its axis vertical, and the vibration which it is desired to examine takes place in a horizontal plane, but at right angles to the motion of the tuning fork; a polished speck on the vibrating body may serve as object. If the tuning fork is suited to the nominal frequency of the vibration, or has a frequency a small multiple or sub-multiple of it, one or other of the figures in Fig 16 will be traced, and will be visible (owing to optical fatigue) as a whole in the microscope. Any slight change in the ratio of the frequencies will be at once detectable by the "wandering" of the form of the vibration from its regular evolution, so that it no longer follows out a recurring sequence. When both vibrators are large bodies (e.g., two tuning-forks) and can be made to carry a light mirror, the figures can be projected on a screen, by allowing light from a source to be successively reflected from the two mirrors before falling on the screen. Lissajous' curves may be conveniently demonstrated on the cathode ray oscillograph (cf. p. 203).

Undamped Motion. In the simple cases of vibration which we have dealt with up to the present no account has been taken of the loss of energy by friction. When a particle has been given the amplitude a it has been tacitly assumed that it continues to vibrate with this amplitude undiminished, or else that energy has been supplied to maintain it so. Let us glance back over the ideal case, in which no degradation of energy takes place. The kinetic energy of the particle is expressed in the product of half its mass and the square of its velocity; in such a case where the elastic force is proportional to the displacement, the potential energy in its turn is proportional to the square of the displacement, because rate of change of potential energy in the direction of the displacement must be equivalent to the force, or alternatively, because the potential energy is the force integrated over the displacement.

If m is the mass of the particle (inertia of the system) and k represents the elastic force per unit displacement, the principle of conservation of energy gives :—

$$\frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}ky^2 = \text{a constant} \quad . \quad . \quad . \quad . \quad (19)$$

By differentiation with respect to time :—

$$m\frac{d^2y}{dt^2} = -ky \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

an expression which we have already obtained (with ω^2 in place of $\left.\frac{k}{m}\right)$ and whose solution we know to be :—

$$y = a \sin\left(\sqrt{\frac{k}{m}}t - \delta\right).$$

The frequency $n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ depends only on the ratio of the elastic to the inertia force, but it must be remembered that this is true only for small displacements. This being assured, we observe that increase of the elastic force or decrease of the mass both raise the pitch of the emitted sound. We may note in passing that the kinetic energy in the motion is proportional to $\left(\frac{dy}{dt}\right)^2$ and therefore to $a^2\omega^2$; this latter quantity is often taken to represent the intensity of the sound.

Damped Oscillations. Under actual conditions, some of the kinetic energy is degraded and appears as heat, either in the system itself or in the surrounding medium. In any case the dissipative force introduced by friction is proportional to the velocity, or rather, to the relative velocities of the rubbing substances, provided these are not so great as to produce vortices in the surrounding medium. Adding such a frictional term to (20) we get :—

$$m\frac{d^2y}{dt^2} = -ky - \mu\frac{dy}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The solution of this equation is :—

$$y = ae^{-\alpha t} \sin\left[\omega t \pm \delta'\right],$$

and may be verified by differentiation. When this is done we find :—

$$\alpha = \frac{\mu}{2m}$$

and
$$\omega^2 = \frac{k}{m} - \frac{\mu^2}{4m^2}.$$

In acoustical phenomena, the rate of decay in amplitude represented by α is comparatively slow, so that the case in which, owing to $\frac{\mu}{2m}$ being greater than $\sqrt{k/m}$, the quantity ω is imaginary, may be

dismissed. Ignoring this case, the effect of friction is twofold :—

(1) Loss of amplitude.

(2) Fall of frequency.

The frequency is $n' = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{\mu^2}{4m^2}}$

instead of $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$,

a change from n to $\sqrt{n^2 - \frac{\mu^2}{16m^2\pi^2}}$.

The effect of viscosity on the frequency is then a quantity of the second order, and in ordinary circumstances we can say that friction has no influence on the period, but only on the amplitude. When the vibrating body is surrounded by a medium which exerts a considerable dragging effect on the motion (e.g., bars and strings vibrating in liquids) we shall see that this approximate statement is insufficient (cf. p. 117).

Free and Forced Vibration. If the system now, instead of being given an impulse and let go, has a sustained periodic force acting upon it, the system reproduces what are called " forced " vibrations. The type described in this chapter up to the present is denoted " free " vibration. For a forced vibration, we must modify (21) by adding a term representing this external force. The simplest type of periodic force is the simple harmonic, so we write our equation :—

$$m\frac{d^2y}{dt^2} = -ky - \mu\frac{dy}{dt} + F \sin pt, \quad . \quad . \quad . \quad . \quad (22)$$

so that F is the amplitude of the force, and $\frac{2\pi}{p}$ the period of its alternation. The particular solution of this equation which denotes the forced vibration is :—

$$y = A \sin (pt - \delta).$$

Substituting in (22) and equating the coefficients of $\sin pt$ and $\cos pt$ we find :—

$$-mAp^2 \cos \delta = -kA \cos \delta - \mu Ap \sin \delta + F \quad . \quad . \quad (23)$$

$$mAp^2 \sin \delta = +kA \sin \delta - \mu Ap \cos \delta \quad . \quad . \quad . \quad (24)$$

From (24), $\tan \delta = \frac{\mu p}{k - mp^2}$.

Dividing (23) by cos δ, and substituting tan δ for this expression and

$$\frac{k - mp^2}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}$$

for cos δ, we get :—

$$A = \frac{F}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}$$

whence $$y = \frac{F}{\sqrt{\mu^2p^2 + (k - mp^2)^2}} \sin (pt - \delta) \quad . \quad . \quad . \quad (25)$$

Another solution is obtained when $F = 0$; this corresponds to the free vibration, and we have already obtained it as a solution of (21). The general solution involves both particular solutions :—

$$y = ae^{-\alpha t} \sin (\omega t - \delta) + \frac{F}{\sqrt{\mu^2p^2 + (k - mp^2)^2}} \sin (pt - \delta) \quad . \quad (26)$$

The general solution includes two periodic functions, one the natural vibration of the driven system, and the other the vibration imposed by the driver. Usually, owing to the diminishing amplitude of the former, the frequency of the driver predominates over the other; but the particular case where the two periods are nearly, if not quite equal, is one of far-reaching importance in acoustics.

The student of electricity may find it interesting to compare these equations, (20), (21) and (22) and their respective solutions with the corresponding cases of circuits containing resistance (R), inductance (L), and capacitance (C). For example, (22) has its counterpart in the equation for the quantity (Q) of electricity at any instant in such a circuit to which an alternating E.M.F. of $\frac{p}{2\pi}$ cycles per second is applied.

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E \sin pt.$$

The comparison shows L to be of the nature of an inertia, R of a viscous force, and $1/C$ of an elastic one. The solution of this is, *mutatis mutandis*, our equation (26), and the solution corresponding to the forced vibration (25), is worked out in text-books on electricity.* In the absence of damping forces due to resistance, the equation becomes :

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = E \sin pt.$$

* *See e.g.* Starling, *Electricity and Magnetism*, p. 353.

In this case the natural frequency of the circuit is $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$; $\left(\text{corresponding to } \frac{1}{2\pi}\sqrt{\frac{k}{m}} \text{ in (20)}\right)$; and resonance is produced when the applied E.M.F. alternates with this frequency, i.e., when $p = \sqrt{\frac{1}{LC}}$.

Resonance. If the effect of viscosity is small, then the amplitude under the action of the driving force is a maximum when the denominator in (25) and (26) is a minimum, i.e., when $k = mp^2$ or $p = \sqrt{\frac{k}{m}}$, or, the frequency of the driver is equal to that of the driven; in fact, if $\mu = 0$, the amplitude would become infinite at this frequency. This enhanced oscillation when the periods coincide is known as "resonance." The amplitude at resonance is (with friction) $\frac{F}{\mu p} = \frac{F}{\mu}\sqrt{\frac{m}{k}}$. A quantity of importance in sound is the "sharpness of resonance," expressing the fall of amplitude with change of frequency on each side of the maximum. When the damping is great this drop of amplitude is very slow; on the contrary, the resonance is sharp when the frictional losses are small. The truth of this apparently paradoxical statement—it appears at first sight as though friction should prevent the attainment of large amplitudes except at coincidence of periods—is verified by the following calculation.

Sharpness of Resonance and Variation with Pitch. Following Rayleigh,[4] we may gauge the response of the particle to sustained forcing, by working out the kinetic energy possessed by the particle at the instant of its passage through the undisturbed position. This is equivalent to finding the maximum velocity in the motion defined by (25), i.e., the velocity when $\cos(pt - \delta) = 1$.

$$\left(\frac{dy}{dt}\right)_{max} = \frac{pF}{\sqrt{\mu^2p^2 + (k - mp^2)^2}}$$

$$\text{Kinetic energy} = \tfrac{1}{2}m\left(\frac{dy}{dt}\right)^2 = \frac{\tfrac{1}{2}mp^2F^2}{\mu^2p^2 + (k - mp^2)^2}$$

Dividing out by $\frac{1}{2}\frac{F^2}{m}$, the mean square of the driving force per unit

mass during the period, we get the kinetic energy per unit forcing, which Barton [5] calls the response (R);

$$R = \frac{p^2}{\frac{\mu^2 p^2}{m^2} + \left(\frac{k}{m} - p^2\right)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Now $\sqrt{\frac{k}{m}}$ represents the natural frequency n of the system in the absence of damping, so that the expression in the brackets represents the extent to which the natural frequency of the system deviates from the forced frequency, and has been called the "mistuning." When driver and driven are exactly in tune, the response $R = \left(\frac{m}{\mu}\right)^2$, i.e., is inversely as the frictional coefficient μ. As the tuning gets

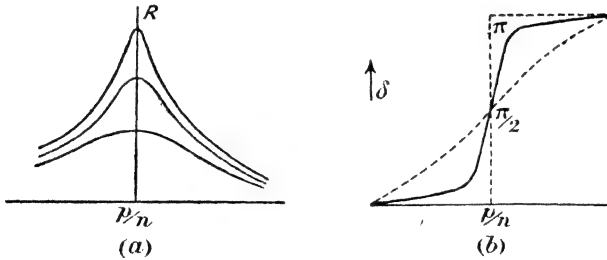



Fig. 17.—Response and Phase at Resonance.

worse, R naturally falls, giving a curve of the type shown in Fig. 17*a*, when R is plotted against the ratio of the frequencies p/n. The sharpness of resonance which is inversely as μ is often expressed by the quality factor (Q). Let p_1 and p_2 be the values of the forcing pulsatance on either side of resonance (p_0) at which the power in the response has fallen to one half. The power is the product of F sin pt and velocity, i.e. of dy/dt derived from (25). The average value of this power is

$$\frac{\mu F^2 p^2}{2[\mu^2 p^2 + (k - mp^2)^2]}.$$

This will be half the maximum (i.e. $F^2/2\mu$) when

$$k - mp^2 = \pm \mu p.$$

The roots p_1 and p_2 of this equation satisfy $p_1 - p_2 = \mu/m$; then

$$Q = \frac{p_0}{p_1 - p_2} = \frac{\sqrt{k/m}}{\mu/m} = \frac{\sqrt{km}}{\mu} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

In practice, the systems we deal with are capable of performing in addition natural vibrations corresponding to a number of members of the harmonic series, i.e., with frequencies which are multiples of n. Provided the damping represented by $\frac{m}{\mu}$ is unchanged for such a harmonic, for a given mistuning, the response will be inversely as p^2 (cf. 27), and therefore will diminish as we go up the harmonic series. In other words, although the maximum response under these conditions will be constant, the resonance curve will droop more sharply at the higher natural frequencies (see Fig. 17*a*).

Phase of Resonance. Having considered the amplitude in equation (25) we will now consider the other factor δ, representing the phase lead of the forced vibration in front of that of the driving force. If p is small, then is δ also small, but grows with p until at resonance, $p^2 = \frac{k}{m}$, $\tan \delta = \infty$, therefore $\delta = \frac{\pi}{2}$, so that the lead has now reached a quarter period. Beyond the resonant frequency this

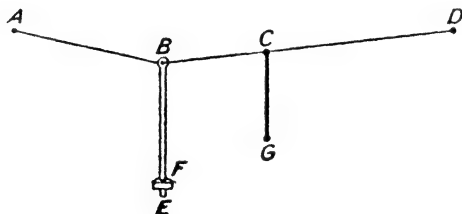



FIG. 18.—Demonstration of Resonance.

lead grows until, when $p = \infty$, δ has reached the value π. If the friction is large, the numerator of the expression for $\tan \delta$ is always considerable. On the other hand, if μp is a small quantity δ keeps low values until quite close to resonance.

These extreme cases are shown in the accompanying Fig. 17*b*. The dependence of amplitude and phase on the relative frequencies of driver and driven may be exhibited by the arrangement shown in the next figure.

BE is a metal stick on which a bob *F* can be adjusted by sliding. *CG* is a simple pendulum of string (rather shorter than *BE*) having a small bob *G*. The points of suspension are secured on the string tied at *A* and *D*. When *F* is pushed to the bottom of the rod and the driving pendulum set in oscillation through a small angle, *CG*

oscillates a little nearly in phase with *EF*. As *F* is pushed higher, resonance is eventually reached, and now *G* moves through a large amplitude in time with *F*, but a quarter of a period later, so that when *F* has reached its maximum displacement on one side, *G* is just moving through its lowest point towards the same side. When *F* is pushed further up, it will be found that the phase difference gradually increases, and the amplitude of *G*'s oscillation diminishes rapidly. The experiment may be made still more striking by placing a series of light pendulums of progressively increasing length between *D* and *B*. If as before *G* is the resonant pendulum, those to the left of *G* whose frequency is higher will lead *G* by progressively increasing phases, while those to the left of low frequency will lag behind *G*.[6]

In such a system it will be found that the resonance though strong is not sharp; as an example of strong and sharp resonance under external forces, the tuning fork (Chap. IV) may be instanced. The strange fact should be noticed that, at resonance, the driving force is exerting its maximum influence at the moment when the amplitude of the subsidiary system is the greatest, and not when it is passing its mean position.

Reaction on the Driver: Coupled Systems. When, at resonance, the amplitude of the resounding system attains unusual dimensions, it is natural to look for the source of the energy required to maintain it. This must evidently lie in the driving system itself; and unless this system possesses some external maintaining energy, such as that of an electric battery or engine, its vibrations must be strongly damped, owing to loss of energy to the driven system. This may be shown, for example, when a tuning fork mounted on a resonance box, i.e., a vessel of air whose natural frequency corresponds to that of the fork, is strongly bowed. Owing to the assumption of the vibration by the air in the box the tone of the combined "coupled system" is much more intense than that of the fork alone; but, for the same initial impulse, the vibrations die away more rapidly.

When the two systems are equal, or nearly equal, in mass or inertia, the rôles of driver and driven may be interchanged. Of course, either of the components of two coupled systems will react on the other, and it is only in view of practical systems that we usually speak of the smaller as the resonator; but the distinction is less obvious when the systems are nearly equal.

A striking experiment illustrates this. Two equal pendulums

AB, *CD* are suspended from two points *A* and *C* and are connected by a thread or light rod *EF* (Fig. 19). If one pendulum *D* be given a small displacement and allowed to swing, it will begin to act on *B*, which now takes up the oscillation, taking energy from *D* until it has reduced *D* to rest. And now the position of affairs is reversed, *B* becomes the driver and *D* again takes up the oscillation until *B* has come to rest. So the interchange goes on, till friction has damped out all vibration. The position of *EF* determines the extent to which the pendulums can react on each other. If *EF* occupies the position

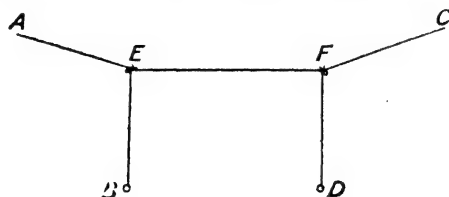



Fig. 19.—Coupled Cord Pendulums.

AC this influence is a minimum, and the coupling is said to be "loose"; as *EF* is lowered each body has more control over the other, i.e., the coupling is tightened.

The effect of one system on the other is that of an added force due to the mass-acceleration of the other. The equations of the two systems (neglecting damping which considerably complicates the analysis) are, cf. (20) :—

$$m_1\frac{d^2y_1}{dt^2} + M\frac{d^2y_2}{dt^2} + k_1y_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$m_2\frac{d^2y_2}{dt^2} + M\frac{d^2y_1}{dt^2} + k_2y_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where the second terms represent the mutual reaction. Differentiating twice :—

$$m_1\frac{d^4y_1}{dt^4} + M\frac{d^4y_2}{dt^4} + k_1\frac{d^2y_1}{dt^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$m_2\frac{d^4y_2}{dt^4} + M\frac{d^4y_1}{dt^4} + k_2\frac{d^2y_2}{dt^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Substituting in (31) the value of $\frac{d^4y_2}{dt^4}$ obtained from (32), we get :—

$$m_1\frac{d^4y_1}{dt^4} - \frac{M^2}{m_2}\cdot\frac{d^4y_1}{dt^4} - \frac{k_2M}{m_2}\cdot\frac{d^2y_2}{dt^2} + k_1\frac{d^2y_1}{dt^2} = 0\,;$$

again substituting in this equation the value of $\frac{d^2y_2}{dt^2}$ obtained from (29), and dividing out by m_1, we get :—

$$\left(1 - \frac{M^2}{m_1m_2}\right)\frac{d^4y_1}{dt^4} + \left(\frac{k_1}{m_1} + \frac{k_2}{m_2}\right)\frac{d^2y_1}{dt^2} + \frac{k_1k_2}{m_1m_2}y_1 = 0 \; ; \quad . \quad (33)$$

with the same equation for y_2. Now $\frac{k_1}{m_1} = 4\pi^2n_1^2$, and $\frac{k_2}{m_2} = 4\pi^2n_2^2$, where n_1 and n_2 are the natural frequencies of the respective systems removed from mutual influence. Further, if the combined systems follow a S.H.M. of frequency N, y_1 (or y_2) $= a \sin 2\pi Nt$;

$$-\frac{d_2y_1}{dt^2} = 4\pi^2N^2y_1 \; ; \quad \frac{d^4y_1}{dt^4} = 16\pi^4N^4y_1.$$

Putting finally $\frac{M^2}{m_1m_2} = \kappa^2$ (33) becomes, on dropping the common factor $16\pi^4y_1$:—

$$(1 - \kappa^2)N^4 - (n_1^2 + n_2^2)N^2 + n_1^2n_2^2 = 0$$

$$\therefore N^2 = \frac{n_1^2 + n_2^2 \pm \sqrt{(n_1^2 - n_2^2)^2 + 4n_1^2n_2^2\kappa^2}}{2(1 - \kappa^2\,)} \quad . \quad (34)$$

κ is called the "coefficient of coupling."

This equation has in general two roots, showing that the complex system has two natural frequencies. In the special case of $\kappa = 0$, $N = n_1$ or n_2 ; otherwise the effect of coupling is to add to the expression under the root in the numerator, and reduce the denominator of (34), so that, assuming n_1 greater than n_2, the roots of N become respectively greater than n_1 and less than n_2 as the coupling gets closer. If

$$n_1 = n_2 = n, \; N = \frac{n}{\sqrt{1 - \kappa}} \text{ and } \frac{n}{\sqrt{1 + \kappa}}.$$

This property is utilized in what is called "multiple resonance," by coupling two resonators in order to get a system having two resonance peaks, which are in general, respectively above and below the upper and lower resonance peaks of the separate systems (cf. p. 210).

Again, an analogy may be shown with two coupled electric circuits whose equations, when resistance can be neglected, are

$$L_1\frac{d^2Q_1}{dt^2} + M\frac{d^2Q_2}{dt^2} + \frac{1}{C_1}Q_1 = 0$$

$$L_2\frac{d^2Q_2}{dt^2} + M\frac{d^2Q_1}{dt^2} + \frac{1}{C_2}Q_2 = 0$$

corresponding to (29) and (30), the coefficient of coupling being $\sqrt{\frac{M^2}{L_1L_2}}$. This M is called the "mutual inductance," and can be found experimentally, and can be calculated for simple configurations. On the contrary there is no means of finding a value of M in the case of two acoustic systems; we can only say that the coupling is "tight" or "loose." M has been calculated for certain pendular and similar systems.[7]

Resonance is both a useful and an obnoxious phenomenon in sound. On the one hand, it is useful in reinforcing feeble sounds, but on the other hand it upsets the delicacy of response of a system which is intended to reproduce or amplify tones of all frequencies indiscriminately. From the conglomeration of tones that fall upon it, a vibrating system picks out those which correspond to one or other of its natural vibration frequencies, and exaggerates them out of all proportion to the rest. There are two remedies for this; either the natural period of the body must lie outside the range of those to which it is submitted, or, better, by a suitable arrangement of multiple resonance frequencies it must be made to resound, more or less, to the whole range. The amplitude-frequency curve of such a system consists of a series of overlapping resonance peaks (cf. p. 202).

Sub-synchronous Maintenance. In our treatment of resonance we have assumed Simple Harmonic Motion in both systems. The driver may, however, provide impulses for a small fraction of the period, and in this case the frequency of the impulse may be a submultiple of that of the driven system. In this way Raman[8] has maintained pendulums in vibration by an electro-magnet which, by its attraction on the steel bob, has the effect of momentarily increasing the gravitational force at the bottom of its swing. The frequency of the interruption of current is equal to, or is a sub-multiple of, that of the pendulum, and if a fork interrupter is used, the frequency of the latter can be accurately calculated if that of the pendulum is known.[9]

A peculiar type of resonance curve is obtained if the maintaining impulse occurs at frequencies somewhat above or below that of the driven system. The curve has no peak, the amplitude falling steadily on one side of resonance and rising on the other, so that, as the frequency of the driver is lowered through resonance and beyond, the forced amplitude continuously increases.

The absence of the peak has been ascribed to breakdown of the simple theory of forced vibrations, so that terms involving y^3 need to be introduced into (21) and (22). In such a case, the amplitude, even of the free

system, would be dependent on the frequency, so that change of amplitude of the freely swinging pendulum would alter its natural frequency; perhaps these changes mask the true resonance phenomenon in the observed curve.

Maintenance by occasional Impulse. Relaxation Oscillations. Instead of being maintained by a continuous force a system may be sustained by giving it a succession of separate impulses at appropriate epochs. Fig. 20(*a*) shows on the left the displacement:

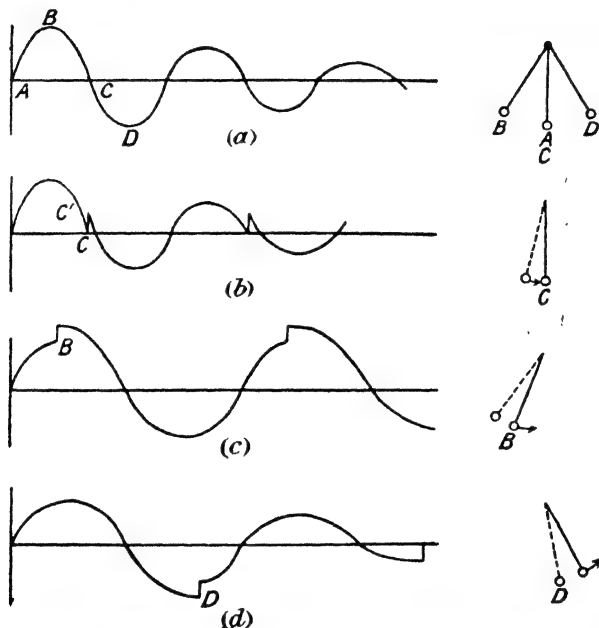



FIG. 20.—Maintenance of Vibration by occasional Impulse.

time graph of the pendulum on the right which is set going and left to itself. Owing to damping, the amplitude, *AB*, *CD*, etc., progressively decreases as time goes on. Suppose that one endeavours to replenish the kinetic energy of the pendulum by giving it a small displacement by hand from right to left at a certain instant, the increment being represented on the graph by a short upstroke at the appropriate point on the graph. If this upstroke is given at the point *C* (Fig. 20*b*) it will delay the movement of the bob to the right and so will increase the time-period, but as it merely repeats the course of events between *C′* and *C* it cannot alter the amplitude at *D*. If

the same movement is delivered when the bob is at B (Fig. 20c), however, and is sufficient to make up for the loss in amplitude since the preceding increment, the motion will be maintained without altering the time-period, insofar as this is independent of amplitude. If the increment of amplitude is delivered at D (Fig. 20d), the motion is more rapidly damped.

A somewhat similar phenomenon has been analysed by van der Pol.[10] First suppose that the damping μ in (21) is a negative quantity so that the solution involves a factor $e^{+\alpha t}$. This will not in fact be a physically realizable case since the amplitude would *increase* with time to infinity. But now suppose that the amplitude is limited by an additional damping factor which grows with it. Van der Pol chooses an equation of the type:

$$\frac{d^2y}{dt^2} = 2\alpha(1 - y^2)\frac{dy}{dt} - \omega^2 y \quad . \quad . \quad . \quad . \quad . \quad (21a)$$

in which the net damping is negative until $y^2 = 1$. He shows that if α is not too small, the square of the frequency is determined by the quantity $\omega^2/2\alpha = k/m \div \mu/m = k/\mu$. The period, $T = 2\pi\sqrt{\mu/k}$ is known as the "time of relaxation" of the system since it determines the rate at which the stress disappears as it is taken out in the resulting shear. The term was introduced by Clark Maxwell[11] in an attempt to relate the viscosity of a system to its elasticity.

In sound, these "relaxation oscillations" occur when a system is subjected to a considerable distorting force, is displaced and then relaxes until the force can again displace it. The method of exciting vibrations in a walking-stick by pushing the point in front of one over a rough surface with which it from time to time engages, and the way in which the breath escapes through the tensioned vocal cords during speech (cf. p. 270) are examples of this type of oscillation.

Interference. We have already spoken of this subject in dealing with the superposition of small motions. Here will be cited some practical instances in which mutual destruction of out-of-phase components is noteworthy. The first of these is the interference-tube, based on a principle first envisaged by Herschel,[12] but put into practical form by Quincke.[13] Sound of any pitch is conducted from opening A (Fig. 21a) of the tube to the ear at B. The orifice at B must be tightly plugged into the ear, and the other ear stopped in order not to hear the note by way of the open air. Beyond A the tube divides; the sound reaches B both by the short tube C and by the

longer tube D, which can be lengthened by sliding it out in the rubber connections E, in the same way as the slide of a trombone works.

If the difference of path *viâ* C or *viâ* D is exactly half the wave-length of the note, the two components arrive at B with a phase difference of π, and no sound is audible at B provided they are of equal intensity. To ensure this, C is made a little narrower than D in order that friction may dissipate energy at a greater rate, per unit length of tube, in C than in D. Fig. 21*b* shows another arrangement in which the alternative path is provided by reflection at the adjustable stopper F. Knowing the frequency and wave-length of the sound, its velocity in the tubes could be estimated but that it will be different in each tube if these are not of the same bore. More must not be

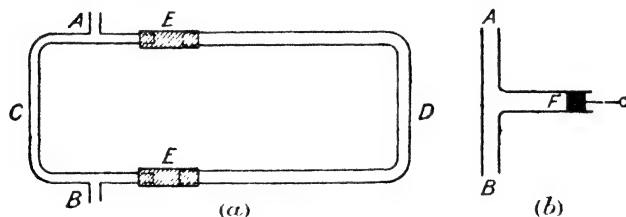



FIG. 21.—Quincke's Interference Tubes.

expected of the apparatus than a qualitative demonstration of interference. Instead of the ear, manometric and sensitive flames (p. 154) have been used at B.

Interference produced by reflection on a large scale may be observed. It we imagine a series of plane waves impinging on a plane non-absorbent wall and being there reflected, there will be zones of no motion in the air (nodal planes), in fact, stationary vibration between the source and the wall, if the position of the source with respect to the wall is suitably adjusted. In actual fact the waves sent out by the source are nearly spherical, but there will be, nevertheless, *points* of no vibration instead of the planes. These points can be detected by any of the instruments, sensitive flame, resonator, etc., described later. In a building, owing to multiple reflection in several directions, the maxima and minima caused by superposition become very complex, and may be detected by an auditor moving about in the building while the tone is sounding.

An interesting use of the interference between a progressive wave

travelling along a pipe and its reflection from the stopper at the end has been made by G. D. West.[14] The source of sound is a high-pitched whistle which can be moved along the axis of the pipe from the open end. Complete interference will occur whenever the returning waves arrive at the source exactly out of phase with those emitted, and nothing will be heard at the mouth of the pipe. By moving the whistle slowly into the pipe and measuring the distance between successive positions of silence, the velocity of sound may be found. The whistle must be blown from a large reservoir of compressed air, otherwise its pitch will change slightly as the pressure falls.

Reflection Tones. When, in place of a simple tone, there falls upon a wall a conglomeration of tones at various frequencies from a complex source, an ear moved to different distances from the wall will hear most strongly that tone, or series of tones, which has a node at the point where the ear is placed. This phenomenon seems to have been first observed between a waterfall (as a complex source) and a rocky wall, by Baumgarten. Above the roar of the fall those " reflection tones " are heard whose frequency, in accordance with this theory is peculiar to the distance of this point of hearing from the wall.[15] The phenomenon has lately again received attention, as it manifests itself in the reflection from the ground of the sounds made by an aircraft. These are noises due to the engine, its turbulent wake, the rotation and vibration of its propeller blades, etc., one or other of which noises is exaggerated to the listener on the ground by interference. Prandtl noticed that the reflection tone rose in pitch as he stooped, and also as the angular elevation of the aircraft changed, since this change alters the angles of incidence and reflection of the sound rays impinging on the ground.[16]

Other demonstrations of interference will be noted under the particular systems to which they are applicable.

Beats. A quite special result of interference arises when two systems of nearly (but not quite) equal frequency react on one another, of which a theoretical study has been made by Helmholtz.[17] Let the displacement of the greater system acting alone be given by $y = A \sin pt$, and of the other alone by $a \sin (qt + \delta)$ where $(p - q)$ is small compared with p. Then the resultant displacement of the particle is given by :—

$$\begin{aligned} y &= A \sin pt + a \sin (qt + \delta) \\ &= A \sin pt + a \sin \{pt - [(p - q)t - \delta]\}. \end{aligned}$$

Put $y = C \sin(pt - \Delta)$, and equate coefficients of sin pt and cos pt

$$A + a \cos[(p - q)t - \delta] = C \cos \Delta$$
$$a \sin[(p - q)t - \delta] = C \sin \Delta.$$

Squaring and adding, we find the amplitude of the resultant given by :—

$$C^2 = A^2 + a^2 + 2Aa \cos\{(p - q)t - \delta\}.$$

The amplitude thus fluctuates between $(A + a)$ when the cosine $= + 1$, and $(A - a)$ when the cosine $= - 1$; with a long period given by $\frac{2\pi}{p - q}$. The phase also fluctuates, for :—

$$\tan \Delta = \frac{a \sin\{(p - q)t - \delta\}}{A + a \cos\{(p - q)t - \delta\}}.$$

Thus the resultant vibration represents a vibration of the same period as the larger force $A \sin pt$, with fluctuating amplitude reaching a maximum equal to the sum of the component amplitudes, with a frequency equal to the difference $\left(\frac{p}{2\pi} - \frac{q}{2\pi}\right)$ of the component vibrations, and phase lag varying from 0 to $\frac{a}{A}$ (fractions of 2π) behind the larger force. These fluctuations in intensity are readily detected by ear, when two systems, whose frequencies differ by a few vibrations per second, are sounding together, and are called "beats." Beats may also occur between a forcing vibration and the natural frequency of the driven system until the latter is damped out. The fact that the number of beats per second equals the difference of the frequencies of the component vibrations, forms a ready method of measuring small differences of frequency, and is accordingly used in tuning instruments, or of testing a unison.[18] The displacement-time diagram of a system producing beats is shown below.

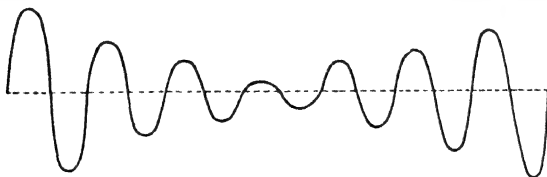



FIG. 22.—Beat Diagram.

There are several instruments capable of showing that these fluctuations actually exist in the air surrounding two bodies of nearby

frequency; they usually consist of membranes which copy the motion of the air, the motion of the membrane being exhibited by one or other of the methods described in the succeeding chapters.

Combination Tones. We now treat of the cases where, at the simultaneous sounding of two tones, a quite new tone is formed, a subject round which a whole literature has collected in the last thirty years. It was noticed almost at the same time by three musicians at the beginning of the eighteenth century (Tartini [19] the violinist seems to have been the first), that when two notes a fifth apart in the middle of the scale were played on the organ, together with them a low note was heard, whose frequency was the difference of the two higher-pitched notes. These are "difference tones," and there have also been observed "summation tones," having a frequency equal to the sum of the primary tones.

Such tones, beside being subjectively studied on the organ as in the original investigation, can be demonstrated objectively by means similar to those used for beats. In the aural examination, using an organ or harmonium, the existence of the difference tone may be made more apparent, by first sounding the low note which the two primaries should combine to give, and then sounding these alone. The original low note will then be recognized in the combination.

With smaller apparatus the difference tone may be heard on sounding (*a*) two tuning-forks of moderately high frequency and some fifty vibrations apart, (*b*) the double whistle consisting of two Pan pipes of high pitch, blown by a blast of air directed across their mouths, or (*c*), Rudolf König's [20] double glass rods, in which longitudinal vibrations of high pitch are excited by a wheel having a wetted cloth rim. To make the experiments effective it is generally necessary that the same mass of air should be violently agitated by the two generators.

The best way to demonstrate that the difference tone can exist outside the ear is to use a suitably adjusted resonator (p. 207) tuned to the expected tone. It will be found on sounding the two primaries that the resonator responds to the (low) resultant tone, reinforcing it. We are here speaking of objective combination tones as against certain tones which exist apparently only in the ear—the so-called subjective combination tones. In point of distinction, this is accepted as the *experimentum crucis* of objectivity, that every vibration which can be picked up by a suitably tuned resonator actually exists in the air.

Beat Tone Theory of Combination Tones. It was suggested, first by Lagrange [21] and by Young [22] soon after Tartini's discovery, that the difference tones had the same origin as beats, i.e., that here we have primary tones so far apart that the beats they produce follow in such rapid succession as to blend into a new tone of low pitch. Beats and difference tones are, on this theory, physically identical; it is the ear which makes the distinction between them, recognizing regular impulses of more than 16 per second as musical notes, and impulses fewer per second as distinct beats. The beat tone theory, or, as it is sometimes called, König's theory, from his exhaustive practical study of these tones,[23] presents, however, some difficulties from the standpoint of the ear as analyser, which will be noted in that connection.

Helmholtz Intensity Theory of Combination Tones. The equivocal existence of combination tones except when the primary generators are intense suggested to Helmholtz [24] that we have here to deal with a case where the simple superposition of two tones can no longer be applied. The indicator, whether artificial membrane, ear-drum or other form, no longer responds to the double forcing of the primaries in a way which is the vector sum of the vibrations they would impress upon it if acting alone. In fact, the restoring forces are not now proportional merely to the displacement. Helmholtz added a term proportional to the square of the displacement. Under the action of the two forces $a \sin pt$ and $a' \sin (qt + \delta)$, the equation for the forced vibrations of the system then becomes :—

$$m\frac{d^2y}{dt^2} + ky + k^1y^2 = a \sin pt + a' \sin (qt + \delta) \quad . \quad (35)$$

The solution of this equation, the working of which must be sought in the original, contains terms involving $\sin 2pt$, $\sin 2qt$, $\sin (p - q)t$, $\sin (p + q)t$, but the amplitude in the difference tone term is a fraction of the product aa', and so requires large values of the primary amplitudes a and a' in order to be appreciable. There should therefore be heard the octave of each primary as well as the difference and the summation tone, when the responding system possesses a non-linear response characterized by the term in y^2. Such a condition implies that a system is asymmetric, for, in addition to the usual restoring force which changes sign with the displacement, this new force inherent in the system does not so change sign; so that the free vibrations of such a system are asymmetric with respect to the equilibrium

position. Helmholtz' theory depends on a *special* species of asymmetry, characterized by elastic forces, $ky + k^1y^2$.

Waetzmann's General Asymmetry Theory. It has frequently been pointed out that the large intensity of the primaries which Helmholtz' theory requires for the production of a difference tone is not borne out by practice, the difference tone often being produced by comparatively weak primaries. Moreover the mathematics of Helmholtz has not escaped criticism. In an attempt to reconcile theory with facts, Waetzmann [25] pictures a *general* asymmetry in a system reproducing combination tones; any asymmetry, in fact, which will make the response of such a system one-sided in the sense that the displacements are no longer symmetrical about the equilibrium position. Such a system Waetzmann realized by loading a membrane

(*a*)

(*b*)

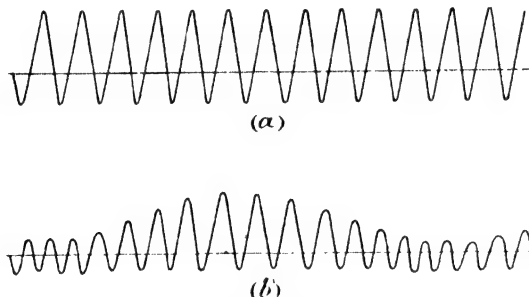



FIG. 23.—Vibrations of Asymmetrically Loaded System.

with a central weight on one side of the membrane only, and found its free vibrations given by a curve like that in Fig. 23*a*.

He next led two simple tones from two tuning forks to this asymmetric membrane, and recorded the vibration of the latter under the simultaneous double forcing. The curve in Fig. 23*b* represents the result. Comparing this with the record of beats (Fig. 22), we observe that the curves are similar, but that the loaded membrane possesses a "rectifying" property, pushing that part of the curve near the maxima to one side of the zero position, as compared to the minima. Performing the Fourier analysis of such a curve, Waetzmann finds the original tones (p, q), a difference tone $(p - q)$, of several times greater amplitude than either primary, a weak secondary difference tone $(2q - p)$, and occasionally a summation tone $(p + q)$. These superposed in the ordinary way make up the recorded curve. This loaded membrane may be taken as a type of the ear drum with its

attached ossicles producing "subjective" combination tones, but similar rectifying properties have been found in other systems, so that we may expect to find "objective" combination tones in the external air due to the same cause, though less definite than at the ear drum.

The theory thus satisfactorily explains the phenomenon, both as regards amplitude and frequency, and, in a way, combines and reconciles the "beat" and the "intensity" theories which failed in generality.

Doppler Effect. This is the name given to the apparent change in frequency of a moving source, or the apparent change in frequency of a stationary source received by a moving hearer. A stationary source gives out n waves per second of wave-length λ, and these reach the stationary hearer with velocity c. If the source moves away with velocity V, still sending out n waves per second, the wave-length or distance between successive maxima is increased in the ratio $\frac{V+c}{c}$, so that the frequency of the waves received by the hearer at velocity c cm. per second is $\frac{nc}{V+c}$. Similarly, if the source is kept still and the hearer moves at a velocity U towards the source, the apparent frequency is $\frac{n(c+U)}{c}$. For both in motion, it is $n\frac{c+U}{c+V}$. This change of frequency was first worked out by Doppler for the optical case. Acoustically it may be observed under the same conditions as the original investigation of Buys-Ballot [26] in Holland. The pitch of a note of a whistling locomotive falls as the locomotive passes, owing to the sudden change of sign of V in the formula. An identical result is heard on the locomotive as it passes a signal bell, also when a projectile passes overhead. Mach produced the same effect indoors by a rapidly whirled bar on the end of which a source is placed so as to approach and recede alternately from the hearer.

References

1. *Hist. Acad. Sciences,* 147, 1753.
2. *Mém. Acad. Sci. Paris,* **4,** 185, 1819.
3. *Comptes Rendus,* **44,** 727, 1857.
4. *Sound,* **1,** 47, 1894.
5. *Phil. Mag.,* **26,** 111, 1913.
6. Barton and Helen Browning, *Phil. Mag.,* **36,** 169, 1918, **37,** 453, 1919, and **44,** 573, 1922.

7. *See* LEES, *ibid.*, **48**, 129, 1924 *and the references therein.*
8. *Ibid.*, **29**, 15, 1915, and **34**, 129, 1917; *Phys. Rev.*, **95**, 449, 1912.
9. RAMAN and DEY, *Proc. Roy. Soc.*, *A*, **95**, 533, 1919.
10. *Phil. Mag.*, **2**, 978, 1926.
11. *Ibid.*, **35**, 33, 1860.
12. *Ibid.*, **3**, 111, 1833.
13. *Ann. Physik*, **128**, 177, 1866.
14. *J. Sci. Instrum.*, **6**, 354, 1929.
15. STARKE, *Ver. deutsch. Phys. Ges.*, **6**, 285, 1908.
16. PRANDTL, *Zeits. tech. Physik*, **2**, 244, 1921, *but see* DÉVÉ, *Comptes Rendus*, **174**, 1,010, 1922.
17. *Sensations of Tone*, App. 14.
18. RAYLEIGH, *Nature*, **19**, 275, 1879.
19. *Trattato di Musica*, 1754.
20. *Ann. Physik*, **69**, 721, 1899.
21. *Misc. taurinens*, 1759.
22. *Trans. Roy. Soc.*, 1800.
23. *Ann. Physik*, 157, 177, 1876.
24. *Sensations of Tone*, p. 152 and App. 12. (*Translation of Ellis.*)
25. *Zeits. Physik*, **1**, 271 and 416, 1920 contains a good summing up.
26. *Ann. Physik*, **66**, 321, 1845.

CHAPTER THREE

LONGITUDINAL AND TORSIONAL VIBRATIONS IN SOLIDS

Longitudinal Vibrations in Rods. The rod in question will be of uniform material and section, i.e., a cylinder, but not of such small diameter as to fall under the classification "strings." As regards longitudinal vibrations without bending, the shape and size of the section are not of importance, though in practice the rod, whether solid or hollow, is circular, for ease in producing the tone. The formula for velocity of propagation of longitudinal waves in a solid is identical with that (3) obtained earlier for similar waves in air, but of course in deducing the value of the elasticity coefficient to be used, the gaseous relations between volume and pressure no longer apply. In the actual rods employed, every elongation δl produces a lateral contraction $-\delta r$. Now:—

$$\frac{v+\delta v}{v} = \frac{(r-\delta r)(l+\delta l)}{rl} = \frac{rl - l\delta r + r\delta l - \delta r\delta l}{rl}$$

$$= \frac{rl + r\delta l - \delta r(l+\delta l)}{rl} = \frac{l+\delta l}{l}$$

approximately, neglecting the term in δr. Under these circumstances, i.e., when changes in length are allowed to take place at the expense of lateral movements, change of volume per unit original volume $\frac{\delta v}{v}$ in the elasticity formula (1) becomes change in length per unit original length $\frac{\delta l}{l}$, provided the appropriate coefficient of elasticity is used. This coefficient is that determined by stretching the wire statically so that lateral contraction can take place and keep the density unchanged, as in the dynamical conditions of vibration, and is known as Young's modulus.

A like method may be used to derive the formula for the longitudinal velocity in rods to that for the speed in gases. Let the longitudinal displacement at section P of co-ordinate x (Fig. 24) be ξ; at Q a little further along the rod, i.e., at $x+\delta x$, it will be, at the same instant, $\xi + \frac{\partial \xi}{\partial x}\delta x$, taking $\frac{\partial \xi}{\partial x}$ as the mean change of displacement

with x, over the slice between P and Q. Then the total elongation of

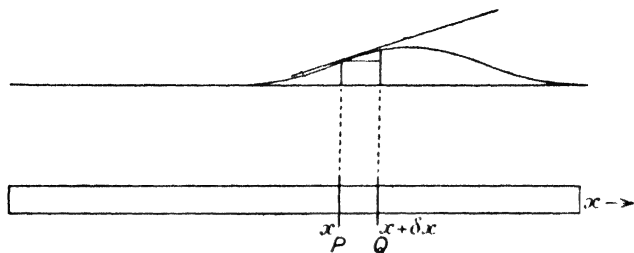



FIG. 24.—Velocity of Longitudinal Vibrations.

the layer is $\left(\xi + \frac{\partial \xi}{\partial x}\delta x\right) - \xi$, and since $PQ = \delta x$, the elongation per unit original length is $\frac{\partial \xi}{\partial x}$, so that, if Young's modulus for the material is $\mathbf{E}$, then $p = \mathbf{E}\frac{\partial \xi}{\partial x}$ (p being the total applied tension per unit area, or negative pressure on the slice). There is a force p per unit area at P directed to the left, and $p + \frac{\partial p}{\partial x}.\delta x$ at Q directed to the right, making a total of $\frac{\partial p}{\partial x}x$ on unit area of the material of the slice, but since above

$$p = \mathbf{E}\frac{\partial \xi}{\partial x},$$

then

$$\frac{\partial p}{\partial x}\delta x = \mathbf{E}\frac{\partial^2 \xi}{\partial x^2}.\delta x.$$

Now we may also express the resultant force on the material as the product mass $\times$ longitudinal acceleration, i.e., $\rho\delta x\frac{\partial^2 \xi}{\partial t^2}$, so that

$$\rho\delta x\frac{\partial^2 \xi}{\partial t^2} = \mathbf{E}\frac{\partial^2 \xi}{\partial x^2}\delta x,$$

or

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{\mathbf{E}}{\rho}\frac{\partial^2 \xi}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{cf. } 2)$$

A solution of this equation is :—

$$\xi = f_1(x - Vt) + f_2(x + Vt),$$

where V^2 is put for $\frac{\mathbf{E}}{\rho}$, and f_1 denotes any function of $(x - Vt)$, and

f_2 denotes any function of $(x + Vt)$. Now $(x - Vt)$ represents a point whose distance from the origin of x is decreasing at the rate of V cm. per sec., so that $f_1(x - Vt)$ is some form of displacement or distortion wave, which is propagated in the positive direction of x with the velocity V. Suppose the displacement ξ is produced at the instant t_0 at the section x_0 of the rod. At a later instant t_1 the function has the same value if x is increased by $V(t_1 - t_0)$, for then :—

$$f_1\{[x + V(t_1 - t_0)] - Vt_1\} = f_1(x - Vt_0)$$

showing that the same displacement ξ is now to be found at the section $x + V(t_1 - t_0)$, representing a transmission of this displacement, in fact, of the whole wave-form with velocity V in the direction of increasing x. Similarly, $f_2(x + Vt)$ represents a wave moving in the direction of diminishing x.

Now if f_1 and f_2 be simple sine functions of the variable, we have seen (p. 40) that two such waves travelling in opposite directions will produce stationary vibration in the rod, with places of maximum vibration (antinodes) and of no vibration (nodes). In particular a clamped section will form a node, and a free end will form an antinode. The position of the clamp will determine the possible modes of vibration of the bar. In practice, we find two types of "end conditions" as they are called, in the longitudinal stationary vibration of a bar. Firstly, the centre may be clamped and the ends left free, which is the state of affairs in the Kundt's tube experiment. Secondly, the bar may be clamped at two points, either at the ends or at certain positions symmetrically placed with regard to the centre.

Since the wave-length of stationary vibration is to be taken as twice the distance between two nodes (p. 40), we should have :—

$$n = \frac{\mathrm{V}}{\lambda} = \frac{1}{2l}\sqrt{\frac{\mathrm{E}}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

for the fundamental or lowest tone of the rod, l being its total length. This gives the frequency of the tone shown diagrammatically in Fig. 25(a and b).

At (c), $$n = \frac{1}{l}\sqrt{\frac{\mathrm{E}}{\rho}}$$

and at (d) $$n = \frac{3}{2l}\sqrt{\frac{\mathrm{E}}{\rho}}.$$

The full "harmonic series" of tones corresponding to frequencies

1, 2, 3, 4, etc. times the fundamental, can thus be obtained by suitably adjusting the position of the two clamps.

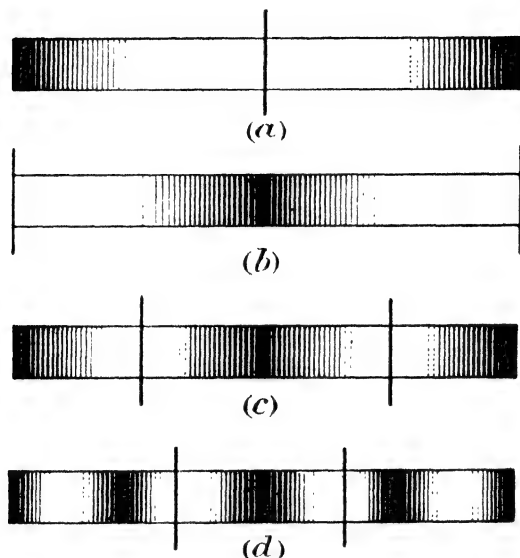



FIG. 25.—Stationary Longitudinal Vibrations of a Bar.

Experimental Methods. Beside metal, glass and wooden rods or tubes of square or round section, wires and rubber cords may be used for the study of longitudinal vibrations. Apart from the Kundt's tube method of determining the velocity of the waves (using air as the standard), it is possible to investigate the position of nodes and antinodes in the solids themselves and to verify the conclusions of the preceding section.

To produce the tones, the usual method is to surround the rod by a cloth (resined for a wooden rod, damp for a glass or metal rod), held not too tightly in the hand and then drawn along the rod in the neighbourhood of an antinode. In "strings" it is possible to get these tones by rubbing a resined bow along the string, in cords by a simple longitudinal displacement. In the latter cases the material must perforce be clamped at the ends, and the tones are usually adulterated with transverse vibrations. These are sometimes present as low notes when the longitudinal tones of rods are carelessly produced by friction--the low inharmonic note resulting is called "*son rauque.*" Unlike the transverse, the pitch of longitudinal vibrations

is independent of the tension along the string. Altberg [1] has shown that it is possible to maintain these tones in a rod by a revolving wheel whose rim is covered with resin, and which continually rubs upon the rod near one end.

The nodes on a rod of square or flat section can be shown by strewing dust, which collects at the places of no motion on the rod. When the rod is of transparent material the vibrations can be studied by a much finer method based on a discovery of Biot.[2] He placed the rod in the polarized light between crossed mirrors, so that the light on reflection from the second mirror was completely extinguished. On exciting longitudinal vibrations in the rod, the light reappeared and remained as long as the tone lasted. Mach [3] used the light which

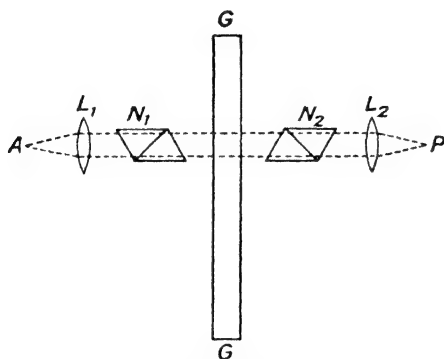



FIG. 26.—Longitudinal Vibrations in a Rod, by Polarized Light.

passed between two crossed Nicol prisms, one on each side of the rod, obtaining in the spectrum of the transmitted light interference bands, which oscillated with the changes in density of the section when the rod was sounding. If the material can be shaped in the form of a long rod, a pulse may be sent along it by a sudden application of tension or pressure at one end. If two polarized beams of light pass through the rod at distant points, they will be successively disturbed by the passage of the pulse, so that a chrono-photographic record of the disturbances will give the time interval between them. In this way Herbolsheimer [4] measured the speed of longitudinal waves in gelatine. Fig. 26 shows schematically the arrangement of the Biot phenomenon. Light from an arc A, made parallel by the lens L_1, passes through the first Nicol N_1, then through the rod GG, the second Nicol N_2, and on to a screen or plate at P. N_2 is first turned to

extinguish as much as possible the light getting through N_1 and GG. On exciting longitudinal vibrations in the rod, light periodically reappears at P.

Davis [5] and also Clark [6] have investigated the tones of continuously rubbed strings by observing the motion of bright points on the strings under a microscope, the string being rubbed by a wheel. The patterns observed conformed to those of strings bowed in the usual manner, i.e., transversely (cf. p. 90), discontinuities in lateral displacement being replaced by changes in density along the strings.

Torsional Vibrations of Rods. The formula which we have already deduced for the velocity of propagation of longitudinal compressions in bodies can also be applied to vibrations produced after giving a twist to one part of an elongated solid, provided the appropriate elasticity be introduced. This elastic force is commonly known as the rigidity (**N**), and is determined from the twist (ϕ) produced at an end by an applied torsional couple, just as Young's modulus is determined from the extension (ξ) produced by an applied longitudinal tension. The formulæ are therefore:

$$\frac{\partial^2\phi}{\partial t^2} = \frac{\mathbf{N}}{\rho}\cdot\frac{\partial^2\phi}{\partial x^2},$$

$$V_{\text{tors}} = \sqrt{\frac{\mathbf{N}}{\rho}}$$

for the velocity, and for the fundamental tone (centre fixed)

$$n = \frac{1}{2l}\sqrt{\frac{\mathbf{N}}{\rho}} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Similar divisions into nodes and antinodes corresponding to Fig. 25 can be obtained by suitable clamping, and shown by sand, if the top is plane. To obtain the tones from such square-section rods one usually employs two violin bows, drawn across in opposite directions, one above and one below the section. As may be imagined, this gives an admixture of transverse and possibly of longitudinal vibrations, therefore Grögor [7] employed a rotating wheel rubbing the end of the rod. With care it was possible to get a note sufficiently pure for pitch estimation. Comparing the fundamental tones of the same rod for longitudinal (36) and torsional (37) vibrations the ratio of the extensional and torsional elasticities of the material can be found. Torsional tones are of little importance in practice.

There is renewed interest in the longitudinal and torsional tones of

short quartz rods (excited by the piezo-electric effect) and steel or nickel rods (excited by magneto-striction). Further details will be found in Chap. X. These tones may also be excited in rectangular rods by a blast of air directed on to a suitably chosen sharp edge, the "edge tone" (cf. p. 157) so produced being tuned to the desired tone of the rod.[8]

References

1. *Ann. Physik*, **11**, 405, 1903.
2. *Physique*, **2**, 15, 1828.
3. *Ann. Physik*, **146**, 316, 1872.
4. *Zeits. Physik*, **3**, 182, 1920; *see also* Srivastava, *J. Acoust. Soc.*, **23**, 232, 1951.
5. *Proc. Amer. Acad. Sci.*, **41**, 691, 1906.
6. *Phys. Rev.*, **7**, 561, 1916.
7. *Phys. Zeits.*, **15**, 788, 1914.
8. Kroncke, *Zeits. tech. Phys.*, **13**, 196, 1932.

CHAPTER FOUR

TRANSVERSE VIBRATIONS OF STRINGS AND RODS

Properties of the Theoretical String. The string is ideally a body having length only, infinitely thin, able to be bent laterally in transverse vibration without bringing into play viscous forces in the material. In so far as natural wires and threads fail in this last respect, we say that they possess "stiffness." In a rod or bar this resistance to bending is the actual cause of the vibration, as it constitutes the restoring force to a displacement. In the string, the tension between the particles, due to the applied tension at the ends of the wire, takes the place of this resistance to bending. In the absence of applied tension, vibration will not take place on displacing laterally a point on the string.

Velocity of Transverse Waves in a String. A string may execute stationary vibration having nodes at the fixed ends, and any number of loops between. The fundamental will be that vibration corresponding to a single loop stretching from end to end. As we have seen, this type of motion may be regarded as made up of two oppositely directed transverse progressive waves moving along the string. It remains to calculate the velocity of such a wave.

Let AB (Fig. 27) represent a portion of the displaced string δx in length, the end A being at a distance x from one end of the string, and its displacement from the undisturbed position A', being y. In the absence of stiffness the stretching force F will be the same throughout the string, but as it acts tangentially at every point its inclination will vary along Ox. If the tangent to the string at A makes an angle θ with Ox, the component of the tension in the direction AA' will be $F \sin \theta = F \tan \theta = F\frac{\partial y}{\partial x}$ when θ is small. The component at B along $B'B$ will then be

$$F\left[\frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)\delta x\right]$$

The net force on AB tending to increase its displacement is therefore $F\frac{\partial^2 y}{\partial x^2}\delta x$ which can be equated to the mass $\times$ acceleration, i.e., to $m\delta x\frac{\partial^2 y}{\partial t^2}$, where m is the mass of unit length of the string.

Finally
$$\frac{\partial^2 y}{\partial t^2} = \frac{F}{m}\frac{\partial^2 y}{\partial x^2}$$

in which we recognize, by comparison with equation (2), p. 3, the velocity of the waves as:

$$V = \sqrt{\frac{F}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Knowing the velocity of transverse waves in the string, we can determine the frequencies corresponding to the wave-length of each possible segmental division of the string of length l, from $\lambda = 2l$ for the fundamental, through all the harmonics or "partial" vibrations of the string. The number and magnitude of these partials is determined by the manner of exciting the string.

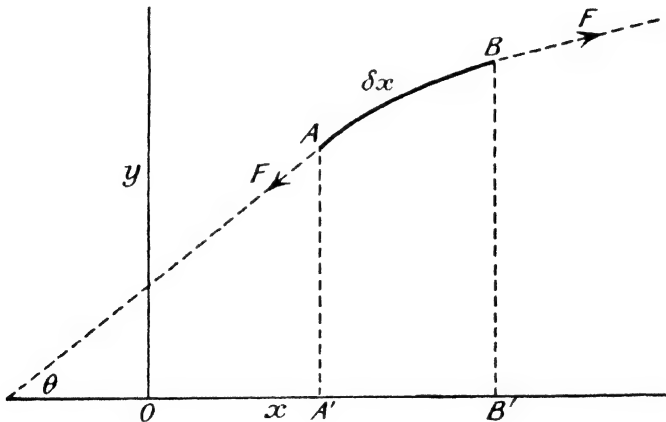



FIG. 27.—Velocity of Transverse Waves in a String.

Apparatus for Strings. The laws expressed by the formula (38) were known in part to the ancients, but the first quantitative verification appears to have been made by Mersenne [1] in 1635. At the beginning of the eighteenth century Sauveur [2] stretched a string between two bridges, applying tension by a weight at the end which hung over a pulley. The experimenter observed the nodes and antinodes ("*nœuds et ventres*") which were formed when the string was vibrating in aliquot parts so as to produce an overtone.

His apparatus was the precursor of the modern monochord or sonometer, which generally holds two wires stretched either horizontally

or vertically between two bridges. One wire of fixed length, screwed up to a certain tautness, gives a standard tone when gently displaced at the centre. In the horizontal form, the other wire passes over a pulley to a scale-pan in which weights are placed. This wire passes over a movable bridge, and the sounding length between the fixed and movable bridges is adjusted to unison with the standard wire. Two methods are available to test whether this condition has been attained. Either the two wires may be sounded together, and the adjustment made so that the beats between them vanish; or use may be made of the resonance principle in the following manner. The standard wire alone is excited, and the second wire, if in tune, vibrates sympathetically so strongly as to throw off light paper riders hung on it for that purpose. In order to increase the "volume" of the sound, as in most stringed instruments the sound-board consists of a hollow box, the air in which is forced into vibration to a certain extent by the string. The vertical form of monochord is perhaps to be preferred, as the friction on the pulley is banished and that on the bridges lessened.

Stiffness of Wire. The ideal string has no stiffness; a rod is a body in which this quality is of prime importance. In an actual wire stiffness plays a definite though subordinate part. Savart [3] first observed that such stiffness could permit a vibration (n_0) even in the absence of tension, and proposed an empirical formula for the actual frequency (n_1) in terms of the ideal frequency (n); $n_1^2 = n^2 + n_0^2$. Rayleigh [4] has pointed out that stiffness alters the "end conditions," the conditions $\frac{\partial y}{\partial x} = 0$ and $\frac{\partial^2 y}{\partial x^2} = 0$ being no longer completely fulfilled at a rigidly clamped end. The various formulæ proposed agree in ascribing to the stiffness a rise in frequency above that given by the simple theory, a rise which is proportionally greater as the frequency grows, so that the partial tones no longer form a harmonic series.

It may be pointed out here that another cause may lead to the violation of the end conditions: the yielding of the end supports, which has the effect of increasing the length of the wire.

Experimental Study of Transverse Vibrations. Before proceeding with the detailed description of the types of vibration of strings under different modes of excitation, it will be as well to describe experimental methods of accurately examining the frequency and amplitude of such vibrations. The methods to be described can be adapted with but little alteration to all transverse vibrations of solid

bodies; in particular to rods. The methods divide themselves into two classes; graphic and stroboscopic. The former are better adapted to complex vibrations containing a number of partials; the latter to a simple harmonic motion.

Graphic Methods. In the simplest of these, the vibrating body carries a style which traces a mark on a piece of paper, which is moved along at right-angles to the direction of vibration. The trace is therefore a wavy line, which in the simplest case of S.H.M. corresponds to the sine wave of Fig. 14, if the paper moves past the style at constant speed. Either the speed of the paper is known, or else a subsidiary time-marker marks dashes at constant intervals of time on the paper alongside the trace of the vibration. In the former case two methods are in vogue. In the one, the strip of paper is wrapped round a drum which is turned by a motor rotating at a constant known speed (usually one of the so-called "phonic motors," cf. p. 116). If then m waves are traced on a length l of the paper, wrapped on a drum of circumference $2\pi r$, rotating at n revolutions per second, these m vibrations occupy a time of $\frac{l}{2\pi rn}$ seconds, whence the frequency of the wave motion may be calculated. In the other, a smoked glass plate is allowed to fall under gravity past the style, which is made to vibrate horizontally by the action of the fork. In consequence of the accelerated fall of the plate, the waves traced are crowded together at first, but gradually open out. If the number of waves between any two points be counted, the distances l_1, l_2 measured from the start, the times t_1, t_2 taken by the plate to fall these distances are found from the formulæ $l_1 = \frac{1}{2}gt_1^2$, $l_2 = \frac{1}{2}gt_2^2$. Hence the time taken for the plate to fall the distance over which the waves have been counted

$$t_2 - t_1 = \sqrt{\frac{2}{g}}(\sqrt{l_2} - \sqrt{l_1}).$$

When the speed of the paper or plate cannot be kept constant by some automatic device, it is preferable to use a time-marker. This may be a suitably arranged metal pendulum, adjusted to beat seconds or some convenient and accurately known interval, the time-period of the pendulum having been determined by comparison with a chronometer. The bob of the pendulum P just dips into mercury in the trough T as it crosses its lowest position, and so establishes a current through the electro-magnet M (Fig. 28) which attracts a spring marker S. The marker leaves momentarily its place of rest, and makes a

"jag" in the straight line which it has been tracing on the drum. A number of cross lines are thus marked on the drum corresponding to intervals equal to half the time of swing of the pendulum. The figure shows diagrammatically this method applied to the transverse vibrations of a bar B clamped at one end C. A style S' has been fixed to the other end of the bar, which traces a sinuous line upon the drum when the latter is rotated, and the bar lightly displaced. As might be expected, the graphic method is more readily applied to a body of large mass, and is not much used for thin strings, where the friction and inertia of the style would appreciably affect both the amplitude and frequency of the motion it is desired to study.

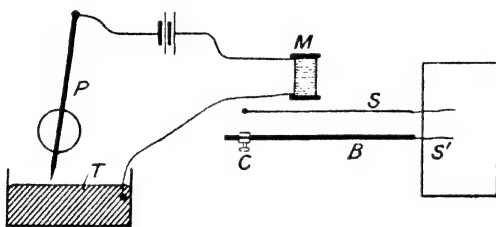



Fig. 28.—Graphic Method Applied to Vibrating Bar.

Photographic Method. When a more permanent record is desirable, a camera is resorted to, and this device has the advantage that the apparatus, if any, which has to be attached to the vibrating body is small and offers the minimum of damping to the motion of the body. In the simplest type, a small mirror is stuck to the part of the body where the movement is large, and this reflects a beam of light from a powerful and concentrated source—like the Pointolite lamp —on to a camera of the "moving plate" variety. Such a camera, which can be simply made and worked, is shown, with the back removed, in Fig. 29.

P is the plate holder and dark slide from an ordinary quarter-plate camera. To this are attached two long loops of elastic cord of equal length, passing over hooks at each end. Under the tension of these cords the holder is normally held in the centre of the camera behind the shutter S. Just before taking a photograph, P is pulled to one end and held by a clip; the shutter S is adjusted to one of three positions, so that the opening will be over one-third of the plate as it shoots past. The reflected spot of light is adjusted to vibrate vertically across this hole. The back of the camera is clipped on, the slide

pulled out so as to expose the plate (it is possible to do this by introducing the hand through a flap in the back), the catch is released and the plate shoots along grooves, past the shutter, until it is brought violently up against the far end with sufficient force to reclose the slide upon the plate ; the holder can then be removed from the camera by taking off the back. If the cords are of equal length there should be no force acting on the holder as it passes *S*, and consequently its velocity at this part of its path should be constant. As this velocity is not known, however, it will be necessary to mark the time in some way ; this is most conveniently done by allowing the bob of a pendulum to cut off the light periodically, leaving a gap in the record of the plate ; the width of this gap is governed by the size of the pendulum bob which is suited to the speed of movement of the plate, which

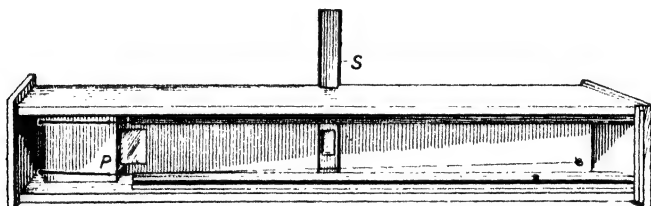



Fig. 29.—Sliding Plate Camera.

speed is, in its turn, determined by the tension put upon the elastic cords.

If the camera can be set up vertically, and the deflection of the beam of light be horizontal, it is often better to do without the cords, and let the plate holder fall through the guiding grooves under gravity. A plate such as is used for flashlight photography is suitable.

In an alternative form of camera a length of cinematograph film is wound round a wooden drum, to which it is clipped, mounted on an axle in a light-tight box. The axle is rotated by a motor and the shutter opened for the time of exposure desired. A tripping gear closes the shutter just before the completion of one revolution of the drum in order to avoid double exposure of the film.

When the vibration to be studied is that of a thin wire it is better to dispense with a mirror, and to place the wire directly in the path of the light, so that it casts a shadow on the plate. The technique of this device has been considerably extended in connection with the Einthoven string galvanometer, where the forced vibrations of a wire

carrying an unsteady electric current in a magnetic field have to be recorded. It is obvious that in any photographic apparatus of this kind the instantaneous or steady projection of the vibrator on the plate must be either a bright spot on a dark background, or a black dot on an illuminated ground. To transform the shadow of our wire from a line to a point, cylindrical lenses are necessary. The arrangement is shown in Fig. 30.

The light from the source *S* is concentrated upon the wire *W*, and the beam made parallel by a second spherical lens; this would give a shadow of the wire in the plane of the paper on the plate, but the cylindrical lens *C* having its axis perpendicular to the paper reduces the shadow to a spot at *P*. The wire is arranged to vibrate in a plane perpendicular to the paper, so that if the plate *P* be shot in the direction of the arrow, the required trace will be made upon it by the spot. The time-marker *T* interrupts the light periodically, as explained.

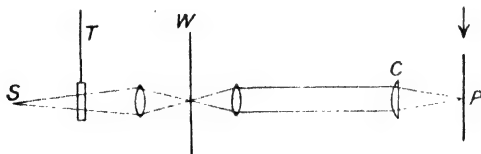



Fig. 30.—Optical System for Photographic Record of Transverse Vibrations.

The Stroboscopic Method. A method which has been of the greatest assistance in studying periodic motion of all kinds, seems first to have been conceived by a number of scientists early in the nineteenth century, but not used for exact measurements till the time of Toepler.[5] The method can be employed in two ways. Either, the vibrating body is illuminated intermittently, or else glimpses of the motion are obtained intermittently. If now the intermittence coincides with the period of the motion, the body will always be seen in one phase, and so will appear motionless. If we can determine the frequency of the intermittence of the light, this will give us that of the motion. If the period of the intermittence is slightly longer than that of the motion, each glimpse will be a little later in phase than the last, and show the body in a somewhat later epoch of its period. In other words, in spite of its actual rapid vibration, the body appears to move slowly to and fro, enabling the vibration to be viewed at leisure.

Intermittent vision and illumination can be furnished by a disc with radial slits (equally spaced round the circumference), passing in

front of the eye, or in front of a light shining on the body. The former is the more usual arrangement. The disc (στρόβος, a top) is fixed to the shaft of an electric motor whose speed is regulated by a resistance. The speed of the disc in revolutions per second multiplied by the number of slits when the stationary appearance is attained, gives the required frequency. The speed of the disc can be read on a cyclometer fastened to the motor shaft, or, more accurately, by another stroboscope. To this end, a pattern or set of patterns is inscribed on the disc, and the pattern is viewed through an interrupter of standard frequency, or is illuminated at fixed intervals. The latter may be adapted from a tuning-fork by attaching two pieces of copper plate to the ends of the prongs in such a fashion that their inner edges are parallel and just opposite each other when the fork is at rest.

This stroboscopic vibrator is placed in front of the disc with the

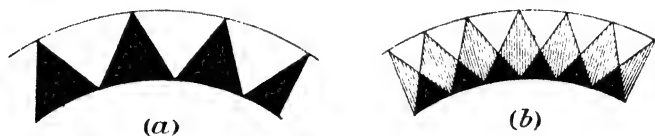



FIG. 31.—Stroboscopic Appearances.

slits the rate of revolution of which is to be determined. The disc recommended for use with the vibrator has inscribed on it a number of concentric rings and equi-spaced black triangles in each ring, whose number increases from 15 to 20 as the rings range outwards. Suppose that the vibrator gives n glimpses per second, and that through it the ring containing p triangles appears stationary. Then between two openings of the slit of the vibrator, the disc has moved on one p'th of a revolution, or makes one complete revolution in p glimpses, occurring in $\frac{p}{n}$ seconds; therefore the rate of revolution is $\frac{n}{p}$ per second. Now n is a standard frequency, and p is known, so that we have found the rate of revolution of the disc. It is not necessary that the disc shall have moved on only one p'th revolution for this ring to appear stationary; the disc might have moved on $2/p$'ths, $3/p$'ths or any integral number of p'ths between the opening and re-opening of the slit, but in that case the disc will be moving at the corresponding multiple of $\frac{n}{p}$ revolutions per second; if, on the contrary, the disc is

travelling half as fast as $\frac{n}{p}$ revolutions per second, then each triangle moves only half the distance between two teeth, so that there appear to be $2p$ triangles standing still in this ring, but overlapping as in Fig. 31*b*. At a speed $\frac{n}{3p}$ revolutions per second there will appear $3p$ triangles stationary. Thus, concentrating our attention on the p ring and starting with the stationary appearance when the disc has moved on only $1/p$'th between two glimpses (this we may call the 1st position), accelerating the disc we shall come to the 2nd position at double this speed, the 3rd position at treble speed, etc. Decreasing the speed from the first position we come to the $\frac{1}{2}$ stationary position at $\frac{1}{2}$ speed, $\frac{1}{3}$ position at $\frac{1}{3}$ speed, etc. Stationary patterns can also be observed at the $\frac{3}{2}$, $\frac{5}{4}$, position etc. Thus if m be the number or order of the position, the speed is $\frac{mn}{p}$ revolutions per second.

It appears that if we restrict ourselves to the observation of stationary positions only, we are limited as to accuracy by the relative closeness of the number of teeth per ring. Usually we shall find no particular set of rings standing still on looking through the slit of the vibrator, but that the outer rings are moving (apparently) in one direction, and the inner rings in the opposite; the stationary position would correspond to some number of teeth lying between the adjacent rings which are moving slowly in opposite directions. We can however work out accurately the speed of the disc by noting the rate at which one of these slowly moving rings "slips" round the disc. For example, one of the triangles on the p ring may be moving in the same direction as the disc and pass completely round in q seconds. This number q may be counted and called the "slip." Instead of the former movement of $\frac{mn}{p}$ revs. per sec., we have now an increased movement of $\left(\frac{mn}{p} + \frac{1}{q}\right)$ revs./sec. Note that if $q = \frac{p}{n}$ we have moved up one position in the scale of stationary rings. Also if a number of rings of teeth are provided, two sets may be simultaneously still, but at different orders. Thus the 4th position in the 20 teeth ring will appear stationary with the 3rd position on the 15 teeth ring, because $\frac{4}{20} = \frac{3}{15}$. This often gives a cue as to the appropriate value of m.

Let us return to the complete apparatus, and take a numerical example. Suppose the stroboscopic disc has 4 slits and is adjusted to the fastest speed at which a vibrating string, seen through the slits appears motionless and single. The vibrator, giving 100 glimpses per second, is now set in motion in front of the disc, and one sees, on looking at the disc through

the chink between the vanes, that the 16 teeth ring is moving slowly in the backward direction at the rate of one revolution in 50 seconds. What is the frequency of the string? We must first find which order of position on the 16 teeth ring we have. On temporarily decelerating the disc, we find it passes through 3 more positions at which this ring is motionless, before it appears as 32 teeth overlapping (half position). We, therefore, had the 4th order of the 16 ring; this may be checked by noting that the 20 ring is slipping slowly backward in the 5th position ($\frac{4}{16} = \frac{5}{20}$). Neglecting the slip, the speed of the disc would have been $\frac{4}{16} \times 100 = 25$, but with the slip of 50 back, the speed is less than this by 1/50 rev., giving 24·98 revs./sec. as the speed of the disc. Finally, as there are 4 slits on the disc, the frequency of the string = 99·92 vibns./sec.

Other methods for obtaining intermittent illumination may be used. The vibrator described above may itself be so used if a beam of light is focussed on the aperture between the vanes, and then allowed to fall upon the disc, which it illuminates. In the other type, originally due to Michelson,[6] the lamp itself flashes intermittently, being of the discharge-tube pattern, such as the well-known Osglim lamp. Connect one of these across the secondary coil of a small induction coil, whose primary coil has a make-and-break of a vibrating reed or tuning fork electrically maintained, and it will provide sufficient light for observing the disc, if the latter is shaded from direct daylight by a hood, or if the whole apparatus is in a darkened chamber. Lately attempts have been made to improve the intensity of such stroboscopic illuminations.

When it is desired to study a periodic motion in detail, it is necessary, as explained above, to have a slow " slip " between the motion and the illumination. An instrument which obviates the necessity of suitably controlling the speed of the stroboscopic disc for this purpose, is the Oscilloscope; in this apparatus a motor is made to run in synchronism by the motion to be studied itself. This motor controls the flashing of the lamp, but the phase of the flashing is continuously advancing on that of the motor, producing the necessary slip, and giving the " slow motion " appearance required for detailed study.

Stroboscopes without Intermittent Light. A number of instruments have been designed, which rely on the peculiar properties of the eye to give a stroboscopic effect; in these, there is no vibrator or flashing lamp. Such an instrument is the apparatus of Mikola,[7] particularly suited to vibrating strings. An image of the centre of the string is projected on to a revolving drum, which contains a

number of white stripes perpendicular to the wire, so that as the latter vibrates, a dark point representing its centre moves up and down the stripes. If the drum is set in rotation, the position of this image on successive stripes follows the motion of the wire, but if the frequency of the stripes (meaning the number of stripes which pass a given point in a second) coincides with that of the wire, successive stripes will always receive the shadow of the wire in the same position. In consequence of this synchronism, and of the after-image effect in the eye, the wave-form of the wire will then appear stationary on the cylinder, whereas, if the latter is turned a little slower or faster, the wave will appear to progress slowly round the cylinder.

Methods based on Rectilinear Vibrations. In Chapter II it was shown that two vibrations compounded together at right angles produce figures whose form can give us information with regard to the periods of the components. Lissajous himself applied the figures formed by a standard tuning-fork together with another transverse vibrator, e.g., a wire or another fork, to the elucidation of the wave-form of transverse vibrations; but, as the figures can be recognized only when a simple relation exists between their component frequencies, they are of very limited application, and the method of obtaining them will be briefly dismissed. Lissajous [8] fixed the object glass of a microscope to the standard fork and oscillated it, keeping the eyepiece firmly fixed. A bright point then appeared drawn out into a line; if the bright point were on a body moving to and fro in a direction at right angles to the motion of the object glass, one or other of the figures was formed in the microscope. This apparatus Lissajous called the Vibroscope. An earlier apparatus on the same principle, the Kaleidophone of Wheatstone, consisted of a metal strip twisted at its mid point so that it formed two parts in planes at right angles, one of them being clamped firmly at the end. The twisted strip was pulled aside and the two parts allowed to swing in rectangular directions; the remaining unclamped end showed the figures, as it was constrained to follow two vibrations at right angles, its own vibration, and that imposed by the clamped part of the strip.

Chattering Method. This method, due to W. H. Bragg,[9] is particularly useful for measuring very small amplitudes such as that of a stretched membrane in vibration. Let *DD* (Fig. 32) represent the body under investigation and let a system consisting of the mass *M* on the end of a spring *S*, whose other extremity is fixed in the mas-

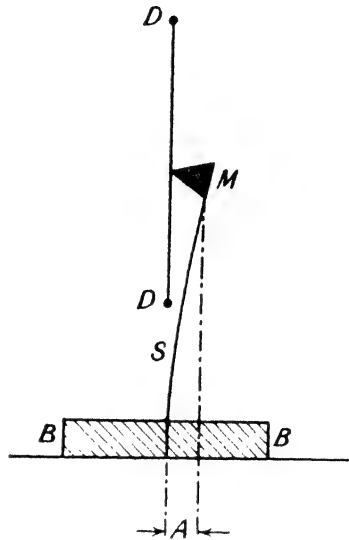
sive block *BB*, be pushed up until the tip of *M* just touches the body *DD*. "Chattering" will ensue, the noise being due to periodic contact between the two systems. The acceleration in the motion of *DD*—whose maximum value is $a\omega^2$ if the equation of its motion is $y = a \sin \omega t$—will throw the mass off, unless the restoring force exerted by the spring is equal to or greater than the ejecting force. The restoring force can be increased by pushing the block *BB* through a further distance *A* towards *DD*, thus bending the spring. In the position shown in the figure, the mass *M* would, if the obstruction of *DD* were removed, execute vibrations given by $y = A \sin \omega_0 t$ in which the acceleration at maximum amplitude, i.e., the actual position, would be $A\omega_0^2$. If then the vibrating system just fails to throw the mass off so that the two move without gap, we must have

FIG. 32.—Chattering Method.

$$a\omega^2 = A\omega_0^2 \text{ or } a = A(\omega_0/\omega)^2.$$

The shift *A* and the ratio of the natural frequencies must therefore be known in order that the amplitude *a* can be determined; ω_0 will, of course, be small compared to ω.

Yet another method for vibrations of small amplitude is that used by Backhaus [10] for an examination of the vibration of sound boards, violin bellies, etc. This consists in sticking a strip of metal foil on the part to be examined and combining it with the nearby metal plate to form a small electrical condenser. The oscillatory changes in capacity caused by the changes of thickness of air gap, while the body is vibrating are measured in a valve circuit such as that of Fig. 50 (p. 134).

Plucked Strings. This is the simplest of the modes of exciting transverse vibrations in strings, which will now be considered in detail. The string is pulled aside by a finger or by a "plectrum" at a particular point, and then let go. From the formula already obtained

for the velocity of small transverse vibrations in strings $V^2 = \frac{F}{m}$ (cf. eq. 38) we can set down for the differential equation of the motion :—

$$\frac{\partial^2 y}{\partial t^2} = V^2 \frac{\partial^2 y}{\partial x^2} = \frac{F}{m} \frac{\partial^2 y}{\partial x^2}.$$

The solution can be written in the form $y = f_1(x - Vt) + f_2(x + Vt)$ representing the two waves travelling in opposite directions, which compose the stationary vibration of the string (cf. p. 66). More explicitly, the equation of the vibration of the string executing a fundamental vibration can be written (after the style of equation (15) Chap. II)

$$y = a \sin \frac{2\pi}{2l} x \cos 2\pi n t \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$= a \sin \frac{\pi}{l} x \cos \frac{\pi V t}{l},$$

or for a partial tone of the j^{th} order :—

$$y_j = a_j \sin j \frac{\pi}{l} x \cos \frac{j\pi V t}{l},$$

since the distance between successive nodes $\left(\frac{\lambda}{2}\right) = \frac{l}{j}$, and $V = n\lambda$. For a note of complex quality, the complete Fourier series is :—

$$y = \sum_{j=1}^{j=\infty} \sin j \frac{\pi}{l} x \left(a_j \cos \frac{j\pi V t}{l} + b_j \sin \frac{j\pi V t}{l} \right). \quad . \quad . \quad (40)$$

the maximum amplitudes of each partial $\sqrt{a_j^2 + b_j^2}$ being obtained from the Fourier integrals representing a_j and b_j. The quality of the note of a plucked string—i.e. the number and variety of the harmonics—depends on the form into which the string is bent before being released, and this again depends on the form of the plectrum. To take an extreme and unpractical case, if the string were bent into a bow of sine-wave pattern and suddenly let go, it would execute the simple fundamental tone of frequency $\frac{1}{2l}\sqrt{\frac{F}{m}}$, according to equation (38). A type which lends itself to scientific study and approaches closely to conditions in certain musical instruments is that in which the string is pulled aside at a single point. An extensive experimental study of this case has been made by Krigar-Menzel and Raps,[11] while

it has received theoretical treatment at the hands of Helmholtz.[12] In the experimental research, light from a narrow slit fell across a wire and was projected on to a rotating drum carrying a sensitized paper strip. The wire executed transverse vibrations along the line of the slit, while the sensitized paper moved at right angles to the slit. Thus when the wire moved the shadow of a given point moved across the bright image of the slit on the drum, and when the drum revolved a sinuous trace was made on the paper by the dark spot, representing the transverse motion of the point on the string, extended

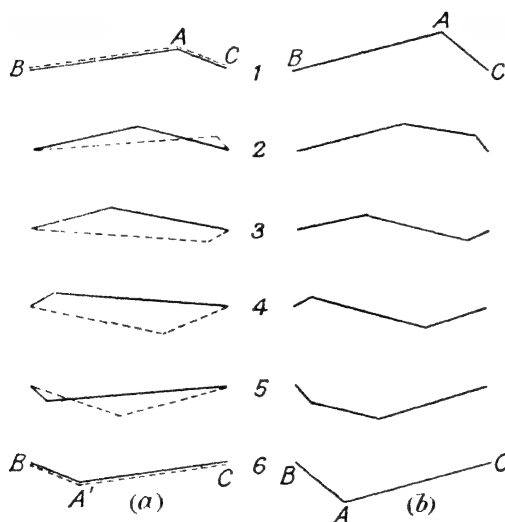



Fig. 33.—Displacement Curves of Plucked String.

on a time axis. Both point of plucking and point of observation were varied. From a set of photographs, dealing with a fixed point of plucking, it was deduced that the wire went through a form of stationary vibration, corresponding to the figure above, representing half a complete period.

The string is plucked initially at A into the triangular shape BAC (Fig. 33*a*). On releasing the string, the bend at A is levelled out by two discontinuities travelling down AB and AC respectively; on reaching the ends of the string, these are reflected as "humps" on the other side of the mean position, meeting again at A', such that $BA' = CA$; and $CA' = BA$. Finally the cycle is completed by a return to the position BAC. At no moment does this string occupy

its undisplaced position; this will only occur when all the partials which make up the vibration are in phase, when, e.g., the string is plucked at its mid point. If X is the distance of the point measured from B, and Y the extent, of the plucking, we have initially $y = \frac{Yx}{X}$ for all values of x up to X, and $y = Y\frac{(l - x)}{(l - X)}$ for values of x between X and l.

We can represent the subsequent motion as the superposition of two discontinuities given by equations having half the amplitudes of the above, and reflected with complete change of phase of π at B and C, the fixed ends of the string. The manner in which these add up to the observed vibration is shown in Fig. 33b. It is possible to obtain coefficients for the Fourier series to represent a "bent" wave of the type of one of these. Such analysis shows that the series contains all the higher partials from $j = 1$ to $j = \infty$, but with amplitudes rapidly diminishing in the inverse ratio of the square of the order of the partial, i.e., as $\frac{1}{j^2}$. Owing to this rapid diminution of amplitude a finite number of partials will in practice suffice to represent the motion.

Krigar-Menzel and Raps found that the effect of plucking the string over a considerable length—instead of at a single point, as by a sharp edge—was to "round off" the sharp corners in the zigzag form of wave. The more the corner is rounded off the less the number of Fourier partials required to make up the wave; in the extremes, a true zigzag, with a discontinuity at the corner, requires an infinite series of components, whereas the true sine wave requires only one. It was observed that the wave-form changed slightly from period to period after releasing the wire. This was ascribed to movement of the ends of the string, due to the bridges not being rigid. The yielding of the bridges of a stringed instrument has another effect; instead of the sounding length of the string ending at the bridge, it is continued a little beyond, in other words, the string vibrates about a point a little beyond the bridge, instead of about a point on the bridge itself, when the latter partakes to a small extent in the vibration. Consequently, all the tones of such a string are slightly lowered by the yielding of the bridge, but all to the same extent, so that the series forming the "note" remains harmonic.[13]

Harp and Banjo. Stringed instruments excited by plucking may

be divided into two classes; those, of which the harp is the architype, having strings of fixed length between two stops or bridges; others, of which the banjo is a common example, in which the pitch is varied by the player "stopping" the strings at different points so as to change the sounding length.

The harp consequently requires a separate string for every note of the musical scale. To reduce space and expense in the modern instrument, strings are provided for playing in one "key" only (C flat major). To enable the player to play in other keys, Erard added a device by which each string can be shortened or lengthened by a small fraction, so raising or lowering the fundamental by a tone or semi-tone; the alteration being applied simultaneously to each of the seven lowest notes and to all its octaves by one of seven pedals. The strings are plucked by the fingers; and if the octave of the fundamental is to be elicited they are touched at the same time in the middle with part of the palm of the hand.

The strings of the banjo are usually plucked by the finger-nail or by a pointed piece of horn (plectrum). As a consequence the resulting waves contain a large number of higher partials, and the tone is more brilliant, but also more metallic, than that of the harp. The points at which the finger is to be placed for shortening the effective length of the wire are regulated by a series of frets, placed along the finger-board over which the strings are stretched. These instruments are provided with a sound box, which by forced vibration amplifies the sound of the strings alone. In spite of this the tone is weak, so that the notes are generally reiterated by repeated touch.

Bowed Strings. The first theoretical and practical investigation of the bowed string came from Helmholtz,[12] who by means of the vibroscope observed the Lissajous figures produced by the motion of the mid-point of the string in combination with a tuning-fork. He found that the form of stationary vibration showed discontinuities similar to those shown in Fig. 33 by a plucked string. A complete experimental study of the vibration curves was undertaken by Krigar-Menzel and Raps, with the apparatus already described. They found that only in special cases were these of the pure zig-zag form; in most cases bending of the theoretical straight lines is apparent.

There are three laws which arose from their investigations as follows.

Young's[14] *Law.* "No overtone is present which would have a node at the point of excitation."

Helmholtz' Law. "When a string is bowed at an aliquot point $\left(\frac{1}{k}\right)$, the part of the string immediately under the bow moves to and fro with [two] constant velocities, whose ratio is equal to the ratio $\frac{1}{k-1}$ of the segments into which the string is divided by the point in question. The smaller of these two velocities has the same direction as that of the bow, and is equal to it."

Krigar-Menzel's Law. "When a string is bowed at any rational point p/q, where p and q are prime to each other, the part of the string immediately under the bow moves to and fro with [two] constant velocities whose ratio depends only on q, and is $\frac{1}{q-1}$."

These laws were established mainly on an empirical foundation, but serve to determine the motion of a bowed string in most circumstances. Young's law applies to strings stretched between sharp bridges in whatever way excited. The other laws deal with the motion of the bowed point, the third being more general, as Helmholtz considered the motion only for the case where the string was bowed at $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of its length from one end. The sudden velocity changes at the bowed point determine discontinuities in the velocity waves which pass over other parts of the string, and so determine the motion of all points in the string, in a manner which will be considered in detail presently.

The action of the bow upon the string as envisaged by Helmholtz and substantiated by experimental tests of the necessary consequences, is then as follows. During part of the period, the bow pulls the string aside with it, so that they have no relative velocity, and the full static frictional force is exerted between the string and the resined hairs of the bow. The tension in the strings is initially at right angles to the frictional force, but as the bowed point is pulled aside, the two parts of the string make a diminishing acute angle with the direction of movement of the bow, and consequently the tensions in the two parts have increasing components in this direction, opposing the frictional force. When these components become together greater than the friction, the string ceases to adhere to the bow. String and bow now possess a relative velocity, and the dynamical friction involved is less than the static. The components of tension now drag the string back against the bow to the original position, and, owing to inertia, beyond this, until the reversed components of tension

bring the string to rest, when it is suddenly caught by the bow, and again dragged with it.

From this theory of the action of the bow, Helmholtz was able to calculate the requisite coefficients of the Fourier analysis of the tone of the bowed string, in the form :—

$$y = A \sum_{j=0}^{j=\infty} \frac{1}{j^2} \sin \frac{j\pi x}{l} \sin 2 j\pi nt, \quad . \quad . \quad . \quad . \quad (41)$$

corresponding to the solution for the plucked string, but with the proviso that in accordance with Young's law, all partials having a node at the bowed point are to be omitted from the summation. In this equation A is the maximum amplitude of the fundamental at its antinode, i.e., in the centre of the string, and depends on the velocity of the bow. The frequency n of the fundamental is given by the usual formula $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$: the amplitudes of the partials, obtained by putting $j = 1, 2, 3$, etc., diminish as the square of the order of the partials, save that those having a node at the bowed point have zero amplitude.

It may be noted that, in practice, the string is bowed near one end, so that q in Helmholtz' law is a large quantity. Consequently, the partials of order q, $2q$, $3q$, etc., only are missing, and as anyway these would have a very small amplitude $\left(\frac{1}{q^2}, \frac{1}{4q^2}\right.$, etc., of the fundamental) the Young proviso may be ignored in this case.

So much for the Helmholtz theory of the bow. Experiment suggests that it is virtually correct; the forward velocity is constant and equal to that of the bow. The constancy of the return velocity does not seem to be so clearly established, but it does seem that we have sudden reversals of the velocity of the string at each end of the forward dragging of the bow.

Raman's Analysis of the Bowed String. Starting from this basis, of two constant velocities at the bowed point, Raman [15] has built up an analysis not requiring the Fourier series, giving the transverse velocity, amplitude and their variation at each point of the string. He treats the stationary vibration of the string as made up of two progressive waves, reflected from end to end of the string, but finds it easier to consider the changes in transverse velocity $\frac{dy}{dt}$

produced by these waves at each point of the string, instead of the displacement. We can then formulate these velocity waves, so that, as they cross the bowed point, they satisfy the known conditions there, i.e., that the resulting velocity of the string $\frac{dy}{dt}$ jumps between two finite and constant values U_1 and U_2. In consequence of the constant velocities at the bowed point, $\frac{d^2y}{dt^2} = 0$ here, except for certain infinitesimal fractions of the period when the jumps occur, and then $\frac{d^2y}{dt^2} = \pm \infty$.

Consequently, if $\frac{dy_1}{dt}$ represents the velocity of the string due to the positively travelling wave, and $\frac{dy_2}{dt}$ that due to the negative wave, their sum at the bowed point should remain constant except at the instants at which the jumps of velocity occur. It is possible to secure this condition, if on one side of the bowed point and distant l from it, the slope of the velocity wave (not that of the string itself) due to the positive wave alone, would be $\tan^{-1}\alpha$, that due to the negative wave only, at the same distance l on the other side of the bowed point, $\tan^{-1}(-\alpha)$ (Fig. 34).

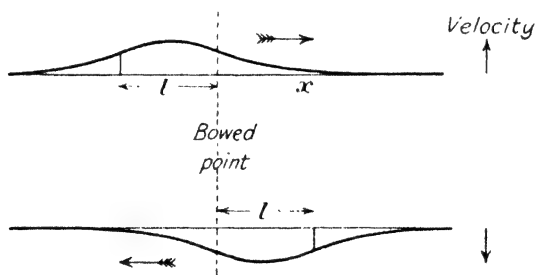



Fig. 34.—Velocity Waves on a String (Smooth Type).

When the waves are thus related to each other, the values of $\frac{dy_1}{dt}$ and $\frac{dy_2}{dt}$ at the bowed point change to equal but opposite extents in the displacements occurring in any small interval of time, and their sum therefore remains constant. It is possible to draw any number of curves to represent the positive and negative velocity

waves satisfying this condition. But there are two additional conditions to be satisfied, viz., that $\frac{dy_1}{dt} + \frac{dy_2}{dt}$ should be always zero at two other points on the string, i.e., its two fixed ends. Such a velocity wave-form as that in Fig. 34 therefore cannot be the solution, in fact, in order to avoid this *impasse* it is necessary to postulate that the waves should have constant slope throughout, with discontinuities where the velocity suddenly drops. The waves are accordingly of this zig-zag type (Fig. 35), in which the velocity wave of amplitude represented by the continuous line is travelling to the right; that represented by the dotted line of equal amplitude, to the left. The note is made up of such waves, having a number of discontinuities proportional to the order of the partial. In our figure the second harmonic is pictured, since there are two discontinuities in twice the length of the string, twice the length of the string being the wave-length of the fundamental tone in the string.

FIG. 35.—Velocity Waves on a String (Zig-zag Type) Forming the Second Harmonic.

A study of these successive phases shows that the velocity at the bowed point (x_0) due to the superposition of these two waves, is always a constant negative quantity between the epochs represented at (*a*) and (*c*) (Fig. 35). At the instant figured at (*c*) occurs a jump to a positive velocity which remains constant between the epochs (*c*) to (*a*), where we return to the original circumstance—it is easier to see what happens at the bowed point if templates are cut out of card to the shape of the velocity waves, and moved past the point in opposite directions. The superposed displacement due to these waves is similar to that shown in Fig. 33 for the plucked string.

In order that the velocity at x_0 may jump from U_1 to U_2 at stated instants during the period, the simplest case to imagine is that in which a discontinuity due to one wave passes over the point at the instant when the other wave would produce, by itself, no velocity of the point. When the point is a node of one of the harmonics of the string, the non-appearance of this harmonic in the complex note requires a number of discontinuities, at intervals equal to half the

wave-length for this harmonic, passing over the bowed point simultaneously in pairs, and so having no part in the formation of discontinuities in the velocity of the bowed point. Apart from these absent harmonics, each of the discontinuities involves a jump in the velocity from U_1 to U_2, as they pass over the bowed point.

If therefore there are j discontinuities in twice the length of the string, the inclination of these waves to the x axis is given by α, where :—

$$\tan \alpha = j\frac{(U_1 - U_2)}{2l}.$$

The superposition of these velocity waves causes the resultant velocity graph along the string to consist of j discontinuities between $(j + 1)$ straight lines inclined at an angle $\tan^{-1} 2\alpha$ to the x axis.

Now the points where the inclined lines of this graph cut the x axis are the nodes of the j'th partial, for as two of them (the ends of the string) are points of continual zero velocity, so also must the others be. Thus A, B, C, D, etc. (Fig. 36), are nodes of the stationary vibration ; E, F, G, etc., are points where the discontinuous jumps from U_1 to U_2 occur. These points, E, F, G, etc., thus form possible bowing points for the maintenance of the motion, and further,

$$\frac{U_2}{U_1} = \frac{ES}{ER}.$$

Therefore

$$\frac{U_2}{U_1 + U_2} = \frac{EB}{AB}.$$

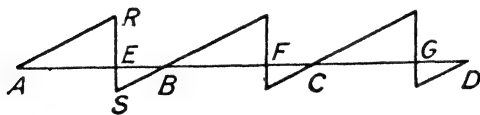



FIG. 36.—Resultant-Velocity Along Bowed String.

Putting $EB = x_j$, the distance of the bowed point from the nearest node of the j'th harmonic, and $AB = \frac{l}{j}$ we find :—

$$\frac{U_2}{U_1 + U_2} = \frac{jx_j}{l}.$$

This is also the fraction of the period for which the string moves with the bow, i.e., with velocity U_2, since the bowed point moves between two definite points with two definite and constant velocities.

From the velocity graph we can calculate the configuration of the string at each instant, since the displacement at any point is the time integral of the velocity wave. The displacement wave for the single discontinuity case consists of the two straight lines meeting at the point which the velocity discontinuity has reached (see Fig 37). The positive and negative displacement waves superposed then give the instantaneous configuration of the string, as in Fig. 33.

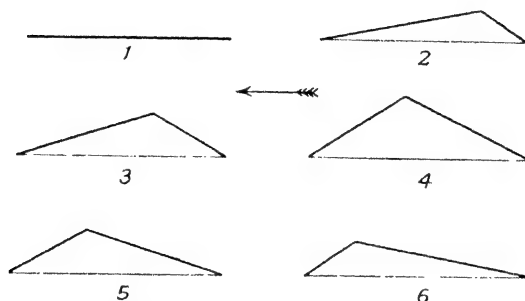



FIG. 37.—Displacement Wave of Bowed String.

Pressure and Width of the Bow. If the pressure of the bow is increased or its velocity decreased, the fraction of the period during which the string adheres to the bow is increased, so that there may be a change in the type of vibration. If the bowing point is near the node of an important harmonic, it will obviously require a greater pressure to produce this harmonic in the subsequent motion than it would require at an antinode of the same harmonic. Raman found experimentally that the pressure needed to produce the fundamental, by bowing near the end of the string, decreased as the square of the distance of the bowing point from this node. Violin teachers usually give instructions that the speed of the bow is to be increased when greater intensity is required, the bow being moved slightly nearer the bridge at the same time. Lippich [16] asserts that the increase of either bowing speed or pressure raises the general intensity, so that, for example, it is difficult to press lightly and play *forte*, or to press heavily and play softly at the same time. The effect of the finite width of the bow will be similar to that of the finger when the string is plucked. It tends to remove some of the higher harmonics, for all those having a node under the bow will be executed with difficulty. From this cause only, the quality of the note produced will be varied according as the bow is near or far from one end. With the bow of a violin,

for example, close to the bridge, the note will be richer in the first half dozen harmonics than if bowed at the usual place.

The Instruments of the Viol Family. In these instruments played with a bow, the four strings are stretched between two saddles along an unmarked finger-board (Fig. 38). The higher saddle is known as the "bridge," the lower on the finger-board is known as the "nut." The bridge stands upon the body, a hollow wooden box which is of a peculiar shape necessitated by the movement of the bow, and is provided with two openings known from their shape as *f* holes. The purpose of the body is to reinforce the sound of the strings by the forced vibrations of the box and the air inside it. That the air in the box does execute vibrations similar to, yet more complex than that of the string, has been shown by Barton and his pupils,[17] who covered one of the openings with a membrane and attached mirror, so that photographs of the corresponding motions could be made on the same plate. The vibrations are communicated to the air *viâ* the bridge and the wooden case of the body, and they have shown that these parts also are set in forced vibration, by means of an optical lever rocked by the bridge or the body. The motion of the air was found to be more complex than that of the intermediate vibrators, showing that the air more nearly followed the forcing vibration of the string. It is mainly in the construction of this sounding system that a good violin with its smooth tone is distinguished from a poor one, or from the sonometer. The best violins were and are those made by the Italians of the seventeenth century. The niceties of their construction, the dependence of the *timbre* on the construction of the body, even on the type of varnish, are not understood thoroughly after centuries of violin making, not even in an empirical fashion; so that it is not strange that a great deal remains to be scientifically elucidated on this subject.

The natural tones of the air in the resonance box might be expected to have considerable influence on the resultant notes of the violin. As a matter of fact, Barton found that in general the vibrations of the enclosed air were due to pure forcing, but Hermann,[18] on the contrary, found on analysis of the vibration curves of the air in the body distinct partials due to that enclosed air (the second harmonic stood out especially in the violin examined, the fundamental in a violoncello). To the incidence of these body-tones Hermann attributes the noticeably different quality of violin and 'cello tone in different parts of their pitch range. The strings also exert a mutual influence

on each other. Morton and Vinycomb [19] found that if two strings were tuned to the same pitch, and one of them was excited, the fundamental only was generally transferred to the second string.

Wolf Note. When the pitch of the tone elicited from the string coincides with the fundamental (or with an important harmonic) of the wood or air of the body, one would expect a large reinforcement of the former, the energy being furnished by the increased bowing pressure required to maintain the same amplitude of the string. In fact, when the pitch is that of certain harmonics of the wooden structure a quite special and undesirable effect is produced, in which the control of the string seems to pass out of the player's hands. The howling effect produced at this pitch has given this note the name of "wolf." With the object of correlating the vibrations of belly and string, White [20] obtained simultaneous records by the method of Barton at the wolf pitch. The belly showed a S.H.M. of large amplitude, whereas at frequencies a little above or below, its motion was quite complex. A fact which does not seem to be explained is that the wolf note is usually observed at the upper harmonics of the wooden system of the violin, and not at the fundamental. The wolf note can also be excited by plucking; in this case the vibration curves of the string and body show simple resonance.

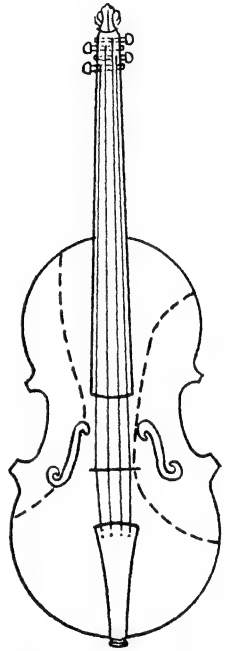



Fig. 38.—The Violin.

According to Kessler [21] who has explored nodal patterns over the back of the violin, a mode with two nodal lines (shown dotted on Fig. 38) is characteristic of the wolf. The *f* holes have little effect on this but movement of the bass bar shifts the body frequency. It is better to reduce the wolf intensity by such adjustments rather than to try to kill it altogether as this would spoil the instrument's radiating properties.

Raman [22] prefers a more detailed explanation of the bowed wolf note. His theory of the bowed string leads him to expect a relatively higher bowing pressure for the fundamental than for the octave, when the string is bowed at a point near its end. Accordingly, at

the wolf pitch the string starts sounding its fundamental, but resonant vibration of the wood drawing energy from it, the pressure between the bow and the string falls, until the fundamental cannot be maintained, and the vibration changes to a type in which the octave predominates; on the consequent cessation of the vibration in the wood, the fundamental reappears. These cyclic changes between octave and fundamental are apparent in the records of the movements of the string.

The Mute. In order to deaden the intensity of the sound it is customary to load the bridge of viol instruments by a metal clamp which grips it, the size and weight of this "mute" depending on the size of the instrument. Not only does it reduce the general amplitude of the vibrations of the instrument, but it gives new quality to the note, a fact long made use of in orchestration. The motion of the bridge is mainly lateral, it is pushed and pulled in the direction of the string. There is also a smaller motion in its own plane about one or other of the feet of the bridge.

Effects of Material and Varnish. Recent Researches. Backhaus [23] in recent experiments shows that if the force of the bow normal to the string exceeds 25 gm. weight, the quality is independent of this force, i.e. of bowing pressure. Beats characteristic of a system with two degrees of freedom are exhibited by the wave-form of the instrument especially near the wolf note. A close coupling between the natural vibrations of the string and the forced vibrations of the wood is characteristic of a good violin; in other words, for these the material acts as a sound-board without those marked resonances such as—in extreme cases—give rise to a wolf note.

Meinel [24] is interested principally in the effect of the thickness of the wood on the tone characteristics. If this is overdone, the spectrum is wanting in the lower frequencies (below 500 c./sec.); very thin wood exaggerates the low at the expense of the high components and the tone lacks brilliance. Putting on the varnish made little difference, but moving the sound post redistributed the resonance peaks besides altering the efficiency of the apparatus, considered as a sound radiator.

Minnaert and Vlam [25] study the vibrations of the bridge, observed by the reflection of a beam of light from a galvanometer mirror stuck thereon. Flexural and torsional vibrations occur as well as pivotal movement—as of a rigid body—about the line of the feet.

Saunders [26] and his students have carried out a long series of

investigations on the violin, particularly directed towards finding those factors which distinguish a good from a bad violin. The response curves of several famous violins were compared among themselves and with those of new violins; the filtering action of the bridge was discussed; a suggestion was made in regard to the meaning of the term "carrying power" so often used by violinists; and an account was given of the results of a test of the ability of an audience to pick out the sound of a Stradivarius, when it and two new violins were played in succession behind a screen.

In the latest work by Watson, Cunningham and Saunders,[27] they excited the violin electro-magnetically with a force oscillating with a single frequency which could be varied over the whole range of the instrument. The new method included direct measurements of the decay of pure tones, from which the damping at a few selected frequences could be obtained. One of these frequencies was chosen to coincide with that of the main vibration of the air inside the body; the others coincided with natural vibrations of the body, which were unaffected by the inside air.

Rohloff[28] has also obtained acoustic spectra of violins. In addition he has measured the rate of damping of notes produced in air and *in vacuo*. If the air is responsible for a high proportion of the total damping and the internal friction a small one, the apparatus is an efficient radiator of sound. This is the case with the old Italian violins examined. By comparing the decrement of beams stressed and unstressed, in transverse vibration, this author concludes that the makers stressed the bellies of their instruments to produce this effect—but compare the remarks of Saunders, above, on this point. The purpose of the varnish was to reduce the internal damping in the wood. Abbott and Purcell[29] have measured Young's modulus and decrement for various specimens of wood used in the manufacture of violins and of pianoforte soundboards.

Struck Strings. It might be expected that the general form of vibration peculiar to a plucked string would be exhibited by the same string when struck by a hammer, since this is merely another method of displacing a given point or region of the string. The motion of the string is in fact similar in the two cases, but more complex in the latter because the struck point is displaced a little before the rest of the string. We could describe the initial conditions of the plucked string as "static," that of a struck string as "kinetic." We have in fact more variables at our disposal in the latter case,

e.g., the relative masses of hammer and string, the striking velocity of the hammer, which have no counterpart in the plucked string.

Theoretical solutions have been attempted from two directions. Helmholtz [30] and others equate the instantaneous force of the hammer on the string to the product mass × acceleration of the struck part of the string. The theory assumes that the time of contact is very short, and that, as a result of the sudden initial velocity imparted to the struck point, waves move in opposite directions along the string and are reflected at the ends.

As a result of his experiments Kaufmann [31] rejected the Helmholtz theory, as experiment showed that the time of contact was always considerable compared with the periodic time. In its stead, Kaufmann based a theory on an assumption that the hammer acts as a massive unyielding particle striking the string. A number of other assumptions

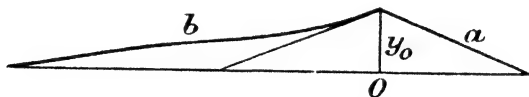



Fig. 39.—Displacement Form of Struck String.

and approximations were necessary, in order that he might finally obtain a solution for a striking point near one end of the string, giving expressions for the time of contact, motion of struck and other points on the string.

The theory has been developed and made to fit the results more exactly,[32] but as the simplest, the original form of Kaufmann will be outlined here. Let m_0 represent the mass of the system moved, i.e., the mass of the hammer plus a mass equivalent to the string or part of the string moved, imagined to be collected at the struck point. Taking this as the origin of co-ordinates, we imagine the string to be bent into a form similar to that in Fig. 39. The short piece of string a, to the right of the struck point, is assumed to remain straight during the impact, whereas the longer piece b is bent into some form of wave given by $y = f(x + Vt)$ such that there is a definite displacement y_0 at O, and a definite slope $\left[\frac{\partial y}{\partial x}\right]_{x=0}$ at the same point on the negative side of O. Now the forces which tend to restore the string to its undisplaced position are the resolved tensions in b and a at O, i.e.

$$-F\left[\frac{\partial y}{\partial x}\right]_{x=0} - F\frac{y_0}{a} = m_0\frac{\partial^2 y_0}{\partial t^2}.$$

As $y = f(x + Vt)$ on the left of O, $\left[\frac{\partial y}{\partial x}\right]_{x=0} = \frac{1}{V}\frac{\partial y_0}{\partial t}$, so that the equation of motion of this point can be written :—

$$m_0\frac{d^2y_0}{dt^2} + \frac{F}{V}\frac{dy_0}{dt} + \frac{F}{a}y_0 = 0 \quad . \quad . \quad . \quad . \quad . \quad (42)$$

This equation is of the form corresponding to damped motion (21), and its solution (as long as the reflection of the wave $f(x + Vt)$ has not reverted to O from the distant end, i.e., while $t < \frac{2b}{V}$) is :—

$$y_0 = \frac{U}{\sqrt{\frac{F}{m_0 a} - \frac{F^2}{4m_0^2V^2}}}\, e^{-\frac{Ft}{2m_0V}} \sin\left(\sqrt{\frac{F}{m_0 a} - \frac{F^2}{4m_0^2V^2}}\right)t.$$

where, at $t = 0$, $y_0 = 0$ and $\frac{dy_0}{dt} = U$, the striking velocity of the hammer (cf. p. 45). Kaufmann put this result into a more practical form by putting in the fundamental time-period $\left(T = \frac{2l}{V}\right)$, thus $F = V^2m = \frac{4lM}{T^2}$, and M = mass of whole string.

$$y_0 = \frac{UT}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}}\, e^{-\frac{M}{m_0}\frac{t}{T}} \sin \frac{t}{T}\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)},$$

assuming that the hammer is thrown back off the string at the instant the latter comes to rest, which occurs when the opposing force due to the string equals the pressure exerted by the hammer. By differentiating the above and putting $\frac{d^2y_0}{dt^2} = 0$, we obtain the time of contact (τ).

$$\frac{\tau}{T} = \frac{1}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}} \tan^{-1}\left[\frac{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}}{-2\frac{l}{a} + \frac{M}{m_0}}\right].$$

When $\frac{M}{m_0}$ is small compared with $\frac{l}{a}$, as Kaufmann claims it is in

practice, the angle in the above expression is approximately π radians and

$$\frac{\tau}{T} = \frac{\pi}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

and

$$y_0 = \frac{UT}{\sqrt{\frac{M}{m_0}\left(4\frac{l}{a} - \frac{M}{m_0}\right)}} e^{-\frac{M}{m_0}\frac{t}{T}} \sin \pi\frac{t}{\tau} \quad . \quad . \quad . \quad (44)$$

It is assumed, as stated above, that the motion at y_0 has not been disturbed by the reflected wave during the time τ.

Though Kaufmann himself gave a few experimental vibration curves for the string, the exhaustive treatment which Raman devoted to the bowed string has been undertaken for the struck string by George [33] in order to discriminate between the Helmholtz and allied theories on the one hand, and the Kaufmann theory and its developments on the other. His hammer consisted in principle of a compound pendulum, so that the velocity of striking, U, could be increased by giving the pendulum a larger swing. The time of contact τ was found by letting the contact make an electrical circuit containing an Einthoven galvanometer, or oscillograph. Certain of the oscillograms showed evidence of a second contact before the hammer finally left the string. By photograms of the Krigar-Menzel and Raps type, George obtains values of τ and y_0 to test the formulæ. Both theories give closest agreement with practice at a ratio of string and hammer masses = 1·7, when the percentage errors are about the same; with a light hammer however (e.g., ratio of masses = 0·267) the discrepancies are greater, but the Kaufmann formulæ fit the experimental values fairly well over the whole range, whereas the Helmholtz theory is very wide of the mark with a light hammer. From the example shown, which illustrates the motion of the struck point before and after being hit by the hammer (Fig. 40), reproduced

FIG. 40.—Motion of struck Point.

from George's photographs, the principal assumption of Helmholtz that the time of contact is inappreciable in comparison with the period, is unjustified. George's results may be summarized as follows: -

(1) The amplitude of the fundamental is greater when the string is struck near one end, and when the relative mass of the hammer is great.

(2) This amplitude varies in a discontinuous fashion, showing maxima and minima as the hammer is moved towards the centre, and the number of maxima and minima becomes less as the mass of the hammer is reduced, until there is only one maximum and two minima.

(3) Early workers in the subject thought it necessary to discriminate between a hard and a felt-covered hammer, but it appears that the distinction is merely one of the finite length of the string struck, similar to that between a narrow and a wide bow.

The Pianoforte. This instrument contains a large number of "strings" in the form of steel wires, one or more for each note of the musical scale, stretched between bridges, one set of which rests upon a "sound-board" and the other set is attached to the frame of the instrument. The strings are struck by hammers at a point distant from one-seventh to one-ninth of their length from one bridge. The vibrations of the strings are rapidly damped, but in order that the vibrations may be stopped as soon as the finger is lifted from the key which operates a hammer, a "damper" comes into action on the majority of the wires in the form of a felt wedge which presses on the string. If desired, all the dampers may be held off by a pedal. Improvement in steel wires and the introduction of the iron frame has made possible greater tensions than formerly, to the enhancement of the intensity of the notes. Nevertheless, three wires are usually given to each note—one or two only when the "soft-pedal" is brought into action. It was formerly thought that the choice of striking-point was governed by the expediency of removing the dissonant 7th and 9th partials—the makers unconsciously following the principle stated in Young's law—and this is no doubt accomplished to a certain extent, but the above experiments on the struck string show that this region is, in general, one of maximum fundamental amplitude for a string so excited, and for this reason is a desirable location for the striking point. Besides, Berry [34] has found that the natural vibrations of the sound-board (which the maker desires to eliminate) are a minimum

when the string is struck in this region, the wood copying the vibrations of the wires. The hammers are covered with felt, but those which strike the short wires forming the upper octaves of the instrument are sharper and more pointed so as not to occupy a relatively larger part of the string than those that act on the long strings. It may be imagined from what has been said about the violin that the quality of the instrument depends upon the material and construction of the sound-board. If the sound is not transmitted quickly over the entire board, different sections of it will be vibrating out of phase, and will impair its effectiveness. This is obviated by the bridges, and by a number of bars fitted tightly to the back of the board to increase its rigidity. Attempts have been made to fit a resonant column of air behind the sound-board, but this is unusual. Hart, Fuller and Lusby [35] in a study of the mystic quality which pianists call " touch " made records of notes as played by virtuosi and those produced by a pendulum which, on release, hit a key. In every case it was possible to match the human record with one of the pendulum's. In fact the only factor which a player can vary is the speed with which the hammer hits the string.

Electro-magnetically maintained Wire. Apart from the use of a bow, transverse vibrations in an iron or steel wire may be maintained by an electro-magnet in which the current is made and broken with the same frequency as the fundamental vibration of the string, so that a part of the wire is attracted towards the magnet once in each period, and then springs back.[36] It is possible to arrange that the wire itself breaks the current which supplies the electro-magnet when the wire has reached its maximum displacement in the direction of the magnet (cf. the electro-magnetically maintained tuning-fork, p. 110). The tones of such wires have been examined by Klinkert [37] by the usual photographic methods. They correspond more or less to the tones produced in the wire when plucked, at the piont where the magnet is placed, by an object of considerable extent, i.e., the resulting wave forms exhibit rounded-off discontinuities, except that the amplitude is maintained, instead of dying away.

When the wire acts as its own current-breaker, the fundamental tone is maintained. The system obeys Young's law, in that the electro-magnet cannot maintain a partial when it is opposite one of the relevant nodes. Even if, with the magnet at an irrational point of division, a harmonic be started in the wire by lightly touching at a rational point, the form of vibration is unstable, and tends to pass

over into one in which the fundamental predominates. It is possible to study the phenomena of resonance in strings by using for the interrupted current in the magnet, that obtained from a contact breaker on an auxiliary wire similarly maintained, or by using a rotary make-and-break. By varying the length of the auxiliary wire or the speed of the rotary make-and-break, one can impress different frequencies on the wire, and study the forced vibrations which ensue. The results verify the phase relationships deduced theoretically (in Chap. II, p. 50) for frequencies near resonance. When the natural frequency of the wire is a little less than that of the impressed force, the phase difference is $\pi/2$. and the amplitude of the forced vibration becomes very large.

Transverse Vibration of Bars. Just as we obtained a general differential equation for the transverse vibration of a stretched string by equating the product mass $\times$ acceleration of any element to the restoring force due to the tension in the string, so we can obtain a differential equation for the bent bar vibrating transversely; only now the restoring force is the bending moment which is a *fourth* order function of the displacement at the point in question. The method of deriving the equation follows Barton's [38] simplification of Rayleigh's [39] treatment. If an isolated force f act on the bar at right angles at any point distant x from the clamped end, it will be equivalent to a bending moment fx about the end. Now, owing to the mutual action of the molecules of the bar, the application of such a force on any part of it will introduce forces on the other parts, or, in other words, it is impossible to bend one small part of the bar by such a single force without bending the whole bar more or less. The effect then of bending down one end of such a clamped horizontal beam, is to introduce various bending moments at various points of the beam, so that the total bending moment about any point can be represented in the form of a sum of products of forces into their distances from the point in question; $B = \Sigma fx$, or $\frac{\partial B}{\partial x} = \Sigma f = F$, say. Further, owing to the mutual interaction of particles of the bar, the value of F at any point will depend on that at neighbouring points, so that F continuously increases along the bar. Writing F as a function of x, and differentiating again:—

$$\frac{\partial^2 B}{\partial x^2} = \frac{\partial F}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

It is assumed that the bar is uniform, and free from forces parallel to its length: and the vibrations so small that rotary effects can be neglected. Consider an element δx of the rod (Fig. 41) having a force F at right angles to its length, which "shears" one face; the corresponding force on the other face will be $F + \frac{\partial F}{\partial x}.\delta x$. The difference between these forces, i.e., $-\frac{\partial F}{\partial x}.\delta x$ represents the force tending to straighten the element if the impressed forces were suddenly removed, and may therefore be equated to the mass × acceleration of the element.

We now require an expression for B in terms of the configuration

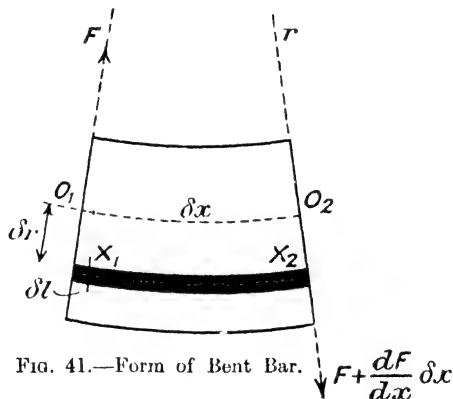



Fig. 41.—Form of Bent Bar.

of the element. Let it be bent into an arc of a circle of large radius r, so that we may take $\frac{\partial^2 y}{\partial x^2}$ to equal the curvature. Owing to the bending, the original slice X_1X_2 (Fig. 41) at a distance δr from the "neutral surface" O_1O_2 (which remains unchanged in length) becomes stretched by the fraction $\frac{\delta l}{\delta x} = \frac{\delta r}{r}$ (by similar triangles, Fig. 41). This is the strain on the slice; the stress is found by multiplying this by Young's modulus E. Then the total force on this slice of cross section δS, becomes $\mathbf{E}\frac{\delta l}{\delta x}.\delta S = \mathbf{E}\delta r\frac{\partial^2 y}{\partial x^2}\delta S$. Multiplying this by δr to obtain the bending moment about the neutral surface, and integrating over the whole area of the element

$$B = \mathbf{E}\frac{\partial^2 y}{\partial x^2}\Sigma(\delta r)^2\delta S = \mathbf{E}\kappa^2\frac{\partial^2 y}{\partial x^2}S \quad . \quad . \quad . \quad . \quad (46)$$

where κ is written for the "spin-radius" of the section about the neutral surface. Returning to (45), the restoring force on the element

$$= -\frac{\partial F}{\partial x}.\delta x = -\frac{\partial^2 B}{\partial x^2}.\delta x = -\mathbf{E}\kappa^2\frac{\partial^4 y}{\partial x^4}S\delta x.$$

Equating this to the mass $\times$ acceleration of the element $S\rho\delta x\frac{\partial^2 y}{\partial t^2}$ we obtain finally :—

$$-\frac{\partial^2 y}{\partial t^2} = \kappa^2\frac{\mathbf{E}}{\rho}\frac{\partial^4 y}{\partial x^4} = \kappa^2 V^2\frac{\partial^4 y}{\partial x^4} \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where V is the speed of longitudinal waves in the bar.

This differential equation combined with the appropriate end conditions will enable us to calculate the frequency of the bar vibrating in fundamental or partial modes. The possible end conditions are :—

(1) Clamped end. Never any displacement; slope of bar always *nil*; $y = 0$, $\frac{\partial y}{\partial x} = 0$.

(2) Free end. Displacement and slope arbitrary, but no force beyond the end to produce curvature or shear. $\frac{\partial^2 y}{\partial x^2} = 0$, $\frac{\partial^3 y}{\partial x^3} = 0$.

(3) End supported on an edge. No displacement, but slope may vary, whereas curvature is nothing, as at a free end. $y = 0$, $\frac{\partial^2 y}{\partial x^2} = 0$.

Various combinations of "ends" are possible, but only two are of practical importance, i.e., (1) one end free and the other fixed, (2) both ends supported.

On the assumption of simple harmonic and undamped motion, a solution of (47) is :—

$$y = a \sin(\omega t + \delta).$$

giving :—

$$\frac{\partial^4 y}{\partial x^4} = \frac{\rho\omega^2}{\mathbf{E}\kappa^2}y = \beta^4 y,$$

so that :—

$$y = A'e^{bx},$$

where b is a root of $b^4 = \beta^4$, i.e. :—

$$b = \pm\beta \text{ or } \pm i\beta,$$

whence :—

$$y = (A\cos\beta x + B\sin\beta x + C\cosh\beta x + D\sinh\beta x)\sin(\omega t + \delta).$$

Bar Clamped at One End. Taking the fixed end as the origin of x, and the free end at $x = l$, we find the multiplier of $\sin(\omega t + \delta)$ in the last equation to be given by values of βl which satisfy :—

$$\sec \beta l = - \cosh \beta l.$$

The readiest way of solving this equation is to plot the graphs of $y = - \cosh \beta l$, and of $y = \sec \beta l$; the solutions will then be given by the values of the intersections of the two curves. From such values the corresponding relative frequencies can be found, since these are proportional to β^2.

Table of βl.

1·88	4·69	7·85	11·00	14·14	17·28	etc.

The fundamental of such a bar is therefore given by

$$\beta l = 1{\cdot}88$$

or $$\sqrt{\frac{\rho}{\mathrm{E}}} \cdot \frac{\omega l^2}{\kappa} = (1{\cdot}88)^2 ; \quad \text{whence } n = \frac{\omega}{2\pi} = \frac{3{\cdot}53\kappa}{2\pi l^2} \sqrt{\frac{\mathrm{E}}{\rho}}.$$

The frequency is inversely as l^2. In other words $n\lambda$ is no longer a constant; the velocity of the transverse waves in a bar is a function of the frequency. This is why we cannot employ Fourier analysis to find the amplitude of the partials. The ratios $\frac{\beta^2}{\beta_1{}^2} \cdot n$ show that the partial tones are inharmonic; their frequencies are not in the ratio of the natural numbers as on a string, nor are the partial nodes equidistant. For this reason vibrating bars or reeds are rarely used alone to form musical instruments; without a resonator to qualify them, their tones are harsh, and in fact are described as "reedy." Note that a bar fixed or free at both ends has a different series of tones, given by :—

$$\sec \beta l = \cosh \beta l,$$

the fundamental being 2·67 octaves higher.

Bar Supported at Both Ends. Considering again the amplitude expression :—

$$A \cos \beta x + B \sin \beta x + C \cosh \beta x + D \sinh \beta x.$$

$$\text{At } x = 0, \; y = 0 \qquad\qquad A + C = 0$$

$$\frac{\partial^2 y}{\partial x^2} = 0 \qquad\qquad A - C = 0$$

which reduces the expression to $B \sin \beta x + D \sinh \beta x$.

$$\text{At } x = l,\ y = 0 \qquad B \sin \beta l + D \sinh \beta l = 0$$

$$\frac{\partial^2 y}{\partial x^2} = 0 \qquad - B \sin \beta l + D \sinh \beta l = 0,$$

subtracting we find $\sin \beta l = 0$. Therefore βl is a multiple of π. The nodes of the partials are now equidistant, β has successive values in the ratio 1, 2, 3, etc., as for a vibrating string, but the partials have frequencies proportional to β^2, and therefore in the ratio 1, 4, 9, etc.

Experimental Work on Bars, especially Reeds. The bar may be encouraged to produce these partials by supporting it symmetrically in aliquot parts. Thus a vibration having three nodes with antinodes at the ends may be produced by supporting the bar at one-quarter of its length from each end (see Fig. 42). The vibrations may be excited by bowing on the end, or by striking with a rubber-covered hammer on the overhanging portion, the bar being prevented

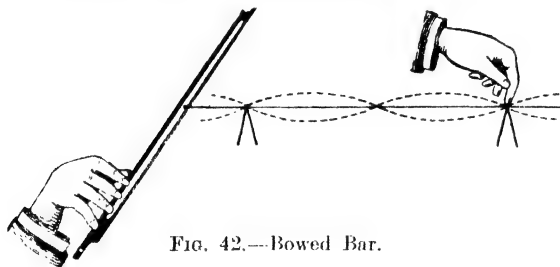



Fig. 42.—Bowed Bar.

from rising off the supports, by pressing it with the finger over the distant support.

Apart from the methods already described for examination of transverse vibrations, the position of the nodes on a flat-topped bar may be roughly indicated by sprinkling sand on the bar.

The use of strips of metal or wood, clamped at one end, in a number of wind instruments, where however the tones of the reeds are generally subordinated to those of a resonator, has led to this form of bar being widely used in experiments. Out of a number of researches on the tones of bars and reeds, having no special interest save to verify the theory outlined above, a few are worthy of selection.

Rayleigh [40] inquired whether it was not possible by means of a single force applied at some point along the bar to bend it statically into the form which it assumed when vibrating in its fundamental mode. He decided that a single force at one-quarter of the length from the free end would be capable of doing this, but a nearer

approximation by Garrett [41] gave one-fifth as the point of application of the force. This was proved by means of instantaneous photographs. Hartmann-Kempf obtained "resonance-curves" for steel reeds by exciting transverse vibrations in them through the medium of an electro-magnet, actuated by alternating currents of various frequencies. When one or other of the natural frequencies of the reed approached that of the intermittent magnetic force, a large response of the reed was, of course, obtained. By means of a set of calibrated and numbered reeds arranged in front of an electro-magnet, he constructed an instrument which indicates the periodicity of the A.C. sent through the electro-magnet, by the reed or reeds which responded.[42] With improved technique it is now possible using either electro-magnetic or electrostatic oscillation to excite modes of vibration of thin metal reeds, such as are used on musical instruments, up to the tenth, or even higher overtone, though the actual motion of any part of the reed is very small (Richardson and Yousef).[43] Agreement between theory and practice for the resonant frequencies is quite good; such discrepancies as exist are probably to be ascribed to the loading of the reed by the ambient air which is also set in motion (cf. p. 117).

The Tuning-fork. This apparatus which now serves as a precision standard of frequency may be regarded as a bent steel bar, vibrating transversely. The effect on a straight bar, vibrating with two nodes, of gradually bending up the ends, is to bring the two nodes closer together, until, on reaching the familiar **U** shape of the tuning-fork, the nodes lie close together at the centre of the bend where the stem of the fork enters. The two parallel bars vibrate with a frequency about two-thirds of that corresponding to the original partial. The rather complex form of the tuning-fork does not lend itself to rigid mathematical treatment, but from what has been calculated for the straight bar we should expect the fundamental tone to be inversely as the square of the length, and directly as the breadth in the plane of bending. The size of the fork is determined empirically by the maker, the final tuning adjustment being done by shaving the ends of the prongs. It is very necessary that both prongs should be exactly alike. Mercadier [44] examined the tones of a number of similar forks of different size, and his results satisfy a formula of the type $n = k.\dfrac{b + b_0}{(l + l_0)^2}$, where b and l are the breadth and length, k, b_0 and l_0 being constants (k contains the velocity of longitudinal

waves and a "form-factor"). He also showed that the thickness of the prongs perpendicular to the direction of vibration was without influence on the pitch. The prongs of low-pitched forks are accordingly made thin and long, those for high pitch, very short and thick. The pitch of the tuning-fork is subject to a small negative temperature coefficient; Rudolf König [45] found the change to be of the order of 1 in 10,000 per degree Centigrade. Magnetization of the steel causes a very small increase in the frequency. The tone of the fork may be elicited by bowing, by striking, or by pressing the ends together, and unless the excitation is done violently, the tone elicited consists almost entirely of the fundamental, with a little admixture of high overtones which are of course inharmonic. The substantial construction and considerable elasticity of the fork discourages the production of the overtones, while some of these are rapidly damped because their nodes lie close together in the material. Chladni [46] found the following tones in a fork weakly excited: 128 (fundamental), 793, 2,340, 4,480, 7,824. The predominance and isolation of the fundamental, as well as its permanent quality, are the advantages which this instrument possesses as a standard frequency source, and may be still further ensured by mounting the fork on a resonance-box, containing a body of air, of size to vibrate with the fundamental frequency. The latter possesses overtones in the harmonic series, so that it will not respond to the inharmonic partials of the fork, if any are present. Nevertheless, under special conditions, a fork may appear to produce other tones than those mentioned above. The octave of the fundamental pitch of the fork may in general be heard. As the transverse overtones of the fork, as well as possible longitudinal or torsional tones would be inharmonic to the fundamental, Lindig [47] concluded that these harmonic tones were formed in the air itself, were in fact terms which arise according to the calculation of Helmholtz when the relation between the restoring force on an air particle is no longer proportional to its displacement simply, but involves the square of the displacement (see Chap. II, p. 61). The vibrations communicated to the air particles are accordingly asymmetric, owing to the construction of the fork. To produce subharmonics of the fork, the latter is struck and the steel stalk pressed lightly on a table. Owing to intermittent contact between the steel and the wood, the former is out of contact for two or more periods of the fork. Seebeck by this explanation ascribed the sounds to successive taps on the table. By using a metal plate in place of the wooden table, and a

string galvanometer connected in series with the contact, this conjecture has been verified by Knapman.[48]

Electro-magnetically driven Tuning-fork. In order to maintain the vibrations of the tuning-fork a source of electrical energy is necessary. It is usual to drive the fork by an electromagnet, the current in which is made and broken intermittently by the motion of the prongs. The contact may be of the platinum point type or may consist of a short tungsten wire fixed to one of the prongs and dipping into mercury, and a similar form of drive may be used to keep a steel reed in vibration. The frequency of the bar or fork may be slightly altered from that of the free vibrations by this form of drive. This may be shown by applying, as Hartmann-Kempf [49] did, an independently interrupted or alternating current to the electro-magnet, keeping constant (maximum) current strength, but varying the frequency of the magnetic impulses on either side of that natural to the fork. From the photographic traces provided by mirrors on the fork, Hartmann-Kempf obtained the resonance peaks (for a number of current strengths) shown in Fig. 43, the dotted lines for interrupted current, the continuous lines for A.C. The effect of increasing the current strength is to lower the frequency of the fork, and to push the resonance maxima to the left of the figure.

Fig. 43.—Resonance Curves of Electro-magnetically maintained Tuning-fork (*Hartmann-Kempf*).

Other experimenters have noticed that the natural frequency of such a vibrating system falls with the amplitude, approximately with the square root of the amplitude, which of course is governed by the attraction exerted by the magnet, and therefore by the strength of the current. Dadourian [50] pointed out that with this type of maintenance, increase of amplitude first lowered and then raised the frequency, so that there is a certain minimum frequency corresponding to a certain amplitude, at which the fork should be run.

In order to use a fork as a precision time standard, the contacts must receive careful consideration, the " cut-off " must be very precise. Further, the fork tends to couple its vibrations, viâ the stem, to

those of its surroundings, and if allowed to do so, these react on the frequency of the fork. Consequently the supports of the fork must be massive. These desiderata are incorporated in the Tinsley fork developed from that of Wood and Ford [51] (Fig. 44). The two long bars form the fork proper, the stem having been replaced by a steel block to which they are fixed. The shorter bars merely support the contacts, which incorporate a stop device, which regulates the travel of the platinum points, keeping the time of contact constant from period to period, as long as the current remains constant. Loads permitting a slight adjustment are screwed on to the ends of the blades. It is claimed that this fork will remain constant to 1 in 10,000.

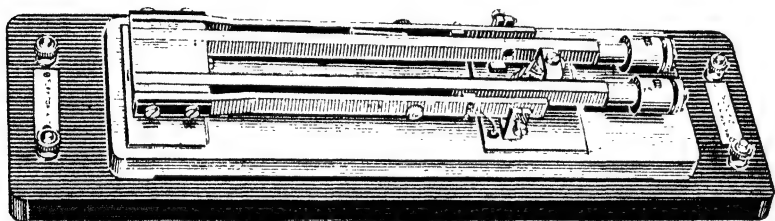



FIG. 44.—Tinsley Standard Tuning-fork.

Valve-maintained Tuning-fork. The development of the triode valve has provided another possible way of utilizing electric energy to maintain a fork. A circuit on this principle was devised about the same time by Eccles [52] and by Abraham and Bloch.[53] The function of the valve is to magnify in the plate circuit, oscillations taking place in the grid circuit, when these are inductively coupled. It is arranged that the two prongs of the fork shall vibrate in front of two coils, one in each circuit, so that, on exciting the fork, the movement of the iron of one prong sets up induced currents in the grid circuit. The magnified oscillations in the plate circuit then act magnetically upon the other prong, with sufficient vigour to overcome the natural damping of the fork, and to maintain the vibration at the expense of the electric energy. For the coils, the receivers from a pair of headphones, with caps and diaphragms removed are useful. Valve-maintained vibrations will be further discussed in the next chapter.

The Phonic Wheel. This device, due to Rayleigh [54] and La-tour,[55] enables the speed of rotation of a motor to be kept at a constant rate controlled by a tuning-fork, and is very useful for driving

a stroboscopic disc. In principle, such a motor consists of a number of iron studs rotating past a corresponding number of electro-magnets. These magnets are energized only at certain definite and constant intervals. If the studs happen to lie opposite the soft iron magnets at the instant of their excitation we have a completed "magnetic circuit," and the studs will tend to move past the magnets at the same rate. If, owing to lagging or gaining by the system of studs, the excitation of the magnets occurs when the studs are a little behind or in front of the magnets, forces of attraction will be brought into play, tending to restore the *status quo*. The studs are formed on the rotor by cutting equidistant slots in a plain cylinder of soft iron. Corresponding soft iron bars line the hollow cylindrical stator, and these are magnetized by a single coil of wire, energized by a battery passing through a make and break on the fork. Such a phonic motor is not self-starting, therefore it must be run up to synchronous speed by hand, or by an auxiliary motor. By counting the revolutions of the motor when this condition has been attained, the frequency of the controlling fork may be accurately obtained.

Musical Instruments involving Bars or Forks. Many musical instruments involve a reed in their sound-producing parts, but in the majority the note of the reed is subordinated to that of the column of air to which it is coupled, so that these do not merit treatment here. Reeds, practically unassisted, are used in the harmonium. They are kept in vibration by the pressure of air which escapes from a reservoir, through an orifice which they cover. The action of the blast of air is rather like that of the bow of a stringed instrument. The pressure forces the free end of the reed outwards until it acquires sufficient potential energy to slip back past the stream, and then it is caught up again after passing its undisplaced position. The reed may be slightly smaller than the orifice, through which it can therefore pass on its return journey, or the orifice may be merely a small hole in a plate, against which the reed presses in its normal position; in the former case it is called a "free reed," in the latter, a "beating reed." The American organ employs free reeds, through which the air is sucked instead of being blown. Possibly because the discontinuities in the action are less abrupt, the latter arrangement produces a tone less rich in upper overtones, and so less harsh.[56]

Instruments containing "supported bars" are to be found in the "toy-shop" of the modern orchestra. The xylophone contains a number of bars of metal or wood, supported at two points near the

ends, and struck with a hammer. In the *glockenspiel* a number of metal tubes are supported on a frame by strings passing through a point near the upper end. On being struck by a hammer near the point of suspension they give out a note intended to resemble that of a bell. They should approximate to "free-free bars," but the quality of their notes is complicated by the tones of the contained column of air; this interesting case appears not to have been treated experimentally. The note of the triangle—a continuous rod of steel of this shape, struck with a straight rod—contains many neighbouring partial tones forming an indefinite noise, and is not tuned to any definite pitch.

Coupled Vibrations of Bar and String—Melde's Experiment. Transverse vibrations may be maintained in a string by attaching

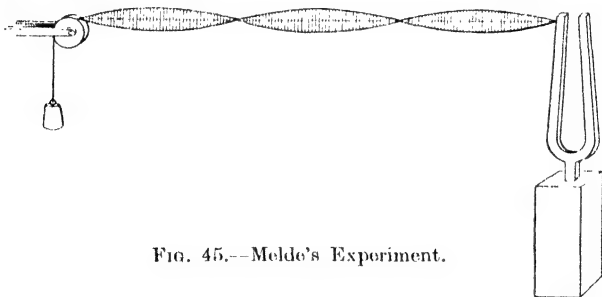



Fig. 45.—Melde's Experiment.

one end to a bar of the same pitch, in which the vibrations are maintained by bowing or by electric means. This was first accomplished by Melde,[57] who gave the apparatus the following form (Fig. 45).

A massive tuning-fork (n about 100) is securely clamped vertically by its stem. A white thread passes from the end of one prong to a tension adjuster in the form of a screw-clamp some yards away, or in the modern form, to a pulley, over which the string passes to a scale-pan containing weights. If the fork is excited, and the length and tension of the string adjusted, presently a condition will be reached, in which one of the partials of the string coincides in frequency with that of the fork. When this state of affairs is reached, the string will exhibit strong resonant vibrations with nodes and antinodes corresponding to the particular partial elicited. These coupled vibrations are possible not only with the fork vibrating in a direction at right-angles to the string, but also with the fork's vibrations in the direction of the length of the string, as depicted in

Fig. 45. In the latter case, as Melde showed, the frequency of the tone in the string is only half that of the fork, for every time that the string passes to its maximum displacement the prong moves to the left releasing the tension a little, while every time the string returns to its medial position, the prong moves to the right tightening the string again. By measuring the distance l between two nodes, we can in theory estimate the note in the fork, for that of the tone in the string is $\frac{1}{2l}\sqrt{\frac{F}{m}}$ (see p. 73), the value of F in dynes being obtained from the sum of the weights hung on the string. To make the nodes visible, a black cloth is hung behind the white string. When the adjustment for resonance is not exact, pseudo-nodes and antinodes will be observed, due to vibrations in the string "forced" by the fork. The resonance curves are always flat, for the light string accommodates itself to the period of the heavy fork. Moreover, in process of accommodation, the note in the string may be any sub-harmonic of that of the fork. This makes Melde's experiment unreliable for estimating the frequency of the maintaining fork. Very interesting are the appearances when the fork is at an oblique angle to the string. These resemble Lissajous' figures as they are made up of vibrations corresponding to both cross and parallel types. In actual experiment, the nodes are only average positions at which the positions of the string in the two extreme phases cross. By stroboscopic examination Raman [58] showed that this crossing point may wander about between two successive loops.

Another point of subsidiary interest is the apparently continuous rotation of the pulley in one direction which usually accompanies the motion. Taber Jones [59] has shown that the motion of the pulley really consists of a series of jerks, and that during the period there are instants when the string will slip past the pulley. When a string is about to slip on a curved surface, there is a definite ratio between the tensions in the two portions of the string where they leave the surface, which ratio is dependent on the angle between the tangents. The string slips when the ratio of these two tensions exceeds a certain limit. This ratio reaches a maximum when the string in the loop next the pulley is uppermost, i.e., when the obtuse angle between it and the vertical part of the string leading to the scale-pan is greatest. It is at this part of the period that slip takes place ; in the rest of the period the pulley is dragged with the string always in the one direction.

Vibrations under a double forcing action may be executed in the

string if it is stretched between two tuning forks of different frequencies (n_1 and n_2). A large number of possible modes corresponding to $j_1 n_1 \pm j_2 n_2$, where j_1 and j_2 can have any integral values, have been detected. Coupled vibrations similar to Melde's form may be observed on rocking one end of a string by a crank of a reciprocating engine, or by fixing a reed to one prong of a tuning-fork.[60]

Decrement of Transverse Vibrations. All vibrations of solids of whatever type are damped by internal friction, but as transverse vibrations of solids involve a considerable motion of the surrounding fluid medium, external friction and resistance is a much more potent cause of the decay in amplitude, which ensues when the system is allowed to oscillate unmaintained. In order to estimate the rate of decay, the ratio of the amplitudes in two successive periods is obtained, a quantity which is known as the decrement. It is the logarithm of this quantity ("log. dec." Δ) which usually figures in calculations. In slowly damped motions, it requires a number of periods to elapse before an appreciable diminution of amplitude takes place. Thus if we measure the amplitude before and after a time equal to p periods :—

$$\frac{a_0}{a_1} = \frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4} = \frac{a_{p-1}}{a_p} = e^{\Delta},$$

$$\log_e \frac{a_0}{a_p} = p\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

a formula which allows us to calculate Δ. If the period be accurately known, it is sufficient to measure, by the microscope, the amplitude at the beginning and end of any accurately measured time interval. Otherwise, a trace by photographic or other means of the vibration must be obtained, and the mean of a number of amplitude ratios over, say, 10 periods measured from the trace on the paper. Valuable work was done by Hartmann-Kempf [61] by this method on electrically-driven reeds and forks.

In determining the dragging effect of a surrounding medium on vibrations, the decrement *in vacuo* is taken as standard, as representing the decay due to internal friction alone. In this case, from equation (26) (p. 47) we have for the exponent representing the decay in amplitude $\alpha = \frac{\mu}{2m}$. The log. dec. being reckoned from half period to half period,

$$\Delta = \frac{\alpha T}{2} = \frac{\mu T}{4m} = \frac{\pi}{2} \frac{\mu}{\sqrt{mk}}$$

approximately. The surrounding medium may increase both the effective mass and the coefficient of viscosity. The increase in the mass to be set in motion by the vibration will, apart from the damping, lower the natural frequency of the vibrating solid ; this is most marked when a bar or wire is maintained in vibration in a liquid. The theory is complicated by the fact that, as Hartmann-Kempf found for a tuning-fork, the frequency changes somewhat as the amplitude diminishes, and along with this goes a diminution of the decrement. The correct delineation of the curve of decay for a tuning-fork in air is of importance in connection with experiments on the sensitivity of the ear. A number of investigators have determined the decrement by obtaining resonance curves of the vibrating system, a method adapted from alternating-current technology.

The damping of a wire has been measured by Florence Chambers [62] by impressing on it the oscillations furnished by the plate circuit of a valve containing self-inductance and capacity. Instead of allowing these impressed oscillations to pass through an electro-magnet placed near the wire, the current itself passed through the wire which was placed in a permanent magnetic field. Then the alternating current in the wire caused transverse oscillations of the same period in virtue of the deflection of a conductor, carrying a current in a magnetic field. From equation (25) (p. 47), we see that the amplitude of the forced vibration is

$$A^2 = \frac{F^2}{\mu^2 p^2 + (k - mp^2)^2}.$$

A is a maximum when $\mu^2 p^2 + (k - mp^2)^2$ is a minimum. Differentiating this denominator with regard to p^2 and putting the result equal to zero, we find the minimum when

$$p^2 = \frac{k}{m} - \frac{1}{2}\frac{\mu^2}{m^2}.$$

This value of p is in fact the resonant frequency of the system when damping is included. The ratio $\frac{A^2}{A^2_{max}}$ was measured for different applied frequencies p by altering the inductance and capacity, and so the value of μ, the damping coefficient, could be approximately found. It varied from 0·001 to 0·003, with the character of the sonometer-bed on which the wire was stretched. By a similar method Martin [63] found the decrement to vary inversely as the square root of

the frequency of the natural tone of the wire, but that it was otherwise independent of the length or tension. Unexpected difficulties presented themselves when he used large amplitudes in order to render the observation of the amplitudes more convenient. Apart from the anomalous resonance peaks (cf. p. 110), observation in two directions at right angles showed that a string plucked or magnetically excited at large amplitudes possessed two slightly different frequencies, which loosely coupled together in the style of a Lissajous figure made up the inharmonic vibration of the string.

Wires and Bars in a Liquid. The lowering of pitch of a tuning-fork when plunged into a liquid was mentioned by Chladni [64] in 1802. Auerbach [65] commenced an extensive study; in his earlier experiments the vibration was not maintained; the lowering of tone produced in a vibrating wire by plunging it from air into water was estimated at 1·11 to 1·18, depending on the frequency. Since then, the subject has become the especial province of women physicists. Experiments in which the body was maintained in vibration and the frequency determined objectively were begun by Lizzie Laird [66] (using steel strings), and continued by Mary Northway and Mackenzie [67] (with steel bars) using a number of liquids. The arrangement for maintaining the vibration was similar to the electro-magnetic device of Eustis [36] (see p. 102). The intermittent current through the electro-magnet also operated a style writing on a revolving drum (cf. Regnault's apparatus Chap. I, p. 6), so that the frequency of the intermittent current (= that of the wire or bar) could be counted. The latter experimenters state that the density has considerable influence, but the viscosity of the liquid has a small effect on the period. The lowering is directly proportional to the width of the bar, but inversely as the thickness. It appears then that the string or bar is loaded with a column of liquid, proportional to the surface which it presents in vibration to the liquid. The reason for the diminished lowering when the thickness, i.e., the dimension in the plane of vibration, is large, is rather obscure; possibly it is connected with the fact that the relative mass of the liquid to that of the bar is less in this case. The statement in the penultimate sentence was deduced theoretically by Stokes, and later by Kalähne [68] who gives a formula in the form $\frac{n'}{n_0} = \left(1 - \frac{\rho'}{2\rho}\right)$, ρ' and ρ being the respective densities of solid and liquid medium, n' the frequency in the liquid, n_0 *in vacuo*.

If the resistance opposed by the liquid is very great, $n' = 0$, and

the motion is no longer periodic. At or above this "critical damping," the wire, if plucked, will return to equilibrium without oscillation, a fact which may be demonstrated with a wire stretched in a bath of glycerin.

Alternating Current in a Wire. If a steady electric current be passed through a wire of considerable resistance, and high coefficient of expansion, such as a thin platinum wire stretched on a monochord base, the wire sags if the current is sufficient to raise its temperature by a considerable amount. If the current were rendered intermittent with a frequency n the wire would sag and recover with the same frequency. In the early days of alternating currents it was noticed that the stretched wires sometimes used as rheostats would sound their fundamental or an overtone when in a condition of resonance with the periodic heating produced by the alternating current passing through them. Now this heating is independent of the direction of the current in the wire, so that the wire is heated and cooled twice in each cycle of the alternating current, or the frequency of the impressed force on the wire is double of that of the current. Yet the frequency of the tone in the wire is found to be half of that of the impressed force (as in the longitudinal form of Melde's experiment), so that finally the frequency of the tone and of the current are the same. Analogous effects can arise in a wire which forms part of a circuit in which an intermittent spark discharge is taking place, e.g., the "secondary" of an induction coil.

Krishnaiyar [69] has examined the amplitude of vibration produced in a sonometer wire whose length and tension could be varied, when currents of fixed periodicity and of constant strength are passed through it, and so has obtained the resonance curve of the system. These vibrations being of the type where the impressed force has a frequency double the frequency of the emitted note, a marked resonance peak is not obtained (cf. p. 56). Instead, there is a gradual increase in the amplitude of response as the frequency of the wire is increased up to, and beyond, that of the forcing vibration. Krishnaiyar found the frequency of the wire by calculation from the tension and length, but apparently no comparison with a standard frequency was made.

Thermophone. The periodic heating of a strip of platinum by an alternating current is the underlying principle of an instrument known as the thermophone. Such an instrument was first described by de Lange,[70] who ascribes the actual discovery to a Russian engineer

named Gwodz. It gives a pure but feeble tone and so is usually provided with a resonator to amplify its vibrations. As a source of sound it has been elaborated by Arnold and Crandall[71] using a strip of platinum 0·0007 cm. thick, in which either A.C. or A.C. superposed on D.C. was used. Applying a current of periodicity which was varied over a considerable range, they found for the unassisted thermophone without resonator that the intensity of response, measured by the sound emitted, fell regularly as the applied periodicity was raised. This result is similar to Krishnaiyar's, and the instrument is probably to be regarded as a telephone transmitter in which the vibrations, normally produced by electro-magnetic means, are in this instrument produced by rapid expansions and contractions of the air near the diaphragm strip under the action of the alternating current.

References

1. *Harmonicarum*, 2, 1635.
2. *Mém. Acad. Paris*, 1700 to 1711.
3. *Ann. Chim. Phys.*, **6**, 1, 1842.
4. *Sound*, **1**, 239, 1896.
5. *Ann. Physik*, **127**, 556, 1866.
6. *Phil. Mag.*, **15**, 84, 1883.
7. *Ann. Physik*, **20**, 619, 1906.
8. *Comptes Rendus*, **45**, 48, 1857.
9. *J. Sci. Instrum.*, **6**, 196, 1929.
10. *Zeits. tech. Phys.*, **8**, 509, 1927, and **9**, 491, 1928.
11. *Ann. Physik*, **50**, 444, 1893.
12. *Phil. Mag.*, **21**, 393, 1860.
13. Rayleigh, *Sound*, **1**, 200, 1877.
14. *Phil. Trans. Roy. Soc.*, **90**, 106, 1800.
15. *Ind. Assoc. Bull.* 15, 1918.
16. *Akad. Wiss. Wien., Ber.*, **123**, 1071, 1914. *See also* Kar, *Phys. Rev.*, **20**, 148, 1922.
17. Barton and Penzer, *Phil. Mag.*, **12**, 576, 1906; Barton and Richmond, *ibid.*, **18**, 233, 1909; Barton and Ebblewhite, *ibid.*, **20**, 456, 1910, and **23**, 885, 1912.
18. *Thesis, Königsberg*, 1908.
19. *Phil. Mag.*, **8**, 573, 1904.
20. *Proc. Camb. Phil. Soc.*, **18**, 15, 1915.
21. *J. Acoust. Soc.*, **19**, 886, 1947.
22. *Phil. Mag.*, **32**, 391, 1916, and **35**, 493, 1918.
23. *Zeits. tech. Phys.*, **8**, 509, 1927; **17**, 573, and **18**, 98, 1937.
24. *Ibid.*, **19**, 279 and 421, 1938; *Akust. Zeits.*, **4**, 89, 1939.
25. *Physica*, **4**, 361, 1937.
26. *J. Acoust. Soc.*, **17**, 169, 1946.
27. *Ibid.*, **12**, 399, 1941.
28. *Ann. Physik*, **38**, 117, 1940; *Zeits. Phys.*, **117**, 64, 1940.
29. *J. Acoust. Soc.*, **13**, 54, 1941.

30. *Sensations of Tone*, App. V.
31. *Ann. Physik*, **54**, 675, 1895.
32. RAMAN and BANERJI, *Proc. Roy. Soc.*, *A.*, **97**, 99, 1920. DAS, *Proc. Indian Assoc.*, **9**, 297, 1925, and **10**, 75, 1926; GHOSH, *Phys. Rev.*, **24**, 456, 1924, and **28**, 1315, 1926, and *Phil. Mag.*, **1**, 875, 1926.
33. *Phil. Mag.*, **47**, 591, 1924; **48**, 34 and 48, 1924; **49**, 92, 1925; **50**, 491, 1925; GEORGE and BECKETT, *Proc. Roy. Soc.*, *A*, **114**, 111, 1927; **116**, 115, 1927.
34. *Phil. Mag.*, **19**, 648, 1910, **20**, 652, 1910, and **22**, 113, 1911. GRÜTZMACHER and LOTTERMOSER, *Akust. Zeits*, **1**, 49, 1936; VIERLING, *Zeits. tech. Phys.*, **18**, 103, 1937; GHOSH, *Ind. J. Phys.*, **12**, 317 and 437, 1938; **13**, 23 and 277, 1939; **14**, 475, 1940; **15**, 1, 1941; DAVY, LITTLEWOOD and McCAY, *Phil. Mag.*, **27**, 133, 1939; BILHUBER and JOHNSON, *J. Acoust. Soc.*, **11**, 311, 1940.
35. HART, FULLER, LUSBY, *J. Acoust. Soc.*, **6**, 80, 1934. *See also* BRYAN, *Proc. Phys. Soc.*, **25**, 147, 1912.
36. EUSTIS, *Proc. Boston Soc.*, **7**, 218, 1880.
37. *Ann. Physik*, **65**, 849, 1898.
38. *Phil. Mag.*, **14**, 578, 1907.
39. *Sound*, **1**, 267, 1896.
40. *Ibid.*, **2**, 254, 1896.
41. *Phil. Mag.*, **8**, 581, 1904.
42. *Ann. Physik*, **13**, 124 and 271, 1904.
43. *Durham Phil. Soc. Trans.*, 1953. *See also* LANDON and DAVIES, *Phil. Mag.*, **26**, 816, 1938.
44. *Comptes Rendus*, **76**, 1198 and 1256, 1874, and **79**, 797, 1876.
45. *Ann. Physik*, **9**, 394, 1880.
46. *Akustik*, 1802.
47. *Ann. Physik*, **11**, 31, 1903.
48. *Proc. Roy. Soc.*, **74**, 118, 1904.
49. *Ann. Physik*, **13**, 124 and 271, 1904. *See also* APPLETON, *Phil. Mag.*, **47**, 609, 1924; MALLETT, *Proc. Phys. Soc.*, **39**, 334, 1927.
50. *Phys. Rev.*, **13**, 337, 1919.
51. *J. Sci. Instrum.*, **1**, 166, 1923. *See also* GALL, *ibid.*, **6**, 18, 1929; SCHUCK, *J. Acoust. Soc.*, **10**, 119, 1938.
52. *Proc. Phys. Soc.*, **31**, 269, 1919. *See also* BUTTERWORTH, *ibid.*, **32**, 345, 1921; DYE, *Proc. Roy. Soc.*, *A*, **103**, 240, 1923, and HODGKINSON, *Proc. Phys. Soc.*, **38**, 24, 1923; DYE and ESSEN *Proc. Roy. Soc.*, *A*, **143**, 285, 1934.
53. *J. Physique*, **9**, 225, 1920.
54. *Nature*, **18**, 111, 1878; *Phil. Mag.*, **13**, 331, 1907.
55. *La Roue Phonique*, 1878. *See also* WOOD and FORD, *J. Sci. Instrum.*, **1**, 166, 1923.
56. TABER JONES, *Rev. Sci. Instrum.*, **5**, 192, 1934.
57. *Ann. Physik*, **109**, 192, 1859; **111**, 513, 1860; **24**, 496, 1885.
58. *Phil. Mag.*, **24**, 513, 1912; *Phys. Rev.*, **32**, 309, 1911, and **4**, 12, 1914.
59. *Phys. Rev.*, **11**, 150, 1918, and **27**, 622, 1926.
60. LISSAJOUS, *Ann. Chim. Phys.*, **30**, 385, 1850.
61. *Ann. Physik*, **36**, 74, 1911.
62. *Phil. Mag.*, **48**, 636, 1924.
63. *Ann. Physik*, **77**, 727, 1925.
64. *Physik*, 1802.
65. *Ann. Physik*, **3**, 157, 1878.

66. *Phys. Rev.*, **7**, 102, 1898.
67. *Ibid.*, **13**, 145, 1901.
68. *Ann. Physik*, **45**, 657, 1914, and **46**, 1, 1915.
69. *Phys. Rev.*, **14**, 490, 1919, and *Phil. Mag.*, **43**, 503, 1922.
70. *Proc. Roy. Soc.*, *A.*, **91**, 239, 1914.
71. *Phys. Rev.*, **10**, 22, 1917. *See also* WENTE, *ibid.*, **19**, 333, 1922.

CHAPTER FIVE

MEMBRANES AND PLATES

We now turn to the transverse vibrations of bodies having extent in two dimensions, and just as we considered the transverse vibrations of the theoretically one-dimensional bodies under two headings, i.e., strings and bars, so we divide the two-dimensional bodies into membranes in which the restoring force is the tension alone and plates, where the restoring force is the bending moment. The former must have a fixed boundary along which the tension is applied, and the only mathematically simple and practically important case is that in which the tension is equal all along the boundary.

Supposing displacements to take place in the z direction, while the membrane at rest lies in the x, y plane, the equation of motion, by analogy with that of one-dimensional waves (2), may be written :—

$$\frac{\partial^2 z}{\partial t^2} = V^2\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

where the velocity of propagation of transverse waves is as for strings $V = \sqrt{\frac{F}{m}}$, but m is now the mass per unit area $= \rho d$ (ρ = density, d = thickness in the z direction).* Equation (50) with the boundary condition that $z = 0$ at the boundary of the membrane, may be used to specify the motion at any point (x, y) of the membrane.

Rectangular Membrane. If a, b are the lengths of the side, in the x, y direction respectively, and the origin of co-ordinates be taken at one corner of the rectangle, our boundary conditions are, $z = 0$, when $x = 0$ or a, and when $y = 0$ or b. These conditions and equation (50) are satisfied by :—

$$z = \sin\frac{j_a \pi x}{a}\sin\frac{j_b \pi y}{b}\sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

On differentiation and substitution it appears that :—

$$\omega^2 = V^2\pi^2\left(\frac{j_a^2}{a^2} + \frac{j_b^2}{b^2}\right).$$

The series of partials of the membrane accordingly have frequencies

* Cf. Lamb, *Dynamical Theory of Sound*, p. 141.

given by putting $j_a = 1, 2, 3$, etc., $j_b = 1, 2, 3$, etc., in the formula :—

$$n = \frac{\omega}{2\pi} = \frac{V}{2}\sqrt{\frac{j_a^2}{a^2} + \frac{j_b^2}{b^2}} = \sqrt{\frac{F}{4\rho d}\left(\frac{j_a^2}{a^2} + \frac{j_b^2}{b^2}\right)} \quad . \quad . \quad (52)$$

In consequence of (52) some of the partials are harmonic to each other. Owing to the lower tones all lying close together in the scale, the general note is a noise in which the lowest tone ($j_a = j_b = 1$) predominates. For the other partials, we have nodal lines instead of the nodal points of a string; their equation is given by putting one or other of the amplitude coefficients in (51) equal to zero, i.e., when $\frac{j_a x}{a}$ or $\frac{j_b y}{b}$ is a whole number.

Circular Membrane. Equation (50) can here be employed more readily in the polar (r, ϕ) form :—

$$\frac{\partial^2 z}{\partial t^2} = \frac{V^2}{r^2}\left(\frac{\partial^2 z}{\partial \log r^2} + \frac{\partial^2 z}{\partial \phi^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (53)$$

with the boundary condition, $z = 0$, for $r = r_0$, the radius. This leads to a solution involving Bessel functions, and to the frequency of the lowest partial $\frac{\cdot 764}{r_0}\sqrt{\frac{F}{4\rho d}}$.* The nodal lines are composed of concentric circles and of radii. All the partials are inharmonic.

The existence of these nodes can be demonstrated, if the membrane in question is large enough, by Chladni's device of strewing fine sand on the membrane; the sand tends to collect in the still places. The membrane may conveniently be excited by tapping with a hammer. By comparing the emitted tones with a sonometer the nineteenth-century workers endeavoured to test the theoretical formulæ. For this purpose, membranes of paper, rubber, skin, etc., were employed, but all these substances possess stiffness in greater or less degree, involving departures from the condition of no vibration in the absence of tension. The difficulty of sustaining a constant tension along the boundary also troubled these experimenters. The nearest approach perhaps to the ideal membrane, and one whose movements may be optically projected, is the soap-film or glycerine-film. Owing to casual changes of density and tension, the natural vibrations of such membranes are very inconstant.

The formula (52) shows that the lowest partial of a membrane

* *See* Lamb, p. 146.

can be made high in the scale by making the tension sufficiently large, the radius and breadth on the contrary small. Membranes under these conditions are often used as accurate recorders of sound, since their resonant range can be placed beyond the average musical note, and at the same time, if the tension is not too great, they are sufficiently sensitive for the purpose. Examples in plenty will appear in the following chapters, in which the forced vibrations of a membrane are thus employed. The Western Electric Co. have made the diaphragm of their reproducing instrument so rigid that it has a natural frequency of 3,000. We have already mentioned the loaded "asymmetrical membrane" of Waetzmann designed to represent the ear-drum (p. 62).

A membrane of opposite characteristics is the conical papier-mâché diaphragm, a foot or so in diameter, employed as a loud speaker for amplifying broadcast music; this is made large and fairly thick to give a marked resonance in the bass of the musical scale, and bring out those low notes to which the usual small diaphragm cannot resound.

Drums. It might be thought that this inharmonic series of overtones proper to a membrane would preclude its employment as a musical instrument. As a matter of fact, most types of drum are mere rhythm markers; they are not intended to take part in the harmony. Exceptionally, the kettle-drums, hemispherical shells containing a cavity of air which resonates to the fundamental of a skin stretched over the top, are tuned to a definite pitch. The tuning is done by altering the tension in the membrane, which is struck by a soft hammer at about a quarter of the diameter from one edge; this is the point at which experiment has shown the inharmonic overtones to be most stifled. The bass drum has two skins one at each end of a cylindrical cavity. These skins may vibrate in or out of phase; only in the latter case when both move inwards together, is the air within compressed and rarefied.

Ghosh [1] has drawn attention to certain Indian drums, in which, owing to progressive decrease of the mass per unit area of the membrane from circumference to centre, the partial tones are made to approximate to a harmonic series. Rayleigh [2] attributes to the drum proper another function besides resonance; it serves to spread its sound more uniformly, whereas an unassisted membrane would concentrate the sound in the direction of the vibration.

Plates. Theoretically the most complicated type of transverse vibration, the tones of plates represent an extension into two dimen-

sions of the case of the transversely vibrating bar. Analysis[3] shows the analogy to hold good in the calculation of the lowest partial tone of a plate, which is directly proportional to the thickness, and inversely as the square of the diameter or other linear dimension, depending on the shape of the boundary.

For a circular plate, clamped at the rim, Lamb[4] deduces $\frac{\cdot 47 Vd}{r^2}$ for the lowest partial, V being the velocity of longitudinal waves in the material. When the material is in the form of a plate, as opposed to a rod, $V^2 = \frac{E}{(1 - \mu^2)\rho}$, where μ here is the ratio of lateral contraction to longitudinal elongation (Poisson's ratio). The upper

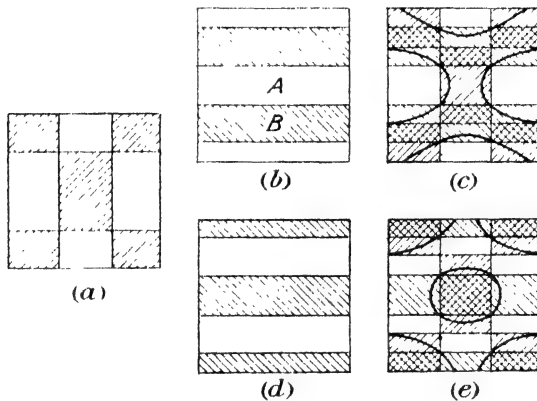



Fig. 46.—Vibrations of Plates (*Zeissig*).

partials, which are of course inharmonic, differ only in the numerical multiplier.

Experimental work on large plates originated with Chladni,[5] and the method of Chladni's Figures survives to this day. The plates used by Chladni were of brass, and fixed to a central pillar. This makes the central point a node; to obtain a vibration corresponding to a given partial it is sufficient to hold the plate lightly at some point on an appropriate nodal line, and to bow where an antinode intersects the edge. Both square and circular plates can be used, and the nodal lines shown by sand. Types of vibration are shown in Fig. 46. That in such cases neighbouring segments vibrate in opposite phases was shown by Lissajous,[6] who held a Y-shaped interference tube over a pair of adjacent segments such as *A* and *B*; in this position little or

no sound can be heard on putting the ear over the main branch of the Y tube, the other ear being stopped. As segment *A* is moving up while *B* is moving down, their motions are in fact in dead opposition of phase, so that the resulting sound waves in the branch tubes completely interfere on combining. The Chladni Figures of plates of other shapes possess academic interest merely. Of special importance is the small thin metal plate clamped at its circular edge, on account of its widespread use in the telephone.

Chladni Figures of steel plates may readily be produced by a method first used by Hort and König.[7] An electro-magnet is held near the plate and alternating current of variable frequency fed to it. As each overtone frequency is reached the corresponding pattern appears in the dust on the plate. Alternatively a taut wire attached to the centre of the plate may have the alternating current passed through it. A telephone receiver held with the diaphragm horizontal may be dusted while the coils are supplied by the current. "Chattering" of the sand on the plate may be produced under certain conditions (cf. p. 83), but lighter particles may remain in permanent suspension over the plate following the vortex currents which are produced in the air above (see also p. 215).

Zeissig [8] has given a simple explanation to the shape of the nodal lines on a rectangular plate. He considers the convolutions of the plate to be produced by the superposition of two sets of stationary waves on the plate; each set having linear nodes and antinodes parallel to one pair of edges. This is shown in the figure, where shaded parts of the plate are depressions, and unshaded parts are elevations; on adding the figures in (*a*) and (*b*), we obtain that in (*c*) with nodal lines approximately represented by the thick line. The original tones, in accordance with theory, have frequencies proportional to the thickness, and inversely as the square of the length of the edges. The complex vibration resulting from the superposition changes with the relative phase of the components, so that in general the nodal forms pass through a cyclic change, cf. Fig. 46*e*, formed by the superposition of (*a*) and (*d*).

Trevelyan Rocker. When Trevelyan [9] in 1831 accidentally discovered that a hot iron fork laid on a block of lead gave rise to musical sounds, he set to work to devise a model which should best exhibit the effect, and produced a prismatic block of copper having a groove on its under surface so that it would rock about the ridges on the lead block, the other point of support being a knob at the end of a thin

round handle, generally known as Trevelyan's rocker. Leslie suggested a theory of the action, which was accepted by Trevelyan. Trevelyan found it necessary to have the ridges very smooth and clean, but the lead was best roughened. Heat being communicated to the lead by the copper, the rugosities of its surface were supposed to expand, to push up each ridge in turn, and then to contract as the heat diffused through the lead, the rocking being due to inequality of inertia of the portions of the rocker on opposite sides of the ridge, causing a lateral movement. Faraday, as a result of experiments with various pairs of metals, went further and said that the success of the experiment depended on the difference of conductivity of the two metals; the hot one must readily transfer its heat to the cold one, but the heat must not be able to diffuse rapidly into the latter, but remain near the point of contact causing local expansion; it was immaterial which one formed the rocker, cold lead would vibrate on a hot copper block.

In further support of the theory that the phenomenon was maintained by heat, Page [10] made a light rocker vibrate on two rails connected to the terminals of an electric battery, the heat being produced by the thermo-electric effects at the points of contact.

This theory was put to test by the writer,[11] and also by Bhargava and Ghosh,[12] using different methods. In both researches, corresponding values of frequency and amplitude were found, resulting in satisfactory confirmation of Leslie's theory. The writer's method of measurement was as follows.

A block of lead with rounded top is screwed down to the bench. A small hole is bored through the rocker, through which a thick steel knitting needle is thrust and held in position by a small screw, so that the needle lies horizontally across the rocker, when the latter rests with its two ridges across the lead block. The end of the needle or indicator is observed through a microscope. The light by which the field of the microscope is illuminated comes from a little electric lamp placed behind the slits of a stroboscopic disc, driven by an electric motor. When the disc is rotating, therefore, the illumination is intermittent, and its period may be made to coincide with that of the rocker, and setting the disc slightly out of step with the rocker, the amplitude at the end of the indicator could be readily observed as the needle appeared to move slowly up and down. No resonant vibration of the indicator was observed, so that the amplitude at the ridge could be directly calculated from the observed motion of the

end of the needle (Fig. 47). Observation confirmed that the rocker oscillates about a point halfway between the ridges, gravity being the restoring force. The time-period of oscillation was proportional to the square root of the amplitude of motion.

Mary Waller [13] has discovered an interesting extension of this method of maintaining a solid in vibration, viz., by contact with solid carbon dioxide. The conditions for success are the same, i.e., a good conductor placed on a colder bad conductor of heat. With the solid carbon dioxide, however, another factor intervenes which makes for greater efficiency and allows of the maintenance by heat transfer of the vibrations of objects of quite high natural frequency which would

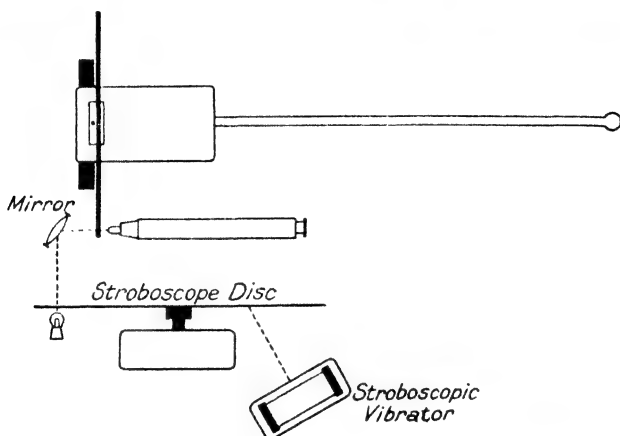



FIG. 47.—Apparatus for Tones of a Trevelyan Rocker.

be difficult to manage with the usual lead block. This factor is that the heat transfer sublimes the carbon dioxide, setting up considerable gas pressures which help to tip the plate over. In this way she has been able to excite the Chladni figures of small plates of circular and other shapes.

Telephone Diaphragm. The thin steel plate commonly employed for telephony may be considered as a stage between the theoretical membrane and the theoretical plate, with a decided bias towards the latter. Without entering into all the electrical details it is sufficient to say that the steel diaphragm in the common form of transmitter is placed in the field of a magnet. Aerial waves impinging on the diaphragm cause it to vibrate and to produce changes in the neighbouring magnetic field, by its displacement. Enclosing part of

the field is a coil forming part of an electric circuit, in which alternating currents are induced by the changing magnetic field. At a distant point in the same circuit is a similar coil, magnet, and diaphragm forming the receiver, where the reverse process takes place, the electric waves being more or less faithfully reproduced as aerial (sound) waves. Usually a battery is placed in the electric circuit, so that the induced currents are mere ripples on the steady current, but the battery is not essential. By suitable arrangements, either in the direction of amplifying the oscillatory current at the receiving end and so increasing the amplitude of vibration of the diaphragm, or by increasing the size of the latter so as to present a larger vibrating surface, or by adding a horn, or usually by a combination of all these methods, the " loud-speaker " is produced, to the end that the ultimate air-waves may be of sufficient intensity to be heard at a distance from the receiver.

The electro-magnetic effect is not the only one which has been used for telephones and loud-speakers. The " effects " which have been used are :—

(1) The electro-magnetic ; currents set up in an electro-magnet by changes in the magnetic circuit.[14]

(2) The electro-dynamic ; currents produced in a conductor by its movements across a magnetic field.

(3) Magneto-striction ; changes in length of a nickel wire in a magnetic field make changes in the latter.[15]

(4) Condenser ; the diaphragm forms one plate of a condenser, whose capacity is changed by its movement, and causes current oscillations in the circuit containing the condenser.

(5) Induction coil ; one coil (incorporating the diaphragm) moves in and out of the other.[16]

Corrugations and conicality lower the natural frequency of the diaphragm. By using the second principle enumerated above, the Siemens-Halske firm has produced a loud-speaker whose diaphragm, a very thin corrugated aluminium leaf, has a frequency (about 20 vibrations per second) below the audible limit of pitch. The familiar pleated paper diaphragm of Lumière has also a low frequency, and a cone-shaped membrane has similar properties.[17] A diaphragm of low frequency free from overtones can be attained by loading a plate with a central heavy boss.[18] It is to be noticed that the action of these instruments is reversible ; if supplied with a sinusoidal current, they excite vibrations in the diaphragm, so that the instrument then acts

as sound-emitter or signal; while if the aerial sound waves impinge upon them, they excite corresponding currents in the circuits to which they are connected, acting as receivers.

Investigations on the acoustic side of the response of telephone diaphragms have been directed towards two desiderata, (1) faithfulness of reproduction of the aerial waves by the diaphragm, and (2) sensitivity of response—two aspects which are however indissolubly connected. This point will be further elaborated in connection with the analysis of sound, but it should be obvious that it is the resonance curve for the diaphragm which must be obtained for a study of this question, which means to say that we have to apply forcing vibrations of constant intensity but progressively changing frequency to the diaphragm, and to measure the corresponding amplitude of the forced

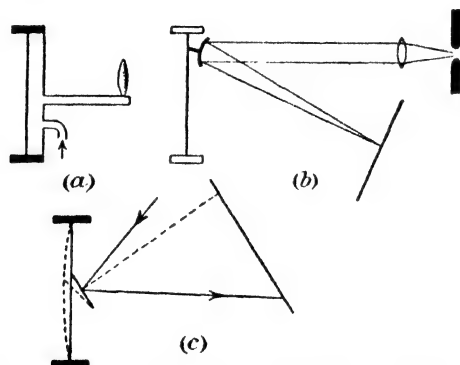



Fig. 48.—Methods of Recording Vibrations of Diaphragms.

vibration. The forcing vibration is usually applied by an alternator of constant output producing oscillations in the electro-magnet over which the diaphragm is placed.[19] The response can be measured in a number of different ways.

(1) The diaphragm is covered with a capsule to which a supply of gas is admitted, and burnt at a pinhole burner (Fröhlich).[20] By this means a "manometric flame" is constructed (cf. p. 183) whose oscillations copy those of the diaphragm, but the manometric flame is unfortunately unreliable for quantitative measurements of amplitude (Fig. 48*a*).

(2) A mirror affixed at a point near the edge, so that movements of the diaphragm cause angular deviations in a beam of light reflected by the mirror on to a scale (Hartmann-Kempf) [21] (Fig. 48*b*).

(3) A mirror placed at the centre would move only parallel to itself and would not be tilted, but by connecting this point to the leg of an optical lever, its oscillations may be recorded (Kennelly and Taylor) [22] (Fig. 48*c*).

(4) A peg may be arranged to stick out at right angles in the centre of the mirror, and to cut off periodically during the oscillation an exceedingly narrow beam of light passing over the edge of the peg (Siegbahn).[23]

Recent methods of recording the movement of membranes are electrical in principle:

(5) A strain gauge in the form of a wire element sandwiched between the membrane and a paper cover stuck on with adhesive has its electrical resistance changed as the resistance of the wire fluctuates. This is naturally suited only to large and fairly thick diaphragms. Fig. 49*a* shows diagrammatically such a gauge stuck to the diaphragm on the right and forming one arm of an A.C. resistance bridge.

Fig. 49.—Recording Membrane Vibrations (*a*) by Strain Gauge; (*b*) by Change of Capacity of Condenser.

(6) The diaphragm may be made to form one plate of an electrical condenser by metallizing part of its surface, if necessary; the other plate being a fixed piece of metal in its proximity. (See Fig. 49*b* and cf. the condenser microphone, Fig. 51.) Both strain gauge and condenser lend themselves to the recording by deflection of the beam of a cathode ray oscillograph (p. 203).

All these methods (except 1 and 6) are open to the objection that they change in some degree the natural period of the diaphragm by adding inertia. For this reason Sell [24] preferred to measure the response in the neighbouring air itself by a resonator, but the introduction of this additional receiver of unknown sensitivity is a remedy probably worse than the disease.

Results tally with the theory of the circular plate as regards dependence on radius and thickness; the numerical factors given by theory are, however, more or less departed from, because of the stiffness and plasticity of the diaphragm. The amplitudes at

the centre are of the order 0·00001 cm. (telephone) to 0·05 cm. (loud-speaker).

The Carbon Microphone. In Bell's original telephone both transmitter and receiver were of the type outlined above ; Edison and later Hughes developed a new form of transmitter, which has remained in principle to the present day. Instead of producing the varying currents in the " line " by induction, they broke the circuit at the transmitting end, and completed it through pieces of carbon loosely packed together. The electric resistance between carbon blocks varies considerably with the pressure—since the granules are elastically deformed thereby—and by allowing a boss on the diaphragm to press upon the conglomeration of carbon, changes of the current from the line battery are set up by the oscillations of the membrane. These are re-converted into movements of the diaphragm at the receiving end in the original fashion, or, for experimental purposes, measurements of the original vibrations of the transmitting diaphragm may be made by studying the " wave-form " of the current in the line. In this way Powell and Roberts [25] have added a further method for the study of diaphragm vibrations by observing the oscillations set up in a circuit containing a carbon-contact microphone touching the diaphragm.

Ribbon and Crystal Microphones. These are the commonest types for broadcasting. The ribbon microphone has the moving element a very thin (about 3 microns thick) band of aluminium moving between pole pieces on the electro-dynamic principle, so that currents are induced in it as it moves to and fro under the influence of sound pressure. It has a uniform response over the range 100 to 6,000 cycles/sec. and has such polar characteristics that it responds mostly to sounds coming from straight ahead and less to those arriving obliquely, a desirable type for broadcasting.

The crystal type has a rochelle salt specimen which, sandwiched between two electrodes, is bent under the action of sound waves which are directed upon it laterally. The potential set up between the electrodes (see Chap. X) is the " response." The response after being independent of frequency rises in the neighbourhood of 5,000 cycles/sec. Consequently, by a suitable combination of electro-dynamic and crystal microphones it is possible adequately to cover the whole audible gamut except the lowest frequencies.

Sensitivity of Microphones. Calibration of microphones at a given frequency may be carried out by traversing them along a

column of air in which stationary waves have been set up, assuming that the pressure or velocity amplitude is given by a formula such as (62), p. 166. It is, of course, essential that the presence of the microphone shall not upset the stationary wave system, which restricts this method to small microphones and wide air columns.

To study the effect of damping on the sensitivity two diaphragms are constructed, as nearly as possible identical, and the damping of one is varied without, if possible, altering the natural frequency, and its sensitivity of response at different values of the damping coefficient is compared with the unaltered one as standard. The comparison is effected by placing them in telephone receivers at equal distances from a source of constant frequency. The one which responds to the greater extent is " shunted "—part of the current induced in it is not allowed to pass through the detecting instrument—until the registered current response in the instrument is the same from both the shunted and the unmodified receiver. The relative response can be calculated from the value of the shunt required.[26]

When a diaphragm is bent by pressure the frequencies of all the partial tones are altered. King [27] has turned this fact to account in order to tune a diaphragm to respond to a signal. The dimensions of a plate cannot be conveniently varied in order to alter its frequency, so that diaphragms to resound to a signal of given frequency were formerly constructed on the " cut and try " method ; but by covering the diaphragm on one side with a reservoir into which air is pumped, a diaphragm at a distance—under the sea, for example—may have its frequency lowered until this is equal to that of the signal.

Valve-maintained Vibrations. In connection with acoustic experiments in general, and with sound-signalling in particular, a simple maintained source of sound is often required. Formerly an organ-pipe or siren was used, but when a source of moderate power taking up a small space is required, it is often convenient to use a telephone receiver of the simple cell type and maintain the diaphragm in oscillation by an alternating or intermittent current in the coil. Such a system may be compressed into a cubic inch, and forms the nearest approach to a " point source " for sound, as the apparatus supplying the current can be placed at any distance from the diaphragm. The latter is too small to permit of an intermittent contact like that applied to the tuning fork, but the source of energy may be an alternator, running at the appropriate speed to produce forced vibrations of the diaphragm at the desired frequency; if this

is one of those natural to the diaphragm the sound will be loud. The "triode valve" offers to the modern experimenter a more convenient means of maintaining the diaphragm in oscillation. A circuit for this purpose is shown diagrammatically in Fig. 50.

The branch between plate P and filament F contains self-inductance L and capacitance C in parallel. It is well known that such a circuit has a natural period of oscillation for alternating current of $2\pi\sqrt{LC}$ (cf. p. 48). Then, with a given value of L and C, oscillations of the frequency $\frac{1}{2\pi\sqrt{LC}}$ are produced by the valve, which react by means of the mutual inductance M in the grid circuit, on the diaphragm D with its accompanying electro-magnet in the plate circuit. Thus by fittingly adjusting L or C, vibrations of any frequency can be impressed on the diaphragm. The mutual inductance M must be "negative." In practice this means the connections to the terminals of one of the coils of M may need interchanging. The same apparatus will furnish alternating current of a desired frequency to the coil of a maintained electro-magnet. When a source of high frequency is looked for, a limit appears to the frequency at which an untuned diaphragm may be set in forced vibration. The high-frequency source must be driven at a natural frequency and the diaphragm is then replaced by a slice of crystalline quartz set in piezo-electric oscillation or a rod of magnetic material which can be driven in magneto-striction (cf. pp. 250, 327).

Fig. 50.—Valve-Maintained Oscillations.

The Condenser Microphone. Most of the above types of microphone suffer to a greater or less extent from the disability that their response is non-uniform over the musical scale. Recently an electrostatic form of microphone, with transmitters and loud-speakers on the same principle, has been developed out of the original form due to Wente. In this microphone a layer of air is sandwiched between a thin metal diaphragm clamped at its edges and another fixed metal electrode embedded in an insulator such as ebonite [28] (Fig. 51). A steady potential difference is applied to the electrodes causing the diaphragm to be attracted inwards into the

shape of a paraboloid, leaving an air space only about 10^{-3} cm. thick between. Any motion of the membrane will cause a ripple in the potential difference between the plates due to the change in capacity of the electrostatic condenser, which can produce a current in a string galvanometer or valve amplifier in whose circuit it is connected. The reason for the distortionless response of this microphone is to be sought in the action of the air film. At low frequencies this provides viscous damping, as the air surges to and fro from centre to circumference with the motion of the diaphragm. At high frequencies the motion is too rapid for the air to follow so that it simply acts as a buffer. At low frequencies then, the system provides extra *resistance* above that natural to the diaphragm, at high frequencies comes extra *stiffness*, which will be a function of the frequency (cf. p. 227). Finally,

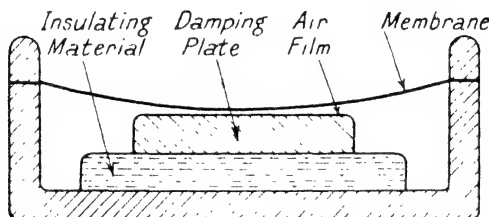



Fig. 51.—Condenser Microphone.

these two factors are found to act in such a way as to give uniform response over a considerable region of frequency (300 to 5,000 c./sec as usually manufactured).

Both diaphragm and air film must be extremely thin. Jakoweff [29] notes that resonance in the air cavity is innocuous if its natural frequency is two or three times that of the highest note to be recorded and if the damping factor is 1·35 times this natural frequency.

In an attempt to produce a small acoustic measuring instrument which should not unduly disturb the field in which it may be placed, Hall [30] has succeeded in constructing a condenser microphone, the diaphragm of which consists of a sheet of aluminium foil 0·0004 in. thick and 2 cm. in diameter, put under tension by screwing down a brass ring over the edge of a hard rubber bushing. The brass shell into which the system was screwed served as the other plate of the condenser. Used as a transmitter the system has an output of about one millivolt per bar and is practically independent of frequency up to 6,000 cycles/sec. Such an apparatus—even smaller ones might

be constructed—offers great possibilities for the plotting of sound fields where larger microphones would produce distortion of the field.

Velocity or Pressure-gradient Microphone. Instruments such as those just described measure pressure or pressure amplitude. Occasionally it is desired to measure the particle velocity instead of or as well as the pressure. If a mass m is acted on by a force f of frequency $n = \omega/2\pi$, and has a displacement y, the ratio of f to the particle velocity $\partial y/\partial t$ is ωm. If, then, the response of the mass is to be proportional to $\partial y/\partial t$ at all frequencies f must be proportional to n. Olson [31] secured this condition in the ribbon microphone in which a thin corrugated metal ribbon, usually of aluminium, is suspended in the magnetic field between flat pole pieces and exposed to the sound field *on both sides.* The force acting on the ribbon due to the oncoming sound is proportional to the difference of pressure over the two exposed sides of the microphone, that is, to the velocity if the distance in question is small compared to the wave-length (cf. p. 221). Such a microphone has marked directional characteristics, especially when placed in a baffle plate.

Bells. Plates are used as instruments of percussion in music under a number of different forms, with or without air cavities, as cymbals, gongs, etc. That shape which possesses most interest from a physical point of view is the bell. The theory of this instrument has been attacked mathematically only by making several reservations. On the one hand the bell may be regarded as a development of the hollow rod, on the other as a bent plate of, generally, non-uniform thickness. The use of hollow cylinders in imitation bells has already been referred to (*glockenspiel*). The form of the church bell is more nearly approached by the half-shell, a form which has been considered by Chladni [32] and Rayleigh.[33] The latter made experiments on church bells proper, of various sizes, determining by ear the inharmonic tones which make up the sound. The commercially made bell is of special metal (bell metal, about 80 parts of copper to 20 of tin) ; a common form is shown in Fig. 52*a*. The clapper hangs loosely inside, and smites the bell on the " sound-bow " *SB* near the open end, where the inside and outside sections have opposite curvature. Only the smallest bells are rung by hand, a handle being placed at the centre of the upper surface for the purpose. Large bells are " swung " by means of a wheel and axle, passing through the same point. Bells may also be kept still while they are struck, usually on the outside, by a hammer,

Lord Rayleigh,[34] and latterly Biehle,[35] examined a large number of bells, and found that the various partial tones formed by division into a number of segments, formed a series approximately as follows :—

Approx. Relative Pitch.	*Nodal Lines.*
1. Fundamental	4 sectors
2. Octave	4 sectors and ring
3. Octave and minor third	6 sectors
4. Twelfth	6 sectors and ring
5. Double octave	8 sectors etc.

It is to be understood that these relations are approximate only, as the tones of a bent plate are inharmonic, but the bell-founder, by adjusting the thickness at various sections, strives to make the lower tones as nearly harmonic as possible. In practice only one nodal ring

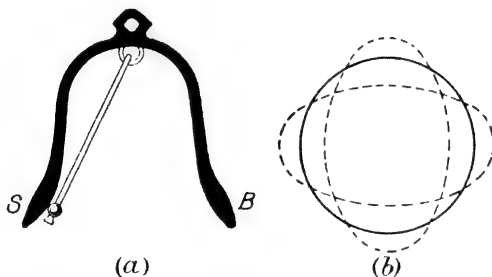



Fig. 52.—The Church Bell.

is formed on a bell ; this eliminates some of the discordant overtones pertaining to a plate. Beside these tones which can be approximately predicted by applying the theory of plate vibrations, there is another tone which, immediately after striking, overpowers these but decays more rapidly. This is known as the " striking note " and its pitch, by which the founder names the bell, seems to lie near that of the octave or second in the series of partial tones ; in a good bell the striking note is made coincident with or harmonic to this second overtone, even if the other overtones have to be left inharmonic to each other. The occurrence of this striking tone is very curious, and has so far baffled explanation. The partial tones can be elicited by resonance with a tuning fork, but not so the striking note ; nor can the latter be picked up by a resonator, or made to produce beats with a neighbouring tone. Its origin may be subjective—formed in the ear itself—but its abnormal intensity is against this idea ; however

Biehle found that it was most prominent when the lower tones of the bell were loudly produced, and as it dies out rapidly compared with the normal tones of the bell, it may be formed in the ear by the large "forcing" of the initial stroke. On the other hand Taber Jones [36] thinks that the phenomenon is an aural illusion; that the striking note is really the fifth partial (double octave) but that its tone location is masked by the lower overtones, making it seem to be in the lower octave.

Further work on bells has been done by Jones and Alderman,[37] also by Curtiss and Giannini.[38] The former have determined the acoustic spectrum of a bell at various intervals after striking. Since the bell is theoretically a struck anisotropic plate we find analogies with the struck string in the rate of decay of partials. The variation in this respect is one of the characteristics of bell tone. The latter workers have succeeded in examining the nodal lines by the method of Tyzzer,[39] viz., excitation of each partial separately by means of a valve oscillator and loud-speaker, and exploration of the vibrating surface with a sort of stethoscope. Whereas, immediately after striking, the fifth partial is the strongest, after 1 sec. it has practically disappeared leaving the third partial master of the field. In 12 secs. this too has nearly fallen below audibility, leaving the fundamental which shows the least attenuation of all.

RELATIVE AMPLITUDES OF BELL OVERTONES AT VARIOUS TIME INTERVALS AFTER STRIKING. (*Jones and Alderman.*)

Order of Partial	1	2	3	4	5	7
At strike	1	·7	2·4	·7	4	2·8
After ½ sec.	1	·6	1·8	—	·8	·75
,, 1 sec.	1	·6	1·8	—	·1	·4
,, 2 sec.	1	·5	1·1	—	·1	—
,, 3 sec.	1	·4	1	—	—	—
,, 7 sec.	1	·4	·8	—	—	—
,, 12 sec.	1	·2	·4	—	—	—

This is for a tenor bell of 345 cycles/sec. In the treble the bells have a thicker waist (zone of minimum thickness of metal) and the attenuation of the partials is more nearly uniform. Giannini thinks, however, that the large bells with finer waists which have the acoustic properties outlined above have a better and more characteristic bell

tone and that this pattern (thick sound bow and thin waist) should be adopted throughout the peal.

The nodal lines are positions of no motion normal to the surface, but a considerable circumferential movement may take place at points on these lines. This is shown by the accompanying figure of a section of the bell when sounding its fundamental (Fig. 52*b*).

Another point of interest noted by Rayleigh is that these nodal "lines of longitude" tend to move slowly round the bell causing a waxing and waning of intensity to a stationary observer as the antinodes and nodes are directed towards his ear. The nodes may be traced out by touching a suspended pith-ball to various points on the surface; the normal motion will throw the ball off, but at the nodes it will remain undisturbed. For such experiments a hollow hemispherical glass bowl suffices. The note may be excited either by striking, bowing the edge, or rubbing a wet finger along the surface, or by simple resonance with an appropriate fork. With the first two methods the point of excitation naturally becomes an antinode; with tangential rubbing the point is usually, at first, a node.

Attempts to imitate bell-tone by gongs made of steel wires bent into a spiral and struck at the free end with a hammer have been made by clockmakers with a certain amount of success. Investigations have been made by Pomp and Zapp [40] to determine what is the best type of steel for this purpose. Carbon steel (0·63 per cent.) tempered for 15 mins. at 300° C. gave the maximum duration of note.

Musical Sand. An interesting phenomenon, which at first sight seems to involve the elastic vibrations of spheres, is the production of sound from certain grains of sand when struck by a rod. This was discovered by Miller at Eigg in 1850. In England there is, or was until recently, a narrow patch of sand running along the shore of Studland Bay between Sandbanks and Swanage which would emit sounds of definite pitch. The grains must be more or less of the same size, and can be excited by blows with a stick. The objection to the theory that the tone is due to elastic vibrations of a granule in a number of sectors as in a solid bell, is that the fundamental of such small spheres would be of very high pitch. Carus-Wilson,[41] who has made a detailed study of the Studland sand, ascribes the note to simple friction between highly polished grains of quartz, but it is not clear why the pitch of the emitted note is then so definite.

Musical Bubbles. The sounds of air bubbles breaking in water, to which the purling of a brook is due, cannot, for the same reason

that the pitch would lie outside the audible limit, be ascribed to compressional vibrations of the air pocket. Minnaert [42] has, however, shown that the frequencies of possible *radial* elongations and contractions of the liquid envelope agree closely with those heard. Moreover, the tones are practically independent of temperature, which would not be the case if the vibrations of an air resonator were in question.

Meyer and Tamm [43] have continued this work in a laboratory research in which the bubbles were produced by electrolysis of the liquid in a vessel which also contained a source of high-frequency vibrations (1,500 to 35,000 per sec.). When the source was in action, they noted that its frequency governed the size of the bubbles liberated in such a way that the product of this frequency and the mean diameter of the bubbles was constant. This is what one would expect if the vibrations of the bubble were radial. It was possible to see under a microscope when resonant vibrations of the bubble took place, because it then took on a matt instead of a sharply defined appearance.

While the bubble was maintained in forced oscillation at this resonant frequency, it was allowed to grow in size by further electrolysis so that its natural frequency decreased. While this was happening the response of the bubble was measured by its reaction on a nearby microphone in the liquid. On the assumption that the natural frequency was inversely as the radius of the bubble, its response to various degrees of mistuning with the forcing vibration was obtained and so its damping coefficient calculated by applying an equation like (27) on p. 49. Larger bubbles—in resonance to frequencies below 7,000 cycles per sec.—had their amplitude of forced vibration directly measured by casting their images on a photo-electric cell, in which an alternating current proportional to the amplitude in question was set up.

An increase in the viscosity of the liquid naturally increased the damping of the vibrations of the gaseous cavity. The damping also increased in linear fashion with the natural frequency of the bubble. Finally, the noise emitted in the hiss of bubbles escaping from a fine jet submerged in water was analysed for frequency. The maximum noise level comes from those bubbles (as judged by their natural frequency) which have a diameter equal to that of the orifice. Again, the product of the frequency showing maximum intensity and the diameter of the orifice was a constant.

The presence of many bubbles in water through which sound is passing lowers its velocity owing to the diminished elasticity.

References

1. *Phys. Rev.*, **20**, 526, 1922.
2. *Phil. Mag.*, **7**, 149, 1879.
3. cf. Rayleigh, *Sound*, **1**, 293, 1877.
4. *Proc. Roy. Soc.*, *A*, **98**, 205, 1920.
5. *Entdeckung. in d. Theorie d. Klanges*, 1787.
6. *Comptes Rendus*, **40**, 133, 1855.
7. *Zeits. tech. Physik*, **9**, 373, 1928.
8. *Ann. Physik*, **64**, 360, 1898. *See also* Wood and Smith, *Proc. Phys. Soc.*, **7**, 149 and 794, 1935.
9. *Phil. Mag.*, **3**, 321, 1833.
10. *Amer. J. Sci.*, **105**, 1856.
11. *Phil. Mag.*, **45**, 946, 1923.
12. *Phys. Rev.*, **22**, 517, 1923.
13. *Proc. Phys. Soc.*, **45**, 101, 1933; **46**, 116, 1934; **49**, 522, 1937; **50**, 70, 1938; **51**, 831, 1939; **52**, 452, 1940.
14. Bell, *Brit. Ass. Repts.*, 1876 and 1877.
15. Ader, *Comptes Rendus*, **88**, 575, 1879.
16. Meissner, *Zeits. Physik*, **3**, 11, 1920.
17. Gaumont, *Comptes Rendus*, **175**, 1051, 1922.
18. Gerdien, *Phys. Zeits.*, **22**, 679, 1921.
19. Hahnemann and Hecht, *Ann. Physik*, **60**, 454, 1919, and **70**, 283, 1923.
20. *Elekt. Zeit.*, **10**, 65, 1889.
21. *Ann. Physik*, **8**, 481, 1902.
22. *Proc. Amer. Phil. Soc.*, **54**, 96, 1915.
23. *Ann. Physik*, **42**, 689, 1913, and **46**, 298, 1915.
24. *Wiss. Veröff. Siemens Konzern*, **2**, 349, 1922.
25. *Proc. Phys. Soc.*, **35**, 170, 1922.
26. Knudsen, *Phys. Rev.*, **21**, 84, 1923. *See also* King and Maguire, *Acustica*, **1**, 123, 1951.
27. *J. Frank. Inst.*, **187**, 611, 1919, and *Proc. Roy. Soc.*, *A*, **99**, 163, 1921, and *J. Sci. Instrum.*, **3**, 241, 1926.
28. Riegger, *Zeits. tech. Phys.*, **8**, 510, 1927.
29. *Jahrb. drahtl. Tel.*, **30**, 151, 1927.
30. *J. Acoust. Soc.*, **4**, 83, 1932. *See also* Sacerdote, *Alta Freq.*, **2**, 516, 933.
31. *J. Acoust. Soc.*, **3**, 56, 1931.
32. *Akustik*, 192, 1802.
33. *Sound*, **1**, 420, 1897.
34. *Phil. Mag.*, **29**, 1, 1890.
35. *Phys. Zeits.*, **20**, 429, 1919; **22**, 337, 1921; **23**, 80, 1922.
36. *Phys. Rev.*, **16**, 247, 1920, and **31**, 1092, 1928; *J. Acoust. Soc.*, **1**, 373 and 382, 1930.
37. *J. Acoust. Soc.*, **3**, 397, 1931, and **4**, 331, 1933.
38. *Ibid.*, **4**, 245, 1933, and **5**, 159 and 211, 1933.
39. *J. Frank. Inst.*, **210**, 55, 1930.
40. *Engineer*, 105 and 120, 1934.
41. *Nature*, **81**, 59 and 109, 1909.
42. *Phil. Mag.*, **16**, 285, 1933; Smith, *ibid.*, **19**, 1147, 1935; Gorbatschew and Severny, *Koll. Zeits.*, **73**, 146, 1935.
43. Meyer and Tamm, *Akust. Zeits.*, **4**, 145, 1939; *see also* Lauer, *Akust. Beiheft.*, **1**, 12, 1951; Exner, *ibid.*, **1**, 25, 1951.

CHAPTER SIX

VORTEX FORMATION AND JET TONES

Before the production of tone in a column of air or in a solid body by aerial vibrations can be properly understood, it is necessary to make a digression, and to discuss the growth of vortices in a fluid, as far as it has been clarified by recent physical research.*

Potential Streaming past an Obstacle. Consider a solid cylinder placed in a moving fluid, like that shown in section in Fig. 53. The ideal motion most amenable to mathematical treatment is that in which the particles are supposed to exercise no dragging effect on each other. The path of particles in an infinite channel in which uniform flow is taking place will then be represented by parallel straight lines; the so-called " stream lines." On dipping the cylinder into such a fluid we introduce the " boundary condition " that at the surface between the fluid and the cylinder there can be no normal velocity. This leads to the requirement that the surface must itself be a stream line. The paths of particles which pass near the cylinder are shown in the figure. The pattern is identical with that of the equi-potential lines of an electrostatic field containing a conducting cylinder, and can be mapped out practically as such; for this reason this type of ideal flow is called " Potential Streaming." The assumption of no friction in the fluid requires that the kinetic energy gained by particles passing from the front to the mid-point of the side of the cylinder shall be just lost when they reach the stern, so that they leave the cylinder with the same velocity with which they struck it. The fluid which actually passes in contact with the body therefore " slips " past the surface without any retardation. This leads to the paradox that there is no force on the cylinder due to potential streaming. This is in obvious contradiction to fact, and yet, except very close to the cylinder, the lines shown in Fig. 53 do closely resemble those observed experimentally by the method of Hele-Shaw [1] provided that the velocity of flow is not too great. This method consists in allowing coloured filaments of liquid to enter a small observation tank in which the flow of clear liquid is taking place. The mistake arises evidently in neglecting to take into account the friction, not only between the

* The subject is more fully discussed in the author's *Dynamics of Real Fluids.*

solid boundary and the adjacent fluid which prevents slipping to the extent postulated, but also between neighbouring layers. The friction approximates the velocity, if any, at the boundary to that in the body of the fluid, so as to produce a steep velocity gradient across the stream lines near the surface of the body. There is thus a shearing force on the fluid tending to retard it, or, if we think of the contrary but dynamically identical case, of the cylinder dragged through the still fluid, there is a resistance offered to the motion of the cylinder, as there would be if it were dragged over a solid surface. The part of the resistance due to shearing effects between the body and neighbouring strata of fluid is known in this country as "skin-friction" and must be distinguished from the resistance due to eddy formation

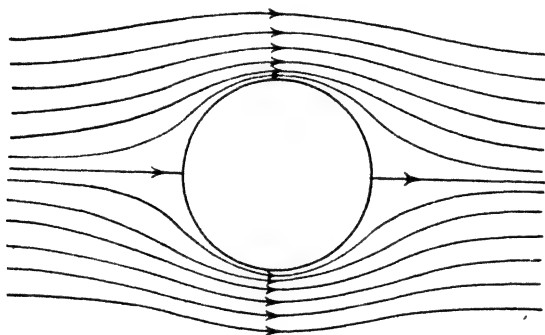



FIG. 53.—Potential Streaming Past a Cylinder.

in the rear of the obstacle. This will be considered later. When the body is without sharp changes of contour and moves at a low velocity through the fluid, skin-friction produces the major part of the drag-resistance.

Boundary Layer. A large number of attempts have been made to introduce viscosity into the mathematical equations of fluid motion, but an exact solution has so far defied all efforts.

The shearing force per unit area between neighbouring layers of fluid is given by the expression

$$F = \mu \frac{dV_x}{dy}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

where μ is the coefficient of viscosity, and $\frac{dV_x}{dy}$ is the gradient of velocity (tangential to the boundary where $y = 0$) measured across the layers.

Now if there were any slipping between fluid and solid, a shearing force would be brought into play, which would be infinitely great compared with that between fluid layers. We believe, therefore, that slip is impossible at the boundary, and take as the boundary conditions $V_x = V_y = V_z = 0$, these being the component velocities in the three rectangular directions. Prandtl remarked that the viscosity of air is a small quantity, and its effect noteworthy only—in accordance with (54)—where the change of velocity from layer to layer is very great. He therefore proposed to neglect viscosity except in a thin layer at the liquid-solid surface. Within this "boundary layer" the tangential velocity rises in a very small distance from 0 at the boundary to the mean velocity of the body of the fluid. Within this layer "stream-line" motion takes place even when the motion outside is unsteady.

For example, we may imagine the motion outside the boundary layer to be simple harmonic with respect to time, and due to aerial waves; or to vary from time to time in an incoherent way about an average value in the fashion denoted "turbulent." In order to calculate the thickness of this layer, Prandtl takes a new co-ordinate ζ, normal to the surface in place of y, such that $\zeta = 0$ at the surface, and $\zeta = \infty$ at the outer edge of the boundary layer. He writes down the equations of motion introducing the viscous term $\mu_1 \frac{dV_x}{d\zeta}$ representing the shearing force between adjacent strata of the layer where μ_1 is a greatly increased viscosity coefficient consequent on the greatly reduced scale of ζ. In the steady state wherein $\frac{dV_x}{dt} = 0$, it appears that the thickness of the boundary layer depends on $\sqrt{\frac{\mu l}{\rho V_x}}$ where V_x is the average velocity in the main fluid, ρ is the density, and l is the length, in the direction of the motion, of the surface on which the fluid has rubbed. On inserting the dimensions it will be seen that the above expression is of the dimension of a length, and that the thickness of the boundary layer increases progressively with the square root of the length of the boundary. It is therefore cumulative. At the end of a wooden board 2 metres long the thickness has been estimated at 1 or 2 cms.

The quantity $\frac{\mu}{\rho}$ is called the "kinematic coefficient of viscosity" (ν), and plays an important part in fluid motion. For the applica-

tion of this theory to the skin-friction of various profiles textbooks on fluid dynamics must be consulted.

Formation of Eddies at the Rear of a Cylinder. The ideal motion depicted in Fig. 54 differs from reality in a further respect. Save at indefinitely slow velocities the liquid would not hug the stern of the cylinder in the manner shown by the stream lines, but would cut off part of the "corner" in the manner shown above (Fig. 54*a*); leaving the cylinder at A, A', and resuming parallel motion at B. The fluid of the shaded area ABA' is "dead water," since it is not carried along by the stream, and the surfaces represented in section by the lines, AB, $A'B$ are "surfaces of discontinuity" to which Helmholtz [2] ascribed the drag resistance of the cylinder. Owing to the shearing effect of the stream on this dead wake, the fluid in it is set in rotation in the form of two eddies (Fig. 54*b*). That the fluid leaves the solid at A, A', may be ascribed to the intense rates of shear

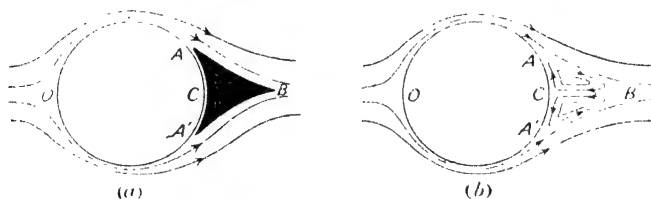



Fig. 54.—Formation of Eddies Behind a Cylinder.

in the boundary layer. Between C and A the fluid in the boundary layer is moving towards A, i.e., opposed to the motion from O to A. At A and A' the tangential velocity in this layer is zero. These points, however, represent regions of considerable instability, and when a sufficient velocity in the body of the stream is reached, vortices formed in the wake no longer remain attached to the cylinder, but on reaching a sufficient size are carried down the stream as if they were solid bodies. It is these travelling vortices with which we are more particularly concerned in sound; but we may point out here their importance in hydrodynamics as they represent energy being dissipated as heat by friction. This energy is carried away from the body, setting up a resistance component far exceeding the skin frictional component. If the points A, A', on a surface where the boundary layer is disrupted, can be pushed farther astern there is less dead water, consequently less eddy-formation and less resistance. That is why

bodies which have to move rapidly through a fluid are "stream-lined," sudden changes of contour in the stern being avoided.

Experiment shows that the main production of vortices occurs behind the body, each vortex being formed and then detached to move down the stream after the others in procession. As long as all have the same initial strength (as we should expect, if the velocity of the stream, and the position of the cylinder are unchanged), this procession of vortex pairs can remain in equilibrium and move down the stream only if the vortices occupy one or other of two orientations, namely, they must be detached periodically from the rear of the cylinder, either in pairs, or alternately from each side. Now although these two possible systems are both equilibrium positions, they are not both stable arrangements.

The calculation of the relative stability of these two equilibrium

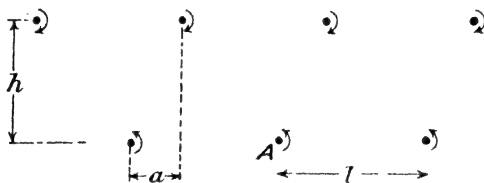



FIG. 55.—"Street" of Alternate Vortices.

systems proceeds on the usual lines. One supposes a small displacement ξ_0 given to one of the vortices in the row due to some accidental disturbance, and calculates the net effect on the velocity $\frac{d\xi}{dt}$ of this vortex due to all the others; the solution of the equation for $\frac{d\xi}{dt}$ may be written $\xi = \xi_0 e^{\alpha t}$. The calculation is too abstruse to be given here, but the result shows for the opposed position, α always positive, any accidental displacement growing with time, system unstable; for the alternate position, α negative, accidental displacements damped out, system stable. Furthermore, in the second case, maximum stability occurs when α has its greatest (negative) value; this occurs when $\frac{h}{l} = 0{\cdot}28$, $\frac{a}{l} = 0{\cdot}5$ (Fig. 55).

Experimental Confirmation. This procession of alternate vortices behind an obstacle in a stream was first noted by Mallock [3] and investigated (independently) by Bénard in 1908.[4] Bénard

allowed the body to dip into a stream of water, and photographed the vortices formed in the stream behind the body. The positions of the vortices were made visible by the light which they scattered as dimples on the surface. The experiments established the fact that the vortices left the obstacle alternately on either side, forming rows with separation depending on the velocity of the stream and the diameter of the obstacle. Kármán [5] investigated mathematically the stability of this vortex system on the lines indicated in the latter part of the last paragraph, and using some further photographs by a collaborator, confirmed the predicted value $\frac{h}{l} = 0{\cdot}28$ for the system formed behind a cylinder: it makes no difference experimentally whether the fluid streams past the obstacle, or the obstacle is dragged through the fluid.

When it is desired to follow the formation of these vortices at frequencies too great for simple counting, Relf and Simmons [6] have found that an electrically heated hot-wire placed in the wake of, and parallel to the cylinder, showed by periodic cooling the passage of the vortices past it. These perturbations were made apparent by a vibration galvanometer tuned to the variations of current through the hot-wire, caused by the periodic change of resistance, which was in turn caused by the cooling. Thus the tuning of the galvanometer to get the best response, with certain precautions, gave the frequency of production of the vortices. Kármán also calculated the effect of this formation on the drag-resistance of the cylinder in a paper [7] of fundamental importance in hydrodynamics, but which does not concern us here.

We are now in a position to discuss these vortex-rows of Bénard and Kármán in the light of certain important acoustical phenomena.

Æolian Tones. The periodic detachment of vortices from alternate sides of the obstacle in a stream imposes periodic cross forces, alternating in direction, on the obstacle. If the obstacle is free to move in a direction at right angles to the stream, it will execute transverse vibrations when the frequency of detachment of a pair of alternate vortices corresponds to one of its natural tones. These are the sounds which Rayleigh called Æolian tones, most noticeable when the wind strikes a system of wires or cords. The phenomenon has been known from Biblical times, harps and even violins being constructed to work on this principle, but the Æolian Harp is not a serious musical instrument, as no proper control can be exercised over the source of

the sound. They are still made as toys, however, and one such is shown in Fig. 56. This is arranged to pivot on a post, and to turn always to face the wind. The wires are usually tuned to a common pitch, but are of different thicknesses, in order to increase the probability of the resonance of the vortex system with one of the wires at any given wind speed.

The first scientific investigation of the phenomenon was made by Strouhal.[8] He stretched a wire between the ends of two rods, and an axle parallel to the wire passed through the mid-points of these rods, so that the rotation of the axle whirled the wire through the air round the circumference of a circle, and in a direction at right angles to its length ; i.e., the wire described the curved surface of a cylinder. Not only the fundamental of the wire, but harmonics also would respond at appropriate speeds. The connection between the diameter D of

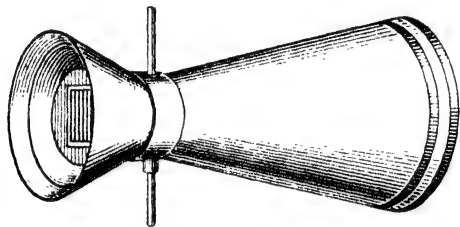



Fig. 56.—Æolian Harp.

the wire, frequency of the tone n, and velocity of the wire through the air V, was :—

$$\frac{V}{nD} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Later Rayleigh pointed out that the vibration was always across the stream, as it must be if it is due to the Bénard phenomenon, and showed that the above formula could be predicted by dimensional analysis. About the same time, Krüger and Lauth,[9] by applying the result found by Kármán and Rubach, showed that the frequency of production of the vortices corresponded to that of the vibrating solid, that $\frac{h}{l}$ was a constant, independent of D and of V ; where h is the distance between the two rows of vortices, and l the length between two successive ones in the same row. Also l was approximately proportional to, and greater than, D, so that $l = bD$ say ; the velocity U of the vortex system relative to the (stationary) fluid was less than,

but proportional to, V—i.e., $U = aV$, where a and b are constants. These facts led Krüger and Lauth to a theoretical basis for Strouhal's formula. The frequency of the vibration represents the number of vortices formed on one side of the wire in one second. If the body swings with the vortices $\frac{l}{V - U}$ is the time between the disengagement of two successive vortices from the same side of the body—i.e., the period of swing :—

$$\frac{1}{n} = \frac{l}{V - U}$$

$$\frac{V}{nD} = \frac{V}{D}\left(\frac{l}{V - U}\right) = \frac{l}{D}\left(\frac{V}{V - U}\right) = \frac{b}{1 - a}$$

which is a constant. From Kármán and Rubach's results for a and b, Krüger and Lauth obtained the value 5·0 for this constant, which is in fair agreement with Strouhal's numbers, which lie between 6·3 and 4·9.

The methods for investigating the laws of Æolian tones are (1) the whirling machine method (Strouhal)—this is open to error if recourse is made to comparison by ear for the pitch ; (2) a pendulum dipping into a revolving tank of liquid ; the pendulum is free to oscillate across the stream, i.e., radially to the tank, and its movements are slow enough to be counted (Riabouchinsky) ; [10] (3) a wire or cord stretched in a frame and held upright in a wind or water channel, the frequency being found by stroboscopic means (Richardson).[11] By allowing light to be reflected from the surface of the water into a camera a photograph can be obtained, which shows the alternate vortices behind the obstacle.

The complete apparatus for the wind-channel method is shown in Fig. 57. The channel had a long glass window in one side, facing a smaller window in the opposite side. The wire was so placed in the channel that it could be observed in a direction inclined about 20 degrees to the axis, through a telescope of a short focus near one end of the long window, the wire being illuminated by an opal-glass lantern, placed behind the small window.

As the noise of the fan and motor prevented all but the louder tones from being distinguished, the vibration of the wire was observed through the telescope, the frequency being found by a stroboscopic method. A motor, carrying a stroboscopic disc, having twelve slits equally spaced round its circumference, revolved directly in front of the object glass of the telescope, so as to cut off the light twelve times

in each revolution. The speed of this motor was adjusted by means of a rheostat, until the fastest speed of the disc, at which the vibrating wire appeared stationary, was obtained. This speed in revolutions per second was known by looking through a stroboscopic vibrator, just beside the telescope, and the speed multiplied by twelve, gave the frequency of the wire, independently of the ear, or of any comparison of pitch. The velocity of the air current was found from a manometer. The wires were fixed vertically in small clamps, with adjusting screws for altering the fundamental frequency.

The modern results establish the constancy of $\frac{V}{nD}$ at a value of 5 for a cylindrical body, except for small values of V or of D. This is

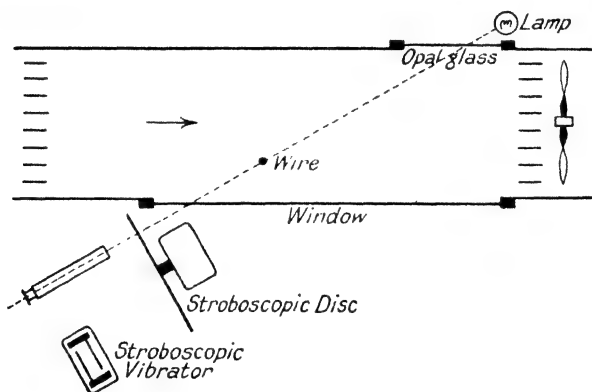



FIG. 57.—Wind Channel Apparatus for Æolian Tones.

noticeable when thin wires (of less than 0·02 cm.) are used; then the "constant" soars up to 8 or more. The lateral extent of the wake depends on the shape of the body, and therefore h alters with the shape, and this affects the value of this constant for bodies of different shape.

Effect of Viscosity on Fluid Vibrations. Experiments made in connection with oscillating wires, or with pendulums in liquids of (kinematic) viscosities varying from 0·01 to 0·5 c.g.s. units, showed little or no effect of viscosity on the quantity $\frac{V}{nD}$, the tones of a wire being produced at practically the same stream velocity in every case. Apparently then viscosity does not appreciably change the rate of formation of the Bénard vortices, though, in recent papers, their

original investigator thinks that a change of $\frac{V}{nD}$ is detectable in his photographs. Viscosity has, however, a very important influence on the initiation of the vortex system. There is a minimum velocity ("critical velocity") below which no vortices, and therefore no tones are produced. If the ratio of velocity to viscosity is small, and if the obstacle is sufficiently narrow and tapered in the stern, the two parts into which the stream is divided at the front of the obstacle can re-unite exactly at the centre of the stern of the obstacle. There is no dead-water region, no discontinuity, and no vorticity. We have in fact stream-line motion corresponding closely to the "potential streaming" of hydrodynamical theory. When the critical velocity is exceeded, the stream lines can no longer hug the stern, stagnation points appear on the sides, and vorticity in the wake.

This critical velocity then depends on the viscosity and density of the fluid, and the width and form of the body, especially of the stern. The two former effects we unite under the coefficient of kinematic viscosity $\nu = \frac{\mu}{\rho}$, the latter pair under a single linear dimension l.

We may write our equation in the general form:—

$$\frac{V}{nD} = C'f\left(\frac{VD}{\nu}\right)$$

where C' now is a non-dimensional constant not involving viscosity The form of the function $f\left(\frac{VD}{\nu}\right)$ could be found from results in liquids of different viscosity. Our results indicate that the function is independent of viscosity over practically the whole range investigated, but that the periodic shedding of vortices to which n is due, starts at a definite value of $\frac{VD}{\nu}$.

The evidence for the last statement is as follows. With a string tuned to a definite pitch n, it is generally possible to get an Æolian tone when $\frac{V}{nD}$ is 5. As the tension in the string is released, n and V for this tone fall, until at a definite value of n, no tone is produced at the appropriate value of V. The inference is, that the motion has become steady—vortices have ceased to be produced. The values of $\frac{VD}{\nu}$ when the tones failed to be heard were collected by the author

and found to cluster round the value 30 for a cylinder, and 60 for a rubber cord of stream-lined section. The latter is designed to encourage steady motion, and we should expect, *ceteris paribus*, to require higher velocities before vortex motion with the consequent vibration of the body would set in.

The principle of dynamical similarity teaches us in this case that for equal values of $\frac{VD}{\nu}$ the motion in the stream will be similar round two bodies of the same shape but of different size. The series of transformations will be identical, though in a different scale, but will take place at different rates. This is what is implied by the statement $\frac{V}{nD}$ is constant. If then the flow is steady (in stream lines) round one wire at a given value of $\frac{VD}{\nu}$, it will be steady round another wire of different diameter, but at the same value of $\frac{VD}{\nu}$. If vortices begin to form behind one wire at a critical value of $\frac{VD}{\nu}$, then vorticity will appear behind the other wire at the same critical value. That, after the initiation of this oscillatory motion, the similarity does not seem to depend on $\frac{VD}{\nu}$, is a consequence of the experimental fact that at all values of $\frac{VD}{\nu}$ above the critical, the vortices form at the same rate, depending on V only.

To sum up, viscosity in the guise of the expression $\frac{VD}{\nu}$ determines when vortical motion shall commence, but, once initiated, has no effect on the period.

The importance of this quantity was first pointed out by Osborne Reynolds,[12] who found that when fluid was in motion through a tube, turbulent motion set in at a definite value of $\frac{VD}{\nu}$. This case has no acoustical importance, but other periodic motions whose initiation depends on this quantity will appear in the following pages. The critical value of $\frac{VD}{\nu}$ is often known as the critical " Reynolds' number " of the particular type of flow.

Tones of Jets. We have considered above the tones which

arise when a stream of fluid passes a linear obstacle; we must now consider the conjugate system of tones produced when a stream issues as a jet from a linear slit in an infinite plate into a stationary fluid. Surfaces of discontinuity arise between the issuing fluid and the stagnant fluid surrounding the orifice; the former tends to curl outwards into the stagnant fluid, and resolve into alternate vortices on each side of the jet, with spacing given by the Kármán formula $\frac{h}{l} = 0{\cdot}28$. As a matter of experimental fact, this system seems to have less stability when, as it does here, the fast-moving fluid lies between the rows of vortices, and there is no certain evidence that such a system is actually produced in the absence of the resonant forcing discussed under organ pipes (p. 161).

The surfaces of discontinuity tend to break up into general vorticity; every vortex pattern seems equally unstable here, and it requires some *deus ex machina* to guide the vortices into a periodically repeated pattern; there is no solid resonator to take up the vibrations corresponding to the wire in Æolian tones.

As a result the tones produced by the issuing fluid are weak, uncertain and fluctuating. They partake of the characteristics of a "hiss," i.e., they correspond to a vortex formation at a high and unsteady frequency. It is to be understood, unless mention is made to the contrary, that we are dealing with homogeneous jets. The issuing fluid has the same properties as that into which it issues, e.g., air into air, or water into water. Jet tones may also be produced when the orifice is circular, and as such were first observed by Cagniard de Latour. This type of motion may be observed at the orifice of a smoke-stack. Such jet tones formed the subject of an extensive investigation by Kohlrausch,[13] who also found that the general frequency of the tone rose proportionately with the velocity of efflux; the relation between n and D (diameter of orifice) however was not a simple one. Similar investigations have been made on the linear slit, the frequency being estimated by comparison with a sonometer, or in the case of a coloured liquid, by endeavouring to count the issuing vortices.[14] Both methods are fraught with so much difficulty that the results must of necessity be inconclusive.

A case of some interest, as it unites the jet and the Æolian tones, is that of the annular orifice formed between a tube and a concentric disc which nearly fills the end, leaving a ring-shaped opening. If the slit is wide so that the wall of the tube can exert little influence on

the flow round the disc, it has been found that the motion corresponds to the Æolian tones for a disc in which the vortex filaments of the cylinder are replaced by embryo vortex-rings, the liquid curling alternately out from and in towards the axis. As the diameter of the tube is reduced, and the wall of the tube approaches the circumference of the disc, it exerts a modifying effect on the frequency of the formation of vortices, expressible by the formula: $n = \frac{V}{d} f\left(\frac{d}{D}\right)$, when D and d are the diameters of the disc and tube (inside measurement) respectively.[15]

Sensitive Jets and Flames. If the progress of a jet be made visible, either by allowing coloured water to emerge into clear water, or by mixing steam with the air which is issuing into the atmosphere, in general a cylindrical filament of issuing fluid will be observed, and this at a definite point breaks up into turbulent motion with radial spreading into the stagnant fluid. The precise cause of this sudden breakdown is obscure, but it is evidently connected with the viscous drag of the stationary on the rapidly moving fluid, which, after the stream has travelled a certain distance, causes such differences of velocity across the section of the jet, that the latter is not able to sustain the large shearing forces brought into play as a consequence, and breaks down into unsteady or turbulent motion similar to that which is believed to be set up in the boundary layer at points of rapidly changing contour. Using very slow speeds of efflux from a nozzle a millimetre or so in bore, and with the nozzle well shielded from accidental disturbance, it is possible to maintain such a jet for a foot or more without breaking up ; but while it is in this condition a very slight increase of velocity applied to the issuing gas at the nozzle, even the act of clapping the hands, or sounding an instrument in a distant part of the room is generally sufficient to make the turbulent point jump back nearly to the nozzle ; a periodic disturbance causes periodic changes in length.

Such a sensitive jet forms a satisfactory detector of sound, and is used in the form of a narrow ignited jet of inflammable gas, wherein the turbulent point can be distinguished by eye. The phenomenon is not altered in principle by igniting the gas, and the striking-back of the turbulent point to the nozzle is made visible by the sudden flare of the flame. Some light is thrown on the mechanism of the phenomenon by measurements of the length of such jets.[16] Coloured water was allowed to stream from a capillary nozzle (bore 0·29 mm.)

into a tank. Corresponding measurements of the length of the jet to the turbulent point l, and the velocity of efflux V_0 were taken. The relation between V_0 and l is approximately a hyperbola.

The shape of these curves can be accounted for on principles of similarity. The point at which the jet breaks up is taken to be that at which the Reynolds' criterion $\frac{VD}{\nu}$ (D = diameter of jet and V = mean velocity at this point, ν = kinematic viscosity) has reached that value at which the motion becomes turbulent. If the initial velocity at the nozzle be increased, the critical velocity V will be reached at a point nearer the nozzle ; l will therefore become smaller as the velocity at the nozzle increases. The effect of altering D or ν may likewise be predicted.

In a jet issuing from a circular nozzle vortices are formed periodically with frequency n, given by $\frac{V}{nD}$ = a constant, D being a linear dimension dependent on the bore of the nozzle.[17] The jet should respond most readily to tones of this frequency, or sub-multiples thereof, and a high-velocity jet to sounds of high frequency. When a jet of gas is ignited the combustion complicates matters, as the visible part of the jet lengthens at first as the velocity increases ; but Rayleigh showed that, over the range for which they are sensitive, such flames behave in most particulars like unignited jets, the progress of which is made visible by smoke. Experiment showed that a sudden small increase in the velocity of the gas feeding a sensitive flame brought the turbulent point—visible as a " flare "—nearer to the nozzle. It is not so much the ignition which divides these from sensitive jets as the absence more or less complete of surface tension in the gas—gas interface.

The other common experience with sensitive jets and flames, that they are sensitive only over a small range of gas pressure, and, therefore, of efflux velocity, is shown by the curves and by theory. It is a consequence of the hyperbolic relation between l and V_0, that at a certain value of the latter a small increase in velocity due to aerial disturbances causes a large change in length, so that in using a sensitive flame it is necessary to work on the steep part of the curve (Fig. 58).

Various types of sensitive flame have been devised from time to time, depending on the range of pitch to which they are intended to respond. Tyndall[18] used pin-hole burners, with gas at high pressure

for tones high up in the musical scale, coal-gas from the normal supply serving for lower notes. Tyndall recognized that the flame was merely an indicator of the evolutions of the jet of gas. These sensitive jets were formerly much employed as detectors, but have been largely superseded by the more precise instruments discussed in Chapter VIII. Rayleigh [19] found that the nodes and antinodes in the stationary waves formed in the air between a source of sound and a wall, could be detected by this method; the flame responded best when placed in an antinode, where the fluctuations in velocity were greatest, whereas the ear detected pressure alterations and maximum sound was heard at a node. A simple sensitive flame can be made by unscrewing the tube of a bunsen burner, and lighting the gas issuing from the nozzle which will be found in the base of the burner. This will respond to tones of moderate frequency. Better still, the gas may be lit above the tube in the usual place, and the air-holes below covered with a thin membrane of tissue paper, to which the sound is guided by a small conical trumpet attached to the bunsen.

Fig. 58.—Variation in Length of a Sensitive Jet.

Recent workers on the sensitive flame have used a loud-speaker source excited by a constant tone oscillator as a source of sound. Both Brown [20] and Zickendrht [21] have taken photographs under long exposure (to show visual appearance) and instantaneously, the former showing a long series of beautiful photographs of the latter type. Symmetrical varicosities (to use the late Lord Rayleigh's expressive term) were seen to travel up flaming jets and smoke jets in much the same way that they travel down water jets. More often than not, the vagarious motion is not symmetrical, being at its maximum in a plane passing through the jet and the source of sound. Brown's work is mainly concerned with the vortices, the passage of which up the smoky column causes it to break away from the steady form and fork outwards with increasing amplitude very much as in edge tones. The observations were easier to interpret since the slit from which the jet emerged was linear instead of circular, as in the earlier

work. It is interesting that although the author finds approximate agreement between the experimental value of the ratio: velocity of vortices/velocity of jet and that calculated by Rayleigh (i.e., 1/2) when the slit is wide, yet narrow jets exhibited much lower ratios. This discrepancy between practice and theory is also found in related phenomena (Æolian tones, edge tones, Kundt's tube, etc.) when the orifice or obstacle falls below $\frac{1}{2}$ mm. in width. Brown has also shown that the origin of the disturbance is at the orifice itself, since if this part is shielded from the sound, the jet becomes insensitive.

Andrade [22] has further shown that the sensitivity arises from a *transverse* relative motion of air and nozzle, since if the sound is directed *along* the jet nothing happens. He has proved that the jet is not in itself selective in its response to frequency and that the selective resonances which Brown had claimed were due to the structure of the building and the air enclosed in it. If these are eliminated an unbroken curve of sensitivity to frequency is obtained as the source is taken through the gamut. Andrade made use of an electromagnetic analogy between the magnetic field round the current in a straight wire and the velocity field round a linear vortex to study the theoretical behaviour of a jet in the form of a thin sheet of flame, subject to transverse disturbance when it leaves the linear orifice. On this basis the observed instability with the development of curls and spurs can be explained on the laws of classical hydrodynamics.

Edge Tones. In order to maintain the stationary vibrations in a column of air, it is customary either to direct a blast of air on to a sharp edge at one end of the tube, or to direct air through a channel periodically closed by a " beating reed " leading to the tube. The first device is employed in the " flute organ pipe," and whistle, the second is applied to all wind instruments, either with an *ad hoc* reed, or pair of reeds, or with the player's lips to act as reeds. It was first noticed by Masson [23] and by Sondhauss [24] that when a blast of air is directed against an isolated sharp edge, tones can be produced in the absence of the column of air. Commonly the air issues from a linear slit a few millimetres wide and several centimetres long, and strikes a sharply bevelled edge of wood or metal placed in the plane of the issuing lamina of air. In the last section we saw that such a lamina has a tendency to form vortex filaments as it emerges into the undisturbed air on either side of the stream, and that, if a stable system of vortices is formed, these vortex filaments will be alternately spaced, one on each side of the issuing air stream.

Now it appears that when the air strikes an edge, the space between slit and edge acts as a form of resonator, so that the length f from slit to edge becomes equal to, or a multiple of, the " wave-length of the vortex system " (applying this term to the distance between successive vortices in the same row). Taking the simplest case, a vortex A (Fig. 59) leaves the outer wall of the orifice as the preceding one on the same side B strikes the edge. This obscure action of the edge has been explained in several ways by the authors in the references cited, but it is probably connected in some way with the secondary vortices formed in the boundary layer alongside the edge itself, and which have to " fall into line " with those coming from the slit. There is a minimum distance f_0 for any given velocity of efflux V at which a tone can be produced. When the separation between edge and slit is a little greater than f_0, the state of affairs is as shown in Fig. 59a. The vortex B has just struck the edge as the next one on the same side is emerging from the slit. The frequency of the " edge tone " is given by the frequency with which the eddies A, B, etc., strike the edge. If they move towards it with velocity U and l = distance between the vortices in the same row, as before (p. 149), then

$$n = U/l,$$

or since $l = f_0$, and $U = aV$, where a is constant,

$$V/nf_0 = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

When V is kept constant, i.e., when constant pressure is maintained behind the slit, as f is increased beyond the minimum f_0, the pitch of the " edge tone " falls in accordance with (56) until at a value f_1 approximately double of f_0, the system becomes unstable, and the tone which is now the sub-octave of the original tends to jump up an octave to what it was at f_0. This jump of an octave we may attribute — and experiment verifies the deduction—to a return to the original spacing of the vortices, but with twice as many between slit and edge. Fig. 59b shows their relative positions just before the transition. l has doubled itself with f, and therefore the width of the " street " in accordance with the Kármán law (p. 147) is also double what it was in Fig. 59a. After the jump (Fig. 59c) l resumes its original value so that $f = 2l$, and, still in agreement with the rule, h is halved, and the original narrow " street, " is recovered. The edge must bisect the two rows of vortices if the tone is to be elicited; consequently, in the shaded area of Fig. 59c, representing the space between the wider and

the narrow " street " at the transition, only the deeper tone can be elicited. In fact after the jump has taken place, the pre-transition tone can be brought back either (1) by moving the edge into the shaded region, or (2) by pushing an obstacle from the side partly into the path of the blast in Fig. 59*c*, and so deflecting the shaded portion of the stream on to the edge.

For experimental purposes the slit may be formed of stream-lined brass plates, let into an otherwise airtight wooden box of about 20 litres capacity, fed with compressed air from a cylinder. The edge is of steel, tapering from about 1 cm. to a razor edge in 10 cm.; it is firmly fixed to a micrometer traverse as fine adjustments across the stream may be necessary. The slit should lie between ½ and 1 mm. Both slit and edge are 5 cm. deep.

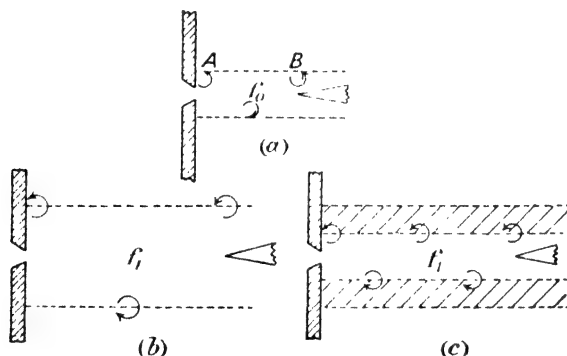



Fig. 59.—Edge Tones.

If f is kept constant and V gradually increased, n increases in accordance with (57), and jumps to the octave above when the vortices rearrange themselves, the original tone being recoverable by the same means. One or other of these devices is used on certain open organ pipes, when the blowing-pressure is increased until the edge-tone and with it the pipe-tone jumps to the octave, and is brought back by deflecting the edge or deflecting the air by a wooden " bridge " pushed into its path from the outside. Owing to the greater blast speed employed, more energy is available for sound production with this artificially lowered octave, and a brighter quality of tone is imparted to the pipe than by that which we may call the natural edge-tone corresponding to the smaller f or V.

The existence of a series of these breaks was disclosed by Wachsmuth,[25] who compared the tones with a sonometer. On the basis of

some further work by Göller, König[26] proposed a more general formula in place of (56), which with slight alteration we may write :—

$$\frac{Vj}{nf} = \text{a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

where $j = 1$ from f_0 to f_1, $j = 2$ from f_1 to f_2, etc., and the values for f at the breaks are connected with the minimum f_0 by $f_0 = \frac{f_1}{2} = \frac{f_2}{3} = \frac{f_3}{4}$ etc. Under favourable conditions, half a dozen breaks may be noticed.

This equation has been verified for various gases by Rieth,[27] measuring the average value of V by the fall of the resistance produced in a thin platinum wire heated electrically, placed in the path of the blast ; and for water by Schmidtke,[28] who was able to observe the formation and path of the vortices at the surface of the water. Most careful measurements in air have latterly been made by Benton,[29] who has been able to measure h by finding the limiting position of the edge where the tone still continues—presumably this is when the edge lies on the dotted lines of Fig. 59, and no longer parts the two rows of vortices. He finds the value $h/l = h/f = 0{\cdot}27$, approximately the value given by Kármán's calculation. In order to get agreement with Kármán's theory in Æolian and edge-tone phenomena, simplicity in the apparatus is essential. Thus Carrière finds himself in disagreement with the applicability of the Kármán formula to the phenomena at the mouth of an organ pipe. Benton points out that it is possible to demonstrate the application of the theory, if the apparatus is sufficiently free from perturbing influences.

Besides the minimum f for tone production, there is a minimum V. These facts are connected with the critical value of $\frac{VL}{\nu}$ discussed in the last chapter, below which vorticity is not present. The linear dimension L is a complex function of f, and of the diameter of the slit, and of the form of the channel through which the fluid has to flow before emerging from the slit. Therefore L is an uncertain quantity, but in confirmation of what was found for Æolian tones, viscosity plays little or no part in the motion once initiated. $\frac{Vj}{nf}$ is practically a "universal" constant for all gases (value 2·0).

An alternative explanation of the edge tone formula may be derived from a consideration of the secondary vortices which may be seen in smoke photographs to form in the boundary layer of the edge

itself as the fluid passes along it. These have to fall into their proper position along the two vortex avenues thus formed on each side of the edge. We have the vortices on one side of the "street" starting from the slit (*S*, Fig. 60) while their fellows on the other side start from the tip of the edge (*E*) with the same velocity, and the only way in which the procession can marshal itself into a Bénard-Kármán avenue of alternate vortices is for one to roll from the edge at the same time as another rolls up from the slit itself. That the vortices do not spring fully grown from these points does not affect the argument if we assume, as visual observation shows, that they all roll up or grow at the same rate.

Edge tones may also be produced by allowing fluid from an annular slit to strike an annular edge. If the latter forms the end of a cylind-

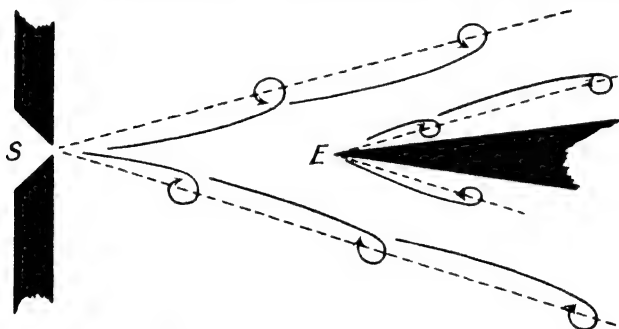



Fig. 60.—Primary and Secondary Edge-tone Vortices.

rical metal tube, an organ pipe may be produced. An organ pipe was so fashioned by Gripon,[30] and the edge-tone formula has been subsequently verified for this case by Krüger and Marschner [31] (but cf. p. 153). Examination of the motion shows that the fluid curls alternately inwards and outwards forming embryo vortices alternately larger and smaller in diameter than the slit; the difference of the radii measures the quantity *h*. Owing to the difficulty of fixing the pipe to the mouth-piece, organ pipes on this principle are not used, but the "bell-whistle" on locomotives is of this type, as it consists of a short bell-shaped resonator, rigidly fixed by a central pillar to the centre of the annular orifice from which it is blown.

Recent Work on Edge Tones. Krüger and Caspar [32] have pointed out the affinity between Brown's photographs of sensitive flames and those of their own and other workers of smoke-jets producing edge tones. Brown [33] has himself extended his work in this

direction. In the edge-tone formula (57, p. 160) he denies that f is an exact multiple of l, but proposes instead the empirical formula $f = 0{\cdot}0466j(V - 40)(1/l - 0{\cdot}07)$, where $j = 1$, 2·3, 3·8, 5·4 for the first four stages of the sub-division of the slit-edge space. He then examines critically the various edge-tone theories and favours a modified form of "escapement" theory, whereby any casual deviation of the jet from the forthright direction sets up a pressure increase in the air on one side of the wedge, forcing the jet back towards the other side, so making it pendulate. It will be noted that this does not completely account for the period of the pendulation. Two other investigations, which are germane to this difficulty, are to be found in papers by Carrière [34] and Coop.[35] Both these authors, acting independently, placed a wire in the path of a jet parallel to the slit from which it was emerging, though they used an ignited gas-jet. Thus, the new wind tone is a sort of combined Æolian and edge tone. In fact, Carrière found that an edge-tone formula satisfied the observations, while Coop considered the sound as an Æolian tone.

The subject has been pursued by Lenihan and Richardson,[36] who used an unignited jet issuing from a linear slit and falling upon a parallel wire. This gives out a note like an edge tone together with the familiar jumps to a higher frequency when the wire is moved further from the jet and the velocity of the air blast kept unchanged. The edge-tone "constant," however, was not the same as that for a wedge and varied considerably with velocity even for one wire. The diameter of the wire had little effect. The frequencies agreed remarkably well with Brown's formula just quoted, provided the velocity of the jet were inserted without subtraction of the figure 40 (Brown's velocities were derived from the pressure and not measured directly as in this research). The wire could be made to vibrate at its own natural frequency if placed a little to one side of the air-stream where there was a gradient of velocity. The experiment seems suggestive in respect of the escapement theory which Brown endorses. For if the wedge against which the pendulation is set up can be replaced by a thin wire without upsetting the mechanism of the edge tone, it seems difficult to see how the postulated differences of pressure can be maintained when there is free communication between the two sides of the jet by way of the far side of the wire.

Hartmann's Jet Oscillator.[37] This may be regarded as an apparatus for producing supersonic edge tones, for it consists essentially of a jet of gas emerging from a nozzle at a speed exceeding the

velocity of sound and impinging on a coaxal ring-shaped edge, which may be the mouth of a small bottle-resonator (Fig. 61) somewhat like the Galton whistle (p. 276). The mechanism is not however quite the same, for Schlieren photographs of the jet show waves of compression reflected from the confines of the jet in criss-cross fashion (dotted lines). If the mouth of the resonator is placed in one of these zones of reflection as shown in the figure, its natural frequency is excited

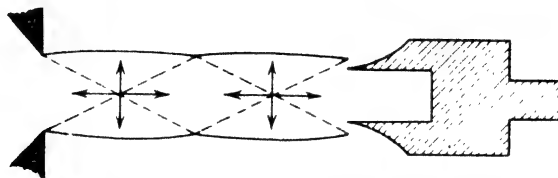



Fig. 61.—Jet Oscillator (*Hartmann and Trolle*).

with considerable intensity owing to the large energy in the jet. Evidently these zones are regions of instability as in other places no excitation of the resonator is produced. The frequency—usually above the audible limit—of the jet itself apart from the resonator is determined by the distance between these zones as wave-lengths, and the velocity of the jet. Besides this there is a slow pulsation of the period of several seconds as the resonator fills with gas up to a high pressure and then exhausts back into the jet. This latter is probably a relaxation oscillation.

References

1. *Brit. Ass. Rep.*, 136, 1898.
2. *Berlin Akad. Sci. Ber.*, 1868.
3. *Proc. Roy. Soc.*, *A*, **79**, 262, 1907.
4. *Comptes Rendus*, **147**, 839 and 970, 1908 ; **156**, 1003 and 1225, 1913 ; **182**, 1375 and 1823, 1926 ; **183**, 20 and 184, 1926.
5. *Göttinger Nach.*, 547, 1912.
6. *Phil. Mag.*, **49**, 509, 1925.
7. Kármán and Rubach, *Phys. Zeits.*, **13**, 49, 1912.
8. *Ann. Physik*, **5**, 216, 1878.
9. *Ibid.*, **44**, 801, 1914.
10. *Aérophile*, **19**, 15, 1911.
11. *Proc. Phys. Soc.*, **36**, 153, 1924, and **37**, 178, 1925. *See also* Holle, *Akust. Zeits.*, **3**, 321, 1938.
12. *Phil. Trans. Roy. Soc.*, **174**, 935, 1883.
13. *Ann. Physik*, **13**, 545, 1881.
14. Krüger and Schmidtke, *ibid.*, **60**, 701, 1919.
15. Tyler and Richardson, *Phil. Mag.*, **2**, 436, 1926.
16. Richardson, *Nature*, **116**, 171, 1925.

17. TYLER, *Phil. Mag.*, **16**, 504, 1933.
18. *Ibid.*, **13**, 473, 1857, and **33**, 375, 1867.
19. *Ibid.*, **7**, 149, 1879.
20. *Ibid.*, **13**, 161, 1932; *Proc. Phys. Soc.*, **47**, 703, 1935.
21. *Helv. Phys. Acta*, **7**, 773, 1934; **14**, 132 and 195, 1941; **15**, 322, 1942.
22. *Proc. Phys. Soc.*, **53**, 329, 1941.
23. *Comptes Rendus*, **36**, 257 and 1004, 1853.
24. *Ann. Physik*, **91**, 214, 1854.
25. *Ibid.*, **14**, 469, 1904.
26. *Phys. Zeits.*, **13**, 1053, 1912.
27. *Diss. Gieszen*, 1917.
28. *Ann. Physik*, **60**, 715, 1919.
29. *Proc. Phys. Soc.*, **38**, 109, 1926. *See also* RICHARDSON, *Proc. Phys. Soc.*, **43**, 394, 1931.
30. *Ann. Physik*, **8**, 384, 1874.
31. *Ibid.*, **67**, 581, 1922.
32. *Phys. Zeits.*, **37**, 842, 1936.
33. *Proc. Phys. Soc.*, **49**, 493 and 508, 1937; *Nature*, **141**, 11, 1938.
34. *Rev. Acoust.*, **5**, 112, 1936.
35. *J. Acoust. Soc.*, **9**, 321, 1938.
36. *Phil. Mag.*, **29**, 400, 1940. *See also* SAVIC, *ibid.*, **32**, 245, 1941; M. MOKHTAR and H. YOUSSEF, *Acustica*, **2**, 135, 1952.
37. HARTMANN and TROLLE, *J. Sci. Instrum.*, **4**, 101, 1927.

CHAPTER SEVEN

COLUMNS OF AIR

Vibrations of Air in Wide Tubes. We have already discussed in Chapter I the plane longitudinal vibrations of air in a wide tube, and have deduced the formula $c^2 = \frac{\gamma p}{\rho}$ for the velocity of sound in such a tube. It will be as well to enumerate the assumptions which are required by the classical theory in order that this formula and those set forth below shall apply. These are :—

(1) That the motion is uniform over the cross-section. This implies that the viscosity effect can be neglected, or the thickness of the "boundary layer" can be neglected in comparison with the width of the tube.

(2) That the oscillations are so rapid that there is not time for heat to be communicated between adjacent layers to equalize the temperature, and consequently the adiabatic formula may be applied.

(3) That vortices or rotatory motion are not set up in the tube.

(4) That the fractional changes of velocity and pressure in the waves are so small that their squares may be neglected.

Under these conditions, the longitudinal vibrations of the air correspond to those of particles of metal forming a rod, and the same equation ((2), p. 3) for the displacement will apply, with the value of the velocity given above :—

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

A quantity frequently introduced in connection with vibrating columns of gas is called the "condensation" s, expressing the fractional change of density of the gas with respect to its original density ρ_0 :—

$$s = \frac{\delta \rho}{\rho_0} = -\frac{\delta v}{v} = -\frac{\partial \xi}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

This is connected with the pressure change on the adiabatic assumption, as follows :—

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma = \left(1 + \frac{\rho - \rho_0}{\rho_0}\right)^\gamma = (1 + s)^\gamma = 1 + \gamma s, \quad (60)$$

approximately when s is small.

The tubes with which we meet in practice have one end open, and the other end either open or closed. When both ends are open, we always have antinodes at the ends, and a node in the middle; the possible modes of vibration are in fact the reverse of those which obtain for the string.

Having regard to the end conditions $\frac{\partial \xi}{\partial x} = 0$, at $x = 0$, and at $x = l$, the possible stationary vibrations of the gas in such a tube are given by (cf. eq. (39), p. 84)

$$\xi = a_0 \cos \frac{j\pi x}{l} \sin \frac{j\pi ct}{l}; \quad . \quad . \quad . \quad . \quad . \quad (61a)$$

so that the displacement amplitude is a_0 at $x = 0$ or $x = l$. The factor j can have any integral value, so that the complete harmonic series is possible.

When the tube is open at $x = l$, $\left(\frac{\partial \xi}{\partial x} = 0\right)$ and closed at $x = 0$ $(\xi = 0)$ the corresponding equation is:—

$$\xi = a_0 \sin \frac{j\pi x}{2l} \sin \frac{j\pi ct}{2l}, \quad . \quad . \quad . \quad . \quad . \quad (61b)$$

a_0 being the amplitude at $x = l$, and $j = 1, 3, 5$, etc. Partials of even order are thus absent. This is a consequence of the fact that we must always have a node at $x = 0$ and an antinode at $x = l$.

The fundamental has thus a wave-length of $4l$, whereas the open tube has a fundamental wave-length of $2l$. The tube with one end closed produces its overtones by unequal subdivision in order to preserve a node and an antinode at its respective ends.

The wave-lengths of partials are shown in the following table:—

	j	1	2	3	4	5	6	7	
Open tube	λ	$2l$	l	$\frac{2}{3}l$	$\frac{l}{2}$	$\frac{2}{5}l$	$\frac{l}{3}$	$\frac{2}{7}l$	etc.
Open-closed tube . .	λ	$4l$	—	$\frac{4}{3}l$	—	$\frac{4}{5}l$	—	$\frac{4}{7}l$	

If, instead of quoting the displacement amplitude at any point, we prefer to give the pressure or density changes, we combine the relation

$-\frac{\partial\xi}{\partial x} = s = \frac{\delta\rho}{\rho_0} = \frac{1}{\gamma}\frac{\delta p}{p_0}$ with (61a) or (62b) above; for example, in the open tube the pressure is given by:—

$$\delta p = \frac{j\pi\gamma}{l} a_0 \sin\frac{j\pi x}{l} \sin\frac{j\pi ct}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

in terms of the normal pressure as unity. The amplitude of the pressure variation (when $p_0 = 1$) at the node in a tube containing air is therefore $1{\cdot}4\frac{j\pi}{l}$ times the displacement amplitude at an antinode.

Open-end Correction. The above theory supposes that the nodes and antinodes are formed exactly at the end of the tube. The assumption for the closed end cannot be gainsaid, for the unyielding material at the end fulfils our condition that $\xi = 0$ at this point. On the contrary the condensation is not zero ($s = \frac{\partial\xi}{\partial x} \neq 0$) at the point where the tube debouches upon the atmosphere, for if $s = 0$, no density changes are possible at the end of the tube. But we know as a matter of fact that the waves are propagated in spherical type outside in the free air, otherwise the sound would be inaudible, so that the inertia of the air in the neighbourhood of the mouth permits a certain amount of density variation there, although less than that possible within the confining walls of the tube. The point where $s = 0$ corresponds to a small negative value of x (if $x = 0$ at the end of the tube) equal to x_0 say, and the wave-length becomes $2(l + 2x_0)$ for a tube open at both ends.

Another way of considering the problem is in terms of the change of type of wave at the open end. If the antinode were strictly at the end, the waves would have to change from plane to spherical (round the middle point of the end cross-section as centre) with a sudden discontinuity. As this cannot be, the wave-front must gradually curve from the plane front in the tube to a spherical one having a centre at $x = x_0$. As a consequence of this loss of energy in waves outside, the reflected wave in the pipe has diminished amplitude, causing, apart from viscosity, a damping and ultimate extinction of the sound in the tube, unless the energy is maintained from an external source.

The calculation of the addition necessary to the theoretical length of the pipe, due to the open end, has been made (first by Helmholtz,[1] then by Rayleigh [2]) only by considering an infinite flange flush with the end of the tube; the result shows that to the length of a tube of

radius r must be added: $x_0 = \frac{\pi}{4}r = 0{\cdot}786r$ (Helmholtz), $x_0 = 0{\cdot}824r$ (Rayleigh).

The experimental estimation of the end correction is usually done by means of a resonance tube. This consists of a cylindrical tube whose distant end is closed by means of a movable piston, or else by a water surface whose level in the tube can be varied. This is used as a resonator to be tuned to a fork. Starting with the "working portion" of the tube as short as possible, the position of the "stop" in the tube is gradually lowered until the air in it resounds to the fork, and the length l_1 is noted. On further lengthening the distance between the open end and the stop, a second resonant length l_2 will be found. In the absence of end correction l_2 would be $3l_1$, according to the table on page 166. Accurately we have

$$l_2 + x_0 = 3(l_1 + x_0),$$

whence x_0 may be determined. Recent experiments give $0{\cdot}58r$ for a flangeless tube.[3] By widening out the open end into the familiar "bell" shape of wind-instruments, Helmholtz showed that a tube requiring no end correction could be made if the "bell" formed a hyperboloid (cf. also pp. 234–238).

Conical Tube. If the cross-section of the tube tapers but slightly the theory for the cylindrical pipe may still be applied. But when the tube forms a cone of large angle open at the wide end, sounds proceeding from the vertex will be propagated along the tube as spherical, rather than as plane waves, and be reflected from an end as such. At a point a little outside the end there will still be an antinode, while the vertex or any intermediate barrier will be a node. But when such a tube emits overtones, the intermediate nodes and antinodes will not be found at the same relative positions as in the cylindrical tube.

The extent of this deviation from the nodes of the corresponding cylindrical tube depends on the distance of the node in question from the vertex of the cone; accordingly the overtones in a wide-angled cone are inharmonic apart from any question of end correction.

The position of the nodes corresponding to different frequencies may be obtained by sinking the cone in water until resonance is shown with a tuning-fork held over the open end, as Zamminer[4] did. As we have to deal with spherical waves diverging from the tip of the cone we should adopt the form of the general equation of wave

propagation which suits this case, and the required form in polar co-ordinates is :—

$$\frac{\partial^2(rs)}{\partial t^2} = c^2\frac{\partial^2(rs)}{\partial r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

r being the radius of a wave, measured from the vertex, and s the condensation. In the actual pipe stationary vibrations will be set up as in the cylindrical pipe, and if we restrict ourselves to restoring forces proportional to rs, so that rs is proportional to $\sin(2\pi nt + \varepsilon)$:—

$$\frac{\partial^2(rs)}{\partial t^2} = -\omega^2(rs) = c^2\frac{\partial^2(rs)}{\partial r^2},$$

and the complete solution may be written :—

$$rs = a_0 \sin\left(\frac{2\pi r}{\lambda} + \delta\right) \sin(2\pi nt + \varepsilon) \quad . \quad . \quad . \quad (64)$$

When the cone is continued to the vertex ($r = 0$) we must have at this point s finite, whether the vertex is open or closed. Therefore from (64) $rs = 0$.

1. *Open Cone.* First consider a conical tube continued to the vertex, and having the base, usually uppermost, open.

At the open end ($r = l$); $s = 0$ so that

$$a_0\left(\sin\frac{2\pi l}{\lambda}\cos\delta + \cos\frac{2\pi l}{\lambda}\sin\delta\right) = 0$$

This, with the vertex condition, combined with the fact that a_0 is not zero, gives :—

$$\delta = 0, \frac{2\pi l}{\lambda} = j\pi, \text{ or } \lambda = \frac{2l}{j},$$

where j is any integer. The harmonics are the same as for a cylindrical pipe of the same length open at both ends, and this, whether the vertex of the cone is open or closed.

2. *Closed Cone.* When the base of the cone is closed the requisite end condition is $\frac{\partial s}{\partial r} = 0$.

$$\frac{\partial s}{\partial r} = \frac{1}{r}\left(\frac{\partial(rs)}{\partial r} - s\right)$$

$$= \frac{a_0}{r}\left[\frac{2\pi}{\lambda}\cos\left(\frac{2\pi l}{\lambda} + \delta\right) - \frac{1}{r}\sin\left(\frac{2\pi l}{\lambda} + \delta\right)\right]\sin(2\pi nt + \varepsilon).$$

Equating this to zero, and adding the vertex condition $\delta = 0$ we get

$$\frac{2\pi}{\lambda}\cos\frac{2\pi l}{\lambda} = \frac{1}{l}\sin\frac{2\pi l}{\lambda}$$

$$\frac{2\pi l}{\lambda} = \tan\frac{2\pi l}{\lambda}.$$

If we put the solution of this, $\frac{2\pi l}{\lambda} = j\pi$, j is no longer integral but may have the following values, 0, 1·43, 2·46, 3·47, 4·47, 5·48, etc., so that the overtones of a cone closed at the base are inharmonic and the nodes of a partial are not equidistant along the pipe.

The Open Cone as Director and Amplifier. It is customary to put the source of sound at the vertex of a conical horn when it is desired to direct the sound, as in the "megaphone." Rayleigh pointed out that this object can be attained only if the mouth, i.e., the base of the cone, is large compared with the wave-length of the tone emitted, otherwise diffraction will ensue at the mouth, tending to produce equal intensity in all directions. To propagate fog-signals over the sea without unnecessary spreading in a vertical direction, he suggested a rectangular aperture for the horn, prolonged in a vertical direction but narrow horizontally. Recent research suggests that a cone's directive or collective properties are small compared with its amplifying properties. Work by Stewart [5] and by Foley [6] has shown that the amplification due to a conical receiver is much less than theory would predict, if the cone "collected" all the energy falling upon its wide mouth. A considerable proportion of the incident energy is reflected back the way it came without penetrating the cone (cf. also pp. 236, 333).

Viscous Damping in narrow Tubes. Consider a driving force $\psi e^{i\omega t}$ per unit area acting axially along a tube of radius r_0.

The equation of motion of an annular ring of radius r and thickness δr is :—

$$\psi = \left[i\omega\rho - \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\dot{\xi}.$$

Substituting $k^2 = -i\rho\omega/\mu$ we obtain a solution of the form

$$\dot{\xi} = -\frac{\psi}{\mu k^2} + AJ_0(kr)$$

where J_0 is a zero order Bessel function.

Applying the boundary conditions $\dot{\xi} = 0$ at r_0 and integrating over the section :

Mean velocity

$$\overline{\dot{\xi}} = -\frac{\psi}{\mu k^2}\left[1 - \frac{2}{kr_0}\cdot\frac{J_1(kr_0)}{J_0(kr_0)}\right]$$

(The equation is now of the form $\dot{\xi} = \psi/Z$.)

When $|kr| < 1$ all the fluid in the tube is under the influence of viscous resistance and Z becomes practically $8\mu/r_0^2$ as for steady (one-way) flow in the tube.

For $|kr| > 10$,

$$Z = i\rho\omega + \frac{2\mu\beta}{r}(1 + i) \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

where $\beta^2 = \omega/2\nu$.

Now in the case of a sound wave

$$\psi = -\frac{\partial p}{\partial x} = \rho c^2 \frac{\partial^2 \xi}{\partial x^2}.$$

Substituting in the original equation

$$\left(1 + \frac{2\nu\beta}{\omega r}\right)\ddot{\xi} + \frac{2\nu\beta}{r}\dot{\xi} = c^2\frac{\partial^2 \xi}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Assuming a solution $\xi = Ae^{-\alpha x}e^{i\omega(t-x/c)}$, we find :—

Absorption coefficient per cm. ; $\alpha = \frac{1}{r_0 c}\sqrt{\frac{\omega\nu}{2}}$.

Velocity of sound $c = c_0\left(1 - \frac{1}{r_0}\sqrt{\frac{\nu}{2\omega}}\right)$, c_0 being the " free-space " velocity, given by equation (65) with the viscous terms removed.

Tubes wide compared to the Wave-length. At the opposite extreme, when the tube is wide compared to the wave-length, modes of vibration may be set up in which the particle motion over a cross-section of the tube is not in phase (as it is in the cases already discussed) but may vary in a manner very similar to that over the surface of a membrane. These phenomena have their analogues in electromagnetic theory in which they are known as " dominant modes " and play an important part in the propagation of electro-magnetic waves in " wave-guides."

The whole forms a three-dimensional standing-wave pattern whose configuration may be explored by the use of a manometric probe or hot-wire anemometer.[7]

Scale of Tube or Pipe. One effect of the width of a tube has already been discussed in the first chapter, i.e. that on the velocity of the sound due to the dragging of the walls. The tubes used as sound sources of the type we are discussing in this chapter are wide enough for this drag to be neglected, but there is another important question linked up with the relation of the width to the length, or

" scale of pipe " as it is termed. This concerns the ease of production of overtones.

Other things being equal, a narrow scale encourages the upper partials at the expense of the lower. The organ builder constructs different stops of pipes on this principle; large scale pipes for the mellow foundation tone, narrow scale for string tone.

The radiation resistance at the open end is proportional to n^2 (p. 234), whereas the internal damping due to viscosity is proportional to $\sqrt{n}$ (p. 171). Consequently the frequency at which the total resistance is a minimum rises as the diameter decreases and the higher partials (for a given length) are elicited with greater effect in the narrow tube.

Tubes of very small scale are employed sparingly as connections between the reeds and the main vibrating column in certain bass instruments. When the diameter is a few millimetres only, the whole of the air in the tube may lie within the " boundary layer," so that its movements are, as a whole, subject to viscous drag. In this case not only is there rapid attenuation of the amplitude as the sound is propagated along the tube, but its slow speed reduces the wave-length and therefore the length of the tube which can " sound " to a given frequency.

A considerable amount of experimental work has been done to test that, in accordance with the formula, the diminution of velocity below the open-air value is inversely proportional to the radius of the tube and the square root of the frequency. In particular Kaye and Sherratt [8] have experimented with tubes of various diameters from 2·9 down to 0·9 cm. made of various materials and filled with four different gases in turn. They are satisfied that the Helmholtz-Kirchhoff formula sufficiently represents the facts. This, of course, is of great importance to know since so many measurements of the velocity of sound involve a " tube correction " based on (66). For very narrow tubes (less than 3 mm. diameter) the correction has been ascertained at high frequencies by Lawley (p. 260). Kastner [14] has measured the decay of waves of large amplitudes in tubes up to 10 ft. in length and from 3 to 1 in. in diameter, using an oscillating piston source. His results concur with Helmholtz-Kirchhoff formula (66, p. 171).

Large Amplitudes in Pipes. Sound waves of finite amplitude have been discussed theoretically by Earnshaw [9] and Riemann [10] and more fully by Rayleigh [11] (cf. p. 23). They all show that if the origin of the disturbance involves an abrupt discontinuity, the form

of the wave must change as it proceeds. Analogies can be instances between this form of sound wave and the wave due to the sudden release of heaped-up water behind a dam when it bursts. Such a wave also changes in type as it proceeds.

Experiments which relate to these theories have usually been made with shock waves, produced by bodies travelling with speeds comparable with that of sound, but only in one instance have measurements been made on the damping of sounds of finite amplitude, and then by an indirect method involving the resonance of the column of air in a pipe. These experiments were done by Lehmann [12] who used as source periodic air pulses at one end of the tube—on the principle of the siren—timed so that their frequency was near that of the natural oscillations of the column of air. The response of the tube was measured by a manometer. By varying the forcing frequency the resonance curve for the column of air was constructed and so the decrement of the system was derived. He obtained values of this factor for pressure amplitudes in the sound waves up to $0{\cdot}8 \times 10^5$ dynes per sq. cm., the mean static pressure in the tube ranging from 1 to 2 atmospheres. Both air and carbon dioxide were used.

The author's experiments [13] were done with "pulses" of sound produced in a brass tube 6·5 cm. diameter by expelling a tightly fitting wooden piston under excess of pressure. At the opposite end of the tube to the piston was a thin aluminium diaphragm, at the back of which a system of tiny levers recorded the motion of the diaphragm by causing a diamond point to scratch a miniature copy of its vibrations on a slowly rotating steel ring. Another diamond gave time marks alongside the pressure record at one-tenth of a second intervals. The ring was rotated by an electric motor and an electrically maintained pressure of about one-third of an atmosphere was built up in a large reservoir from which connection to the main tube was first cut off by a tap. Sudden opening of the tap expelled the piston from the tube and the resulting pulse reverberating up and down the tube recorded its amplitude on the diaphragm.

Brass tubes of diameter 6·5 cm. and of length either 32·5 or 64 cm. were used. Both air and carbon dioxide were used with initial pressures up to 8 lb./in.^{2} above atmospheric. To estimate the absorption coefficient the natural logarithm of the pressure amplitude from successive passages along the tube as the pulse ricochets are plotted against the order of the wave-length, whence the damping per wave-length is estimated. The frequency is also calculated from the record and

from the velocity of sound at the temperature of the experiment, the wave-length itself is derived; hence the damping per centimetre, which is the quantity α in Kirchhoff's formula.

The damping coefficient observed is much greater than Kirchhoff's theoretical value for infinitesimal amplitudes and increases the more, as the amplitude itself is greater. This was found by Lehmann for

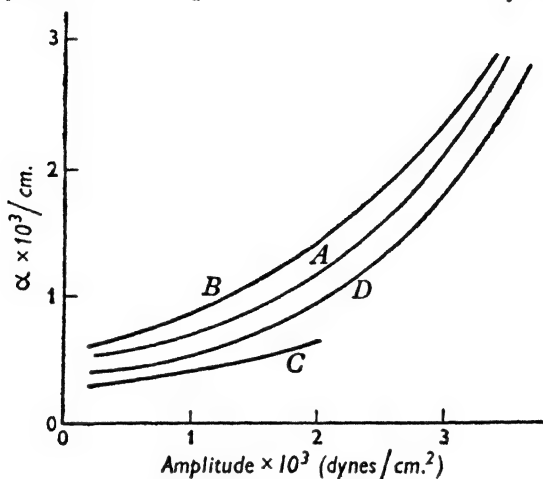



Fig. 62.—Attenuation of Sound Waves of large Amplitude (*Richardson*).

continuous sounds though his range of amplitude was much smaller. Fig. 62 shows four cases for which the data are:—

A. Air at 20° C., 50 per cent. humidity, $\lambda = 243$ cm.
B. Air at 20° C., 40 per cent. humidity, $\lambda = 143$ cm.
C. CO_2 at 16° C., $\lambda = 240$ cm.
D. CO_2 at 15° C., $\lambda = 135$ cm.

The values deduced from Kirchhoff's formula are, for amplitudes, (A) 2·4, (B) 3·0, (C) 1·5, (D) 2·0 $\times 10^{-4}$ respectively. Even at the lowest pressures, these values are exceeded by 2 or 3 times while at the highest amplitudes the experimental damping may be 10 times greater.

One would not expect Kirchhoff's theory still to apply at such large amplitudes, since the Reynolds numbers corresponding to the actual particle velocities much exceed the critical value for the breakdown of laminar flow. Indeed neither diminution of velocity in the tube, according to Kirchhoff's formula for c (p. 171), nor increase according to Riemann's formula for c' (p. 23) was observed, but perhaps the two opposed effects cancelled each other.

Flue Organ Pipe. Having shown how the vorticity in the jet from a linear, circular or annular orifice may be stabilized by making it strike a suitably placed edge, we are now in a position to discuss the maintenance of aerial vibrations in a tube by the edge tones from such an orifice, generally a linear slit. Fig. 63 shows a section of an organ pipe called "stopped diapason" which is of wood and of rectangular section. The wind at a constant pressure of several inches of water enters the mouthpiece M, then emerges from the slit O and strikes the edge E, formed by bevelling the wall of the pipe. An adjustable stop S closes the pipe, so that the "speaking length" is from the neighbourhood of O to S. Wachsmuth [15] was probably the first to recognize the organ pipe as a coupled system, i.e., the edge tones at the mouth coupled to the natural frequencies of the column of air in the tube. He says (1904) "the tone of a flue organ pipe is always one of the possible edge tones determined by f, by the blowing pressure, and by the length of the resonance tube." The f of the edge-tone formula obviously corresponds to OE. At a given blowing pressure, and therefore at a given V, a value of f can be found at which the pipe "speaks" most readily, but the pipe gives the same tone at neighbouring values of f. This is the generally accepted view to-day; before the present century it was thought that the issuing lamina of air vibrated transversely like a reed, at a frequency governed entirely by the column of air, but the notion has been superseded. To Wachsmuth and to Hensen [16] we are indebted for the modern conception of the functioning of flue organ pipes. By mixing smoke with the issuing air and examining it stroboscopically, or by taking instantaneous photographs, van Schaik [17] and also Weerth [18] was able to show the existence of these movements in the issuing air stream, though as Bénard's work was still to be done, they understood the mechanism of the edge tones imperfectly. Carrière [19] has recently drawn the appearance of the smoke under the stroboscope, showing the analogy with the edge-tone photographs of Schmidtke. The organ pipe should be designed so that at normal blowing pressure, the edge tone $n \propto \frac{Vj}{f}$ is equal to the fundamental of the pipe, so that pipe and edge tone are coupled at

Fig. 63.—Flue Organ Pipe.

resonance. In accordance with this principle (though not in recognition of it) organ-builders make f continually decrease from bass to treble on each " stop " of pipes. Such a coupled system as that with which we are here concerned is governed in the main by the less strongly damped component, in this case the column of air. A certain amount of mutual accommodation takes place between pipe and edge tone, but whereas the edge tone may be considerably pulled out of its natural period of vibration in order to secure equality of period, that of the column of air is alterable to a very small extent. Analogies may be found with the coupled vibrations of tuning fork and string which form the basis of Melde's experiment (p. 113). To exemplify the properties of the coupling, we may suppose the blowing pressure to be continuously increased beyond the normal. As V is increased, the natural frequency of the edge tone rises beyond the fundamental of the pipe, but the latter succeeds in forcing its own period upon the edge tone until the natural frequency of the edge tone, if isolated, would be nearer to the first overtone of the pipe than to the fundamental. Up to this moment the frequency of the coupled system has remained in the neighbourhood of the fundamental of the pipe, but now a jump occurs to the overtone, both edge tone and pipe tone rapidly picking up the new frequency which they retain with slight alteration until a jump to the next overtone takes place. Actually the behaviour is more complex in that the overtone may appear before the fundamental has ceased, producing a complex note. The procedure is shown graphically in Fig. 64, where the actual pipe tones of an organ pipe are shown by thick lines, and the natural frequency of the edge tones in the absence of the pipe, by a dotted line.

The increase of V resulting in the formation of overtones is known as " overblowing " the pipe.

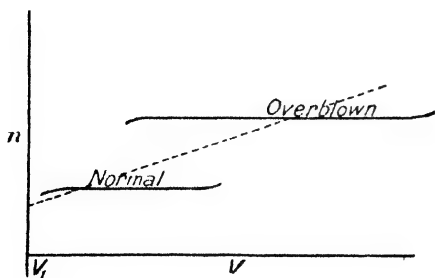



Fig. 64.—Tones of Normal and Overblown open Organ Pipe.

The converse phenomenon of the " underblown " pipe presents several items of interest. Suppose V to be continuously decreased below normal. Very soon the edge tones will be so far below the pipe tone that the latter ceases to sound. But now, the two systems being still coupled, the pipe

is still endeavouring to impose one or other of its own tones upon the vortex production at the mouth, and in an endeavour to conform, the edge tone reduces its pseudo "wave-length" by a transition like that described on p. 159 to $\frac{1}{2}f$, $\frac{1}{3}f$, etc., and the same tone is again, but feebly, elicited. Thus if the fundamental is normally produced at V_1, so that $\frac{V_1}{n_1 f}$ = constant, it may also be produced under conditions given by :—

$$\frac{\frac{1}{2}V_1}{n_1\frac{1}{2}f} = \frac{\frac{1}{3}V_1}{n_1\frac{1}{3}f} = \text{a constant,}$$

or the octave (in the case of the open pipe) given by :—

$$\frac{\frac{1}{2}V_1}{2n_1\frac{1}{4}f} = \frac{\frac{1}{3}V_1}{2n_1\frac{1}{6}f} = \text{a constant.}$$

These transitions (shown in Fig. 65) can the more easily take place if f is large—if the mouth is "cut up high"—as the organ-builder phrases it. Thus we are faced with the curious fact, that overtones can be formed both by extra and reduced pressure in blowing the pipe. Of course the intensity of the sound is very different under the two conditions. The input of energy to the pipe is measured by the product pressure of wind × volume delivered per second, for this is the work which the blower has to do per second. A certain percentage of this—which we may call the "efficiency" of the pipe—is converted into sound. Now V^2 is proportional to the blowing pressure, and the volume delivered per second is V times the area of the slit, so that the intensity of the sound, *ceteris paribus*, varies as V^3. Therefore the tones of overblown pipes are louder than normal, whereas those of underblown pipes are very feeble. They may be heard when the air is first driven into the pipe—this is technically known as "coughing" or "murmuring"—but may be extinguished by so designing the mouth, that the jet is not directed on to the edge until normal pressure is reached (cf. p. 158).

Fig. 65.—Tones of "Underblown" open Organ Pipe.

The stopped pipe is tuned by adjusting the position of the stop; the open pipe may be sharpened by cutting off part of the end of the tube. What is known as "voicing" is an adjustment of the coupled

system to give the note required, and is mainly accomplished by operations on the mouth.

Some of the considerations which determine the quality of the note are enumerated here.[20]

1. The "scale" of the pipe, i.e., ratio of diameter to length.
2. Height of the mouth.
3. Shape of upper lip or "edge"; whether thick or thin, convex or concave.
4. Shape and position of languid (see Fig. 63; *L*).
5. Nicking of lower lip, *P*.
6. Material of the tube.

The explanation of some of these points has been given in our text, others are at present beyond scientific explanation. One is reluctantly compelled to admit that the organ-builder with his empirical methods is years in front of the physicist.

Kuhn [21] has carried out a careful investigation of the effect of changes in the geometry of a typical organ pipe near the mouth on the tone quality. In terms of Fig. 63, he varies (1) height of mouth, (2) diameter of slit orifice, (3) horizontal position of upper lip, (4) breadth of mouth. He found that for each partial tone there was a setting of the upper lip both horizontally and vertically which gave the greatest amplitude to this partial in the complex note of the pipe. This effect is apparently connected with the angle at which the air debouches from the slit, since altering slit diameter had a similar effect. All partial amplitudes increased with the breadth of the mouth and with wind pressure.

Lüpcke [22] has recently made a careful examination of the timbre of the orchestral flute, varying the blowing pressure to get the best output. The fundamental was, of course, very prominent always and the third partial was usually stronger than the second especially when cross fingering was used. As the player runs up the scale the overtone structure increases with a jump to greater overtone emphasis at the first overblown note (*d*). The pressures used by players (measured by a manometer in the mouth) were low compared with those on the organ (about 1 in. water). There was a slight rise of pitch, keeping to one note, with blowing pressure.

Reed Pipes. In this class of sound-producing systems a column of air is closed at the mouth by a "beating reed" (cf. p. 112). Normally the reed stands clear of the orifice leading to the pipe, but when wind under pressure is introduced to the chamber surrounding the

reed, it is deflected by the rush of air into the pipe, until it shuts up the pipe orifice; then, being under tension, it springs back and lets the air into the pipe again, and the cycle is repeated. This coupled system consists of " bar " in transverse vibration, and " air column " in longitudinal vibration. Various possibilities may occur, depending on the form of the " coupling."

Firstly, let us consider the reed organ pipe. The air tube or pipe proper is generally conical. The reed, generally of brass, is at the vertex, and is contained at the end of a narrow tube, to which one end of it is clamped. The reed and this narrow tube are enclosed in a short supply pipe termed the " boot." The conical pipe and the reed are tuned to the same frequency, the tuning being accomplished by a spring which presses on the reed near its clamped end, shortening it or lengthening it as desired (Fig. 66). The resulting note is due to the fundamental of the reed with the coincident fundamental of the pipe, together with some overtones of the latter. As the overtones of the reed are inharmonic, these are not produced because they do not form part of the " note " of the column of air. The vibrations of the reed are always simple harmonic although the air vibrations involve the overtones proper to the form of the air column. Trouton first[23] observed that the length of the supply-tube exerted a control on the tone of the complete pipe, which is really a tripartite system. If the length of the supply tube is of the order of the length of the pipe, the best arrangement is to have the former about a quarter of the wave-length, or an odd multiple of a quarter. The reed apparently vibrates most freely when placed near a node of the column of air in supply-tube and pipe proper. Auger[24] used a cone one metre long having a fundamental frequency of 133 cycles/sec. and applied a beating reed to it at its vertex. Keeping the blowing pressure constant, he gradually increased the length of the reed. As the length increased the frequency fell until at a reed-length of 4·7 cm., the reed and the first overtone of the column were in tune. Any slight further increase of length caused the note to drop suddenly to the fundamental of the cone, at which it remained until the reed-length exceeded 5 cm., when it again began to fall. On reversing the procedure the jump to the overtone occurred at a length of 5 cm. (instead of 4·7 cm.). Next,

Fig. 66.—Reed Organ Pipe.

he kept the reed-length fixed and increased the blowing pressure, showing that a jump to the overtone of the system may occur when the pressure reaches a critical value, which is likewise different for a range of increasing pressure to one of pressure decreasing.

Mokhtar [25] has taken wave-forms of the pressure variation in the pipe and of the vibrations of the reed with which it is coupled. While the wind pressure increases, the pipe gradually grows in richness of overtone structure, although all the while the vibrations of the reed itself have remained simple harmonic. When the critical pressure and consequent jump occur, the fundamental disappears from the record of the aerial vibrations; at a higher critical pressure, the first overtone may also disappear. Under these latter circumstances (i.e., in which the fundamental has disappeared from the pipe's note, but not from the reed note), it is evident that the forcing impulse from the reed occurs once in several vibrations of the air column. The system, therefore, executes *relaxation oscillations* (cf. p. 56), the relaxing being evident when one compares the two records, the amplitude of the aerial vibration falling after each pulse of air from the reed, until the latter reopens the chink between itself and the stop against which it "beats," when the air column is revivified in a sudden increase in amplitude.

Mokhtar's records exemplify the phase relationship between the vibrations. If the wind pressure be kept constant at a value slightly in excess of the minimum to make the pipe sound while the length of the tube resonator is increased so that its natural frequency becomes more out of tune with that of the reed, the peak in the air compression lags more and more behind the reed's motion until they are so at sixes and sevens that the system is reduced to silence. By raising the blowing pressure at this stage it may be made to sound again. The fact that with the shorter length of resonator we have a simple harmonic motion on the part of the driver maintaining relaxation oscillations of the air column shows that the motion is maintained by periodic impulses and not by continuous forcing. With periodic impulses of duration short compared to the full period, maintenance is most efficacious if the impulse of pressure occurs at an epoch when the pressure in the air is a maximum. The length of the tube under these conditions is that for which the compression travels to the open end and back just in the time between two pulses. As the tube is lengthened the system will pass through a series of favourable (corresponding to the condition just named) and unfavourable (silent)

lengths. This type of maintenance will be discussed in more detail in connection with the singing flame (p. 193).

The Effect of Wall Materials on Quality. The role of the material of a pipe on the quality of the note it emits has long been a source of argument, some maintaining as the writer has done, that this factor has little effect, others that it exerts the major control over the quality. We shall assume in what follows that the walls are neither so thin and lissom as to give way under the compressions and rarefactions set up by sound waves in the column nor so strong and elastic that they are easily set in resonant (longitudinal or transverse) vibration with the edge tone or reed to which the column is coupled. Wolf notes of the latter type are not unknown in ill-made metal flutes and clarinets. Even in the absence of a definite " wolf ", Lottermoser [26] disclosed a modulation of the upper partials of the column when they happened to lie near natural (longitudinal) tones of the metal pipe. (The latter are usually too high in pitch to affect the fundamental tone of the column.)

Boner and Newman [27] have made a very painstaking research on this point, in which they were careful to compare flue pipes of different wall material otherwise identical in construction, i.e. having the same mouth, the region most sensitive to slight adjustments. They used eight different materials, including shellaced paper and thin copper. The former pipe improved in speech as the shellac dried and when the tube was grasped by the hand, the latter continued to " ring " in one its natural longitudinal frequencies after the wind was cut off from the pipe.

The general results may be thus summarized :—

(1) the fundamental amplitudes were nearly all equal,
(2) the even harmonics of the paper pipe were weaker than those of the rest,
(3) the wooden cylinder is as strong in harmonic development as the metal cylinder, contrary to what many organ builders say.

The extent to which the walls confining a vibrating column of air may take part in the vibration has also been studied by Knauss and Yeager,[28] using a cornet having a crystal pick-up attached to it, but though metallic vibration was detectable it proved to be insignificant as a contribution to the total sound output of the instrument.

On the whole, then, the effect of wall material on timbre is small.

Examination of Phenomena inside the Pipe. We have already dealt with the behaviour of the column of air in a pipe as given by theory; and it now remains to describe the methods employed to test what is happening to the air in the pipe. The first methods will be qualitative, later some methods claiming to give quantitative results will be described. The methods may be classified in accordance with the particular property of the air which they measure, as displacement, pressure, density or temperature. Most of these instruments were originally used on organ pipes but can be adapted to columns of air maintained in vibration in other ways.

One of the earliest methods of obtaining an idea of the relative displacement in different parts of the pipe was to lower a little paper membrane formed on a ring and covered with sand into the pipe held vertically. Maximum agitation of the sand particles is shown at an antinode, minimum agitation at a node. A corresponding pressure indicator is the manometric capsule devised by König.[29] This consists of a membrane of parchment or thin rubber gripped between two rings, dividing the capsule into two halves. One half can be sealed by glue or wax to any point of the pipe at which a hole is made, through which the pressure variations at the point in question are communicated to the membrane. The other side of the membrane faces a little box, closed save for an inlet by which coal-gas impinges on the membrane from a supply, and a gas outlet ending in a pin-hole burner. The gas lit at this burner acts as a sensitive indicator of the motion of the membrane by its movement up and down; to this end both inlet and outlet should point directly at the membrane. The flame is also able to show the presence and relative intensity and phase of any harmonics which may be present in the pipe, but owing to its inconstancy the instrument is unsuited to absolute measurements of the pressure amplitude. The motion of the flame can be examined stroboscopically, or in the revolving mirror invented by Wheatstone (Fig. 67).

When it is desired to record the note of a pipe, a record of the flame's movement can be obtained, either by passing a strip of paper over the flame, when its soot will form a periodic record on the paper (Brown),[30] or by mixing as much acetylene with the gas as the flame will take without becoming smoky, and photographing the motion on a rapidly moving plate. By this means Merritt[31] obtained records of speech and musical notes directed on to the membrane of the capsule, in the open atmosphere.

Kundt's dust tube method has already been described and may be used for an organ pipe if one side at least be of glass, indeed blowing such a pipe with another gas forms an additional method for finding the velocity of sound in the gas.

The position of the nodes and antinodes may be found by a thin listening tube pushed into the pipe from one end, and connected to the ear. A minimum sound will be heard at an antinode. As the search tube is liable to interfere with the motion in the pipe, König [32] replaced one side by a water surface, and pushed a tube bent twice at right angles through the water into different parts of the pipe.

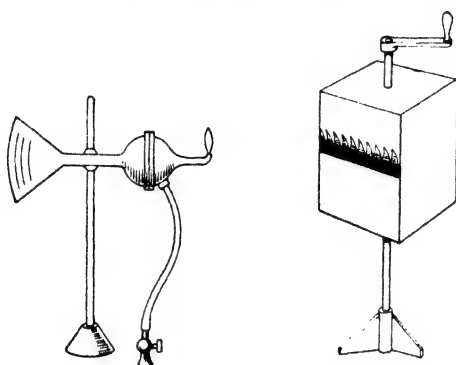



FIG. 67.—Manometric Flame (*König*) and Revolving Mirror.

Measurement of the Pressure Amplitude in the Pipe. A liquid manometer of the bent tube type possesses too much inertia to record the small but rapidly alternating changes of pressure in the pipe. To obviate this, Kundt [33] conceived the idea of admitting the compressions only to the manometer by means of a valve which opened only outwards. This was placed at a node of the pipe, and the manometer showed a steady increment over the atmospheric pressure, by which Kundt claimed to measure the pressure amplitude at the node. Raps [34] endeavoured to improve this valve manometer by a mechanically operated valve which opened at the phase of maximum compression, at a frequency which was made to coincide with that of the pipe.

The valve itself exercises a considerable influence, both on the readings of the gauge, and the performance of the pipe, so that it is not surprising to find that the values by these stroboscopic manometers do not agree with later measurements, being in fact much too

large. A convenient and simple membrane manometer may be made on the lines of the manometric capsule, but with the flame replaced by a mirror as indicator.

Fig. 68 shows a full-size drawing of the instrument convenient for application to a wooden pipe. The capsule widens out conically from a short cylindrical piece to a flange between which and a corresponding ring there is fixed by screws a membrane of the thinnest sheet rubber tightly stretched in order that its natural frequency may be high. The mirror has to be given an angular motion in order that it may deflect a spot of light on a scale. To accomplish this, the mirror may be placed on a little lever which presses on the membrane. It must be remembered, however, that a membrane has its natural frequency considerably lowered by a load of this kind, and one wants to keep this frequency above the range of possible harmonics of the pipe. Because of this it is better to cement the mirror by rubber solution in an eccentric position. On the instrument used by the writer the

Fig. 68.—Manometric Membrane with Attached Mirror.

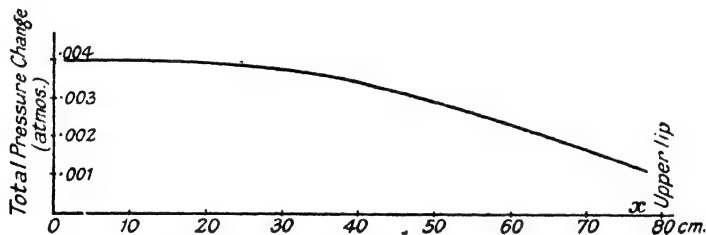



Fig. 69.—Pressure Amplitude along Stopped Organ Pipe.

mirror was made by silvering a tiny fragment, about 1 mm. square, cut from a microscope slip. This was fixed at a distance equal to half the radius from the centre of the membrane. With the lamp and scale a metre away, the instrument had a sensitivity of 1 cm. for a change of pressure equal to a thousandth of an atmosphere. The calibration is carried out statically by applying various small pressures (measured on a water manometer) and noting the corresponding deflection of the image of a narrow slit of light on the scale. When

the apparatus is on the sounding pipe, a hole being made into which the cylindrical portion of the capsule is sealed by soft wax, the pressure amplitude can be read off from the calibration line, when the amplitude of the deflection has been observed. For exact work a correction for the lag of the membrane may be necessary. Instead of attaining the corresponding deflection θ_0 instantaneously, the light approaches it on an exponential curve. As the instrument is lightly damped the approach to the final deflection is rapid. A curve of the pressure amplitude down a stopped pipe obtained by means of the manometer is shown (Fig. 69).

Density Amplitude by Optical Interference Method. Töpler and Boltzmann [35] introduced a method for examining the extent of

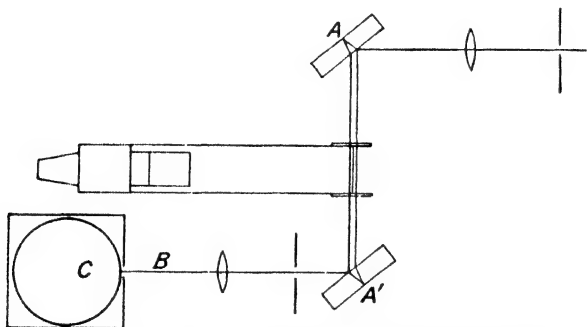



Fig. 70.—Optical Interference Method for Organ Pipe (*Töpler and Boltzmann*).

the density changes at the node at the end of a stopped pipe, based on the Jamin optical interferometer. Light fell on an inclined glass slab A (Fig. 70), so that part entered the glass and was totally reflected by the back surface, passed through a glass window at the end of the pipe, and out again by an opposite window. The rest of the light entering the glass slab was reflected at the front surface, and emerged parallel to the other ray but passed outside the pipe. The two beams were recombined by a second glass slab A', but owing to the somewhat different paths they had traversed, coloured interference bands were observed in an eye-piece placed at B (Fig. 70). The position of these bands depends on the optical distance traversed, and if the air in the pipe increases in density owing to compression the velocity of light in the air is decreased, so that a greater relative retardation of the two beams occurs which results in a shift of the interference bands. Now when the organ pipe sounds, rapid changes of density take place at

the node, so that on looking through the eye-piece the bands seem to be broadened, as the eye cannot follow the rapid movements of the bands following the rapid changes of density. Examined in a stroboscope, the amplitude of the movements of the bands is disclosed, and knowing the dependence of the velocity of light on the density of air, the density amplitude could be calculated. This is connected with the pressure amplitude by the formula $\delta p = \gamma p \frac{\delta v}{v} = \gamma p s$ (cf. 60).

Raps,[36] who repeated the method, found it better to calibrate the interferometer by compressing statically the air in the pipe, and noting the shift of the bands produced. He also photographed the movement of the bands by a revolving-plate camera C, employing both a reed and a flute organ pipe.

The method has the advantage that there is no question of lag, at any rate there is no lag of the order of the period of the tone, but it is extremely necessary that the walls and windows of the pipe shall be quite rigid, a slight vibration of the latter being sufficient to cause shifts of the bands greater than that due to the density change sought.

Measurement of the Temperature Change at a Node. The compressions and rarefactions taking place at a node cause temperature changes connected therewith by the adiabatic law. The amplitude of the change is very small, about 0·1 degree Centigrade, and requires a resistance thermometer of very fine wire to measure it. This difficult feat has been attempted by Neuscheler.[37] He used a Wollaston wire of 0·001 in. diameter, the oscillatory change in resistance being observed. The maximum estimated temperature change was 0·13° C., corresponding to a pressure variation in the node of 0·0155 atmosphere, the pipe being blown at a pressure of 5 inches of water. Pressure and temperature θ are connected by the relation $\delta\theta = \frac{(\gamma - 1)}{\gamma}\frac{\theta}{p}\delta p$. Taking the temperature coefficient of resistance of platinum as 40×10^{-4}, this would produce a resistance change of 4×10^{-4} ohms. It is doubtful whether the indications of a resistance thermometer can be relied on in such small but rapid fluctuations, in spite of the careful technique developed by Neuscheler. In a later paper, Friese and Waetzmann[38] claim that such a "thermometer registers a fraction (depending on the fineness of the wire) only of the temperature changes in an oscillation of 100 periods per second."

Displacement Measurement. The cooling of a hot wire by the current of air in which it is placed forms a more convenient method

of measuring the amplitude in a pipe, since the changes of resistance involved are considerable, and a string galvanometer is unnecessary. It was discovered by Richards [39] that when a hot wire is placed in an alternating draught—actually the wire was placed on the prong of a tuning fork—it assumed a steady resistance corresponding to the maximum velocity in the alternation, i.e., to $2\pi na$, when a is the maximum displacement, and n is the frequency (see p. 33). In other words, the resistance of the vibrating wire as measured on a Wheatstone Bridge with a dead-beat galvanometer is the same as that which it would have if placed in a steady wind of this velocity. By measuring the steady drop of resistance of a nearly red-hot wire (0·001 in. diameter) placed at different positions in the pipe, we are able to find the variation of a along the pipe.[40]

Recently the possibility of a direct measurement of the amplitude of the gas molecules by intermixing with visible particles has been explored. Lewis and Farris [41] allowed dust particles to fall through a sounding tube held horizontally, measuring their sideways displacement in a microscope. Gehlkopf [42] using oil drops and Carrière [43] and Andrade [44] using smoke have been able to hold particles in suspension long enough for their amplitude in vibration—they appear as streaks under the microscope—in a sounding tube to be measured. Naturally the smaller the particles introduced the greater the fraction of the total (molecular) motion which they assume. Both W. König [45] and Andrade have gone into the theory of this method and the latter has shown how to calculate the total amplitude by extrapolation.

König assumed that the Stokes expression for the resistance of a particle of radius r and velocity $\dot{\xi}$, i.e. $6\pi r\eta\dot{\xi}$, would apply and wrote for the equation of motion of a particle of mass m and density ρ in a sound-wave of amplitude a (gas amplitude) and pulsatance ω :—

$$m\ddot{\xi} = 6\pi r\eta(a\omega \cos \omega t - \dot{\xi}).$$

Whence
$$\dot{\xi} = \frac{a \sin (\omega t - \delta)}{\sqrt{\left[\left(\frac{2}{9}\frac{\rho r^2\omega}{\eta}\right)^2 + 1\right]}}.$$

The expression in the denominator gives the ratio of amplitude of the gas to that of the solid particle.

Wood-wind Instruments. The *flute* and *piccolo* are types of organ pipes of variable length. Fig. 71*a* shows a simple flute in section without keys or levers. In the modern form (*cross-flute*) edge tones are produced by the player directing his breath across

a hole M to its opposite edge. A number of side holes are provided and when all these are covered by the finger-tips or by levers and keys the speaking length is from the mouth-hole to the open end, and the instrument sounds "middle C." As each hole is opened in turn antinodes are formed which terminate the effective length of the pipe at a shorter distance from the mouthpiece and raise the tone. As in the organ pipe this reacts upon the edge tones; but the player assists the adjustment by blowing across the mouth with a greater velocity as the frequency goes up; f being an invariable quantity, roughly equal to the diameter of the hole.

Steinhausen [46] found that the vibrating length of the air column did not terminate at the open hole nearest to the mouth hole. Thus,

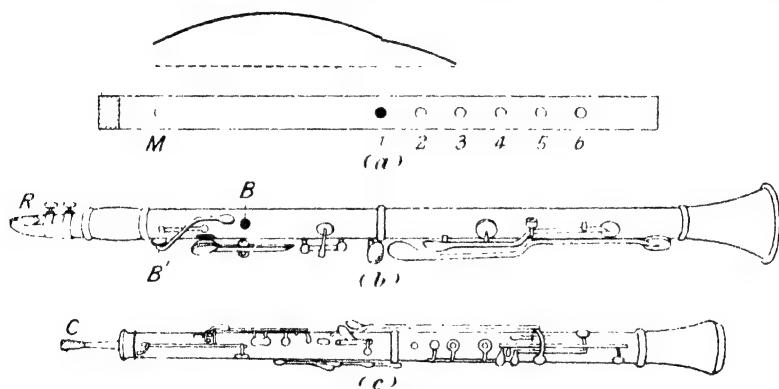



Fig. 71.—Wood-Wind Instruments.

in the figure, hole No. 1 is supposed to be closed while 2 to 6 are all open. If these holes were wide compared to the diameter of the tube, the vibrating column would stretch from one antinode at M to another at 2, but owing to the small diameter of the holes, the vibration is extended a little beyond 2. The upper line exhibits the relative pressure amplitude along the flute. By overblowing, the octave of the whole series of notes is obtained, intermediate semi-tones being formed by keys opening additional holes. The flute is characterized by its pureness of tone and absence of harmonics.

The *clarinet* (Fig. 71*b*) has a beating reed R of cane secured to the mouthpiece by clamps, but the frequency of the reed is at the mercy of that of the cylindrical column of air, which the player varies by opening side-holes. The reed forms a node (cf. p. 180), while the open end of the cylindrical tube, or some intermediate point if the side-

holes are open, is an antinode, so that the series of tones is the same as that in a column of air stopped at one end (p. 166); and overblowing, which is assisted by opening a hole B near the closed end of the tube, produces the twelfth instead of the octave of the fundamental series, and the quality of the note is one rich in the odd partials. The difficulty in playing the instrument is, that unless the mouthpiece is carefully constructed, the reed may escape from its bondage to the column of air and give out that distressing " quack " which characterizes its natural note. The player assists in adjusting the length and pressure on the reed, by his lips, with the flat part of which he grips the reed (see Fig. 71*b*).

As the reed begins to close the aperture a condensation starts down the tube but is reflected as a rarefaction from the open hole or end of the tube and on return finds the aperture closed by the reed. It is reflected therefore as a rarefaction for a second trip down the tube. Again reflected at the open end, this time as a condensation, it returns to push open the reed aperture. Thus in one complete cycle of the reed's vibrations the sound travels four times the length of the tube as in the stopped diapason type. Accordingly, even harmonics should be missing in the timbre. McGinnis, Hawkins and Shera [47] have taken a large number of oscillograms of the clarinet. They ascribe the characteristic quality of the clarinet to the strong odd harmonics, though they find that the even ones are not entirely absent, as simple theory would indicate. In the high register the second, in fact, overpowers all the others but it is well known that the characteristic clarinet timbre cannot be distinguished in playing high notes.

The spectra naturally change with blowing pressure and with the effective length of the reed. The player can adjust this by the grip of his mouth according to the pitch of the note he wishes to elicit, and to a certain extent pull the column out of its natural frequency.

The *oboe* (Fig. 71*c*) and *bassoon* have a *conical* tube, opening out from the vertex, where the mouthpiece is situated. As explained on p. 170 the column of air in such a tube corresponds to that in a doubly-open cylindrical pipe, so that overblowing gives the octave of the fundamental notes formed by successively shortening the effective length of the air column. The reed differs from the clarinet reed, as on these instruments it consists of two thin pieces of cane C with their free ends almost touching, and projecting into the player's mouth. Such a double reed is more flexible than the thick clarinet reed, so that it is not necessary to adjust the pressure on the reed to a nicety;

literal overblowing will produce the higher notes of the instrument, as in the flute. The "note" is very penetrating and rich in high harmonics.

The *saxophone* is a mongrel instrument, invented by Sax in 1840, but not much used till recent years. It has a conical tube like the oboe, but a clarinet reed and mouthpiece. In consequence of its conical shape, overblowing elicits the octave of the air column, which is enclosed in a metal tube. Here we have a proof that it is the shape of the tube which determines what partial is produced by overblowing, and not the type of reed. Although the quality of the saxophone is not unpleasant, the combination produces a rather distressing slowness and uncertainty of "speech."

Redfield [48] raises some important points in connection with the theory of wind instruments. The most telling of these is that the embouchure is not a node of the column of air in the instrument, for this column is effectively continued more or less into the player's mouth, larynx and chest. This explains the discrepancy between the theoretical speaking lengths—plus end-correction at the flare—and the actual frequencies of such instruments. The mouthpiece is in fact to be treated as approximating to an antinode, not as a node.

Ghosh [49] develops the suggestion that the reed never entirely closes the mouthpiece. It is further assumed that the pressure inside the player's mouth remains constant while the reed acts in the fashion of a valve to produce periodic variations of pressure in the embouchure of the instrument. Ghosh obtains approximate equations for the motion of the reed, considered as a wedge-shaped bar clamped at one end. On account of the aerial vibrations, the pressure in the mouthpiece fluctuates cyclically about the value $P \cos \omega t$ where P is the pressure inside the player's mouth. The motion of the reed will impose further pressure variations on these. If the chink opens at an epoch of maximum pressure, more air will flow in and accentuate the peak of pressure, but if the opening occurs at a minimum there will be a tendency to level out the fluctuation, i.e., to make an antinode of the mouthpiece. Theory shows that the best condition for maintaining the coupled system will be that the flow of air into the mouthpiece should precede the phase of maximum pressure therein by a quarter period. Finally, the feeble even numbers in the spectrum are shown to be components in the reed's vibration forced upon the unwilling column of air.

Brass Instruments. The instruments which consist merely of a

simple conical tube like the *post horn* and the *bugle* can of course give only the harmonic series of tones, of which it is not generally possible to sound the fundamental. The mouthpiece consists of a cup or cone-shaped orifice, which fits over the player's lips which form the reeds of the instrument. In order to elicit the overtones of the instrument the player must compress his lips together more tightly and blow harder in order that the "reeds" may vibrate with the higher frequency. Music for these "natural horns" must be confined to the series of notes formed by the partials of the instrument—the "harmonic scale" as it is called. In order that the full range of semi-tones may be obtained, orchestral instruments of this family are provided with three or four branch tubes, normally closed by "pistons." By depressing one or other of these valves the player can lengthen the tube and obtain a new harmonic series based on a fundamental several semi-tones lower than the normal. With three or four of these harmonic series available, the gaps in the scale of the natural horn can be bridged.

The fundamental of the horn can also be lowered by the player partially shading the bell-shaped open end of the instrument by means of his hand, thus increasing slightly the effective length of the vibrating column. It is said to be possible to produce two notes at once if the player blows to elicit one note, and at the same time hums another note into the horn, but it is doubtful whether this has been achieved when the two notes do not form part of the harmonic series, that is to say, the humming probably serves to emphasize one partial tone in the series which is already present in the vibrations of the column of air in the instrument.

The *trombone* exceptionally employs a "slide" which lengthens the speaking portion on the principle of the device figured in Quincke's interference tube (Fig. 21). The player can thus vary the fundamental tone of his instrument continuously, but in practice the slide is employed in eight positions, with each corresponding series of harmonics. The relation between the slide trombone and an instrument "*à pistons*" is analogous to that between a violin and a banjo with its fixed frets for the fingers. The advantage lies with the former instruments, inasmuch as the player can "make his own notes"; a skilful player unconsciously adjusts the sounding length of string or tube to suit the pitch of other instruments in the orchestra, or to allow for temperature variations. On wind instruments without slides one can make small increments of the length by pulling out the

mouthpiece joint before playing, but such an adjustment will be correct for one side hole and therefore for one note only. Thus, if the uncorrected lengths corresponding to a note and its octave are l and $\frac{1}{2}l$, the lengths corrected by an additional l_0 will be $l + l_0$, $\frac{1}{2}l + l_0$, and the ratio of these no longer being 2 to 1, the octave relation will be imperfect.

A somewhat similar fault occurs when more than one piston on a brass instrument is pressed so as to lengthen the tube by adding more than one branch tube; thus, if the first piston depresses the fundamental a semi-tone, and the second a whole tone, the first increases the speaking length in the ratio 15 : 16, the second in the ratio 8 : 9, and hence the two together lengthen it by $\frac{1}{15} + \frac{1}{8} = \frac{23}{120}$ which is less than the increase necessary for three semi-tones. Hence the two branches together are insufficient to obtain the required lowering of a tone and a half. To overcome this fault short compensating tubes are added which come into action only when the appropriate combination of pistons is depressed, and which bring the total added length to the amount necessary. Besides these transitory adjustments of the speaking length, more permanent alterations to suit the composer's requirements may be made by inserting "crooks," U-shaped bends of tubing added to the length of the tube which change the whole series of notes produced by the horn with or without the employment of pistons. The crooks give quite the same effect as the pistons except that they are in use all the time, having no valves to cut them off.

The quality of wind instruments is determined by many factors. Apart from the shape and scale of the tube and the form of the mouthpiece, the material and thickness of the walls of the tube play a part (cf. p. 181). The damping is not a constant quantity for the harmonics of a brass instrument, but changes as the pitch goes up, so that the relations deduced on p. 49 for the sharpness of resonance do not apply. Analysis of the wave-form of the notes from wind instruments shows that not only the relative number of harmonics but also their absolute position in the scale determines the quality. Thus the trombone tends to bring out with emphasis all overtones lying between the notes 485 and 580, whatever the fundamental pitch.[50]

Barton and Laws [51] by placing a little manometer in a corner of the player's mouth, found the blowing pressures necessary for various notes of the scale on a brass instrument. It is rather difficult to

co-ordinate their results with theory, as the latter has not advanced far enough to take all the human factors into account. But the experiments show: (1) that the blowing pressure rises proportionately to the logarithm of the pitch, the intensity being kept as constant as possible; (2) that at constant frequency the pressure rises with the intensity. In view of what was said with regard to overblowing an organ pipe and the energy used therein, these variations are in the direction we should expect.

Transient Sounds. We have thought it best to leave to this point our account of the transients of musical instruments because of its general application, although most of the careful analysis has been done on the violin and the two proto-types of organ pipe.

We have already dismissed as characteristic of the note many of the features of individual instruments formerly thought to be of primary importance, leaving the transients which precede and follow the establishment of the steady state as discriminatory factors, e.g. between the violin, the orchestral oboe, and the oboe-imitation stop on the organ.

Trendelenburg, Thienhaus and Franz [52] were among the first to record the initial transients of organ pipes on their octave filters. During the 1939–45 war, Nolle and Boner [53] in U.S.A., and Cook in England examined oscillograms of organ pipes.

Cook [54] analysed his oscillograms of an open diapason to show the relative intensities of the first and second partials in the transient and also recorded the rise of air pressure.

One notices the intrusion of even partials during the build-up and their eventual disappearance before the steady state is reached. The order in which these partials appear usually depends on the *rate* of augmentation of pressure. When the air is turned on slowly the fundamental appears before the second harmonic and contrariwise. The transient states do not correspond to a series of possible permanent states under sustained forcing but are characteristic of non-linear systems.

The Singing Flame. The fact that a jet of hydrogen, burning in an open tube, would under certain conditions cause a musical note to be emitted, was first observed by Higgens in 1777, and other observers studied various aspects of the phenomenon, without attempting to account for it, until De La Rive advanced his theory, i.e., that the periodic condensation of water vapour by the burning hydrogen caused the emission of the note. That this cannot be the true cause

of the "singing" was shown by Faraday, who was able to replace the hydrogen by carbon monoxide, a gas which produces no moisture in its combustion, without detriment to the effect. Faraday himself put forward an alternative theory, that the note was caused by successive explosions of the gas with the oxygen of the air, the flame then dying out until a further supply of air arrived at the jet, when it was re-ignited. For Wheatstone had shown in his revolving mirror that the flame was not steady, but vibrated up and down, so as to appear in the moving mirrors as a succession of images. As a result of his experiments, Sondhauss [55] considered the cause of the singing to be the heating of the air in the neighbourhood of the jet, the subsequent change of density causing a compression to flow away from the jet, thus starting the air in the large outer tube (hereafter referred to as the air tube) to sound its natural tone. He also found that certain combinations of the air tube and gas-supply tube would not sing, and concluded that the length of the gas tube must vary with different gases, in order that the oscillations of gas and air may be in step near the jet. If these oscillations were stopped by a plug of cotton-wool in the air tube near the jet, the singing ceased.

Rayleigh [56] showed theoretically that, unlike ordinary resonance, the impulses given to the resonator (i.e., the air in the tube) by the impressing force (the heat) must occur at the phase of maximum displacement or condensation, and not in the neutral position. Also that for the continuance of the vibrations the gas tube must be of such a length and in such a position, that a condensation at the jet will travel down it and back again so as to arrive at the jet in phase with a condensation in the air tube (as Sondhauss had previously stated), in fact, that there should be stationary waves in the gas tube as well as in the air tube.

This impulsive type of maintenance is met in the theory of relaxation oscillations (p. 56), wherein it appears that the additional pressure impulse must be given by the flame bobbing up at the phase of maximum compression (corresponding to Fig. 20*c*, p. 55) if the vibration is to be maintained, so as to heat the gas suddenly at this epoch and compress the gas further.

The writer has made some experiments which support this view.[57] A brass tube, 60 cm. by 4 cm., was taken and a gas tube with a jet placed in it in such a position that the singing started spontaneously and vigorously. A hole, 1 cm. diameter, was bored in the brass tube at this point, also a larger hole, over which a piece of glass was fixed

by sealing wax, to serve as a window through which the singing flame could be observed (Fig. 72). Over the small hole a König manometric capsule was placed, the burner of which was brought right round in front of, and close to, the window, so that when lit this manometric flame appeared vertically beneath the singing flame. Both flames were fed by coal gas, and in order to compare the flames they were looked at directly through a stroboscope. By this means it was seen that they rose and fell almost simultaneously and in the same phase. Fig. 73 shows the appearance of the two flames in the stroboscope. The theory of Rayleigh with regard to the phase of the heat supply receives its justification in this experiment.

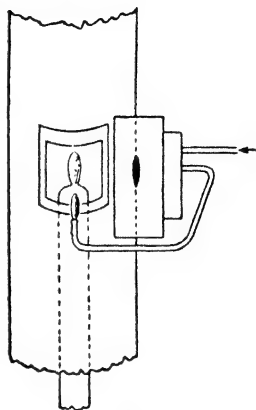



Fig. 72.—Phase Relationship of Singing Flame and Column of Air.

Position of Flame and Length of Supply Tube. Granted that the action of the "singing flame" is to increase the pressure amplitude, it must be placed at a point where the change of pressure is appreciable, i.e., near a node. The ends of an open tube being antinodes for all harmonics, the flame will not sing near these points.

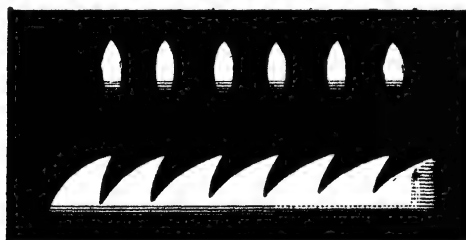



Fig. 73.—Movements of Singing Flame (*above*) and Manometric Flame Compared.

From what has been said with regard to reed-pipes, we should expect to find vibrations in the gas-supply tube, and since the systems are coupled and must have the same frequency, we should expect the maintenance to be most perfect when stationary waves are formed in the gas. By fitting a manometric capsule to the *supply tube* just below the flame, one found that maximum compression occurred there just after the flame struck back to the jet, as if one had turned off

the flame by a tap and the pressure accumulated then in the gas main behind it. When the singing was most powerful, stationary waves could be traced in the gas tube by adjustable manometric flames, a node always being found near the jet and an antinode below at the end of the gas tube in the reservoir. Rayleigh suggested that the gas tube should be of length about $\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$, $\frac{5}{4}\lambda$, etc., for best maintenance, λ being the wave-length of the tone in the *gas*. When this adjustment is made, the violence of the flame's oscillation as it is pushed up the tube is sufficient to extinguish it before the central node is reached. When the adjustment of the length of the supply tube is less perfect (and considerable variations on each side of the "resonant" length are possible, in fact, as Taber Jones [58] points out, a flame will often sing without a supply tube of definite length if it is connected haphazardly to the gas main) the sound is more feeble, and the singing continues without extinguishing the flame, as it is moved from near the bottom to near the top end. When the supply tube has a length nearly equal to $\frac{1}{2}\lambda$, λ, $\frac{3}{2}\lambda$, etc., the phase of the flame is most inimical to maintenance, and no sound is heard.

The Rijke and Bosscha "Gauze Tones." In contradistinction to the purity of the above tones, there are other ways of maintaining the sounds of columns of air, in which vibration is so vigorous and therewith the overtones are so prominent that they have earned the well-merited name of "howling tubes." Besides using thermal means to increase the *pressure* amplitude in a stationary vibration we may employ the source of heat to create a temporary convection current, and so increase the *velocity* amplitude. Curiously enough no external means of rendering the heat-supply intermittent, corresponding to the gas-supply tube of the singing flame, is necessary with these heating effects. The first of these was discovered by Rijke,[59] who placed a piece of metal gauze in the lower half of a vertical tube (best about a quarter of the way up) and heated it red-hot by a flame. On withdrawing the flame the air in the tube sounded until the temperature of the air and the gauze were nearly equalized. This phenomenon was attributed by Rayleigh to a combination of loop and node effects. Under the action of the first cause, every upward movement of the air in vibration brings cold air on to the heated gauze, whereas the downward movements bring the already hot air back on to the gauze. Thus the greatest temperature difference occurs in the "up-

stroke," consequently the greater heat transfer and consequent compression occurs as the air goes up. When the gauze is in the lower half, this upward phase occurs just before the maximum compression (because the air is then moving towards the central node) and the vibration is assisted. When the gauze is in the upper half this phase precedes a rarefaction, and so the motion is damped. This hypothesis, which we have seen verified for the singing flame, explains the complementary phenomenon discovered by Bosscha,[60] that if a cold gauze be placed in the upper half of the tube, and a current of hot air be passed up the tube, a similar sound is produced.

It is possible to maintain the sounds of the air column at the antinode formed at the end of the tube itself, by means of a gauze and a gas supply, with the gas lit *above* the gauze. A Méker burner, being constructed on this principle, will act in the same way.

References

1. *Wiss. Abhand.*, **1**, 303.
2. *Phil. Trans. Roy. Soc.*, **161**, 77, 1870; *Phil. Mag.*, **3**, 456, 1877.
3. Stücker, *Akad. Wiss. Wien, Ber.*, **116**, 1231, 1907; *see also* Higgs and Tyte, *Phil. Mag.*, **4**, 1099, 1927; Bate, *Proc. Phys. Soc.*, **48**, 100, 1936; *Phil. Mag.*, **27**, 195, 1939; King, *ibid.*, **21**, 128, 1936; van den Dongen, *Rev. Acoust.*, **7**, 1, 1938.
4. *Ann. Physik*, **117**, 173, 1855. *See also* Mokhtar and Messih, *Proc. Phys. Soc.*, **62**, 793, 1949.
5. *Phys. Rev.*, **16**, 313, 1920.
6. *Ibid.*, **20**, 505, 1922.
7. *See* Hartig and Swanson, *Phys. Rev.*, **54**, 618, 1938; Hartig and Lambert, *J. Acoust. Soc.*, **22**, 42, 1950; Bogert, *ibid.*, 432; Shaw, *ibid.*, 512; Beatty, *ibid.*, 639, 850.
8. *Proc. Roy. Soc.*, *A*, **141**, 123, 1933.
9. *Phil. Trans. Roy. Soc.*, 133, 1860.
10. *Fortschr. Phys.*, **15**, 123, 1860.
11. *Sound*, **2**, 39, 1878.
12. *Ann. Physik*, **21**, 534, 1934.
13. *Proc. Math. Phys. Soc. Egypt*, **3**, 1, 1947.
14. *Phil. Mag.*, **32**, 206, 1941. *See also* Oberst, *Akust. Zeits.*, **5**, 27, 1940.
15. *Ann. Physik*, **14**, 469, 1904.
16. *Ibid.*, **2**, 719, 1900; **4**, 41, 1901; **21**, 781, 1906.
17. *Archiv. Néerl.*, **25**, 308, 1892.
18. *Ann. Physik*, **11**, 1086, 1903.
19. *J. Physique*, **6**, 52, 1925; **8**, 215, 1927.
20. Mercer, *J. Acoust. Soc.*, **23**, 65, 1951.
21. *Thesis, Berlin*, 1940.
22. *Akust. Zeits.*, **5**, 39, 1940.
23. *Proc. Roy. Dublin Soc.*, 132, 1888.
24. *Comptes Rendus*, **207**, 216, 1938; **209**, 508, 1939.
25. *Phil. Mag.*, **27**, 195, 1939.
26. *Akust. Zeits.*, **5**, 324, 1940.

27. *J. Acoust. Soc.*, **13**, 145 and 149, 1941.
28. *Ibid.*, **13**, 160, 1941.
29. *Ann. Physik*, **146**, 161, 1872.
30. *Phys. Rev.*, **33**, 442, 1911.
31. *Ibid.*, **1**, 166, 1893.
32. *Ann. Physik*, **42**, 549, 1891.
33. *Ibid.*, **128**, 337, 1866.
34. *Ibid.*, **36**, 273, 1882.
35. *Ibid.*, **141**, 321, 1870.
36. *Ibid.*, **50**, 193, 1891.
37. *Ibid.*, **34**, 131, 1910. *See also* JOHNSON, *Phys. Rev.*, **45**, 641, 1934.
38. *Ann. Physik*, **76**, 39, 1925.
39. *Phil. Mag.*, **45**, 925, 1923.
40. RICHARDSON, *Proc. Roy. Soc.*, *A*, **112**, 522, 1926.
41. *Phys. Rev.*, **6**, 491, 1915.
42. *Zeits. Phys.*, **3**, 330, 1920.
43. *J. Physique*, **9**, 187, 1928; **12**, 165, 1931.
44. *Phys. Soc. Discussion on Audition*, 79, 1931; ANDRADE and PARKER, *Proc. Roy Soc.*, *A*, **159**, 507, 1937. *See also* MEYER and KARMANN, *Acustica*, **1**, 130, 1951.
45. *Ann. Physik*, **49**, 648, 1916.
46. *Ibid.*, **48**, 693, 1915.
47. *J. Acoust. Soc.*, **14**, 228, 1943. *See also* McGINNIS and PEPPER, *ibid.*, **16**, 188, 1945.
48. *Ibid.*, **6**, 34, 1934.
49. *Ibid.*, **9**, 255, 1938.
50. HERMANN-GOLDAP, *Ann. Physik*, **23**, 979, 1907. *See also* LOTTERMOSER, *Akust. Zeits.*, **2**, 129, 1937; ASCHOFF, *Akust. Zeits.*, **1**, 77, 1936; YOUNG and LONGRIDGE, *J. Acoust. Soc.*, **7**, 178, 1936; MOKHTAR, *Durham Phil. Soc. Trans.*, **9**, 352, 1938; *Phil. Mag.*, **27**, 195, 1939; BONER, *J. Acoust. Soc.*, **10**, 32, 1938; KNAUSS and YEAGER, *ibid.*, **13**, 160, 1941; WOODWARD, *ibid.*, **13**, 160, 1941.
51. *Phil. Mag.*, **3**, 385, 1902.
52. *Akust. Zeits.*, **1**, 59, 1936.
53. *J. Acoust. Soc.*, **13**, 145 and 149, 1941.
54. *Thesis, Durham*, 1946.
55. *Ann. Physik*, **109**, 1 and 426, 1860.
56. *Sound*, **2**, 227, 1877.
57. RICHARDSON, *Proc. Phys. Soc.*, **35**, 47, 1922.
58. *J. Acoust. Soc.*, **16**, 254, 1945.
59. *Ann. Physik*, **107**, 339, 1859.
60. *Ibid.*, **127**, 166, 1866.

CHAPTER EIGHT

ANALYSIS OF SOUND

In the preceding chapters, grouped under appropriate headings, appear a large number of methods by which the vibrations of particular systems, wires, bars, columns of air, etc., may be studied. In general, the sounds from these bodies are conveyed to our ears by waves in the air. The open air—open, that is, in the sense that there are no partially enclosed spaces which can resonate—gives a faithful reproduction of the vibrations impressed upon it by the source, so that if we can devise an instrument which will faithfully analyse the sonorous air waves, we have a universal instrument for analysing the notes produced by any sound-making body whatever. The construction of such an instrument is by no means simple, and it will be best to consider some of the difficulties in the way, before describing those instruments themselves which analyse sound waves in air.

Requirements of a Sound Analyser. If the air waves are due to a sustained unvarying note, the analysis consists in finding the amplitudes and frequencies of the Fourier components which make up the given note. The instrument will register these by a movement of certain amplitude, e.g., of a spot of light on a scale. This movement is known as the " response " of the instrument. The instrument must then give *adequate and equal response* to tones over the whole audible range, provided the tones are of equal amplitude. It is not essential that the amplitude of the response should be always a constant factor times the amplitude in the air, as long as the law connecting them is known, and is independent of frequency. Some considerations which mitigate against this desideratum are :—

(1) The reproducer has a natural period or periods of its own and gives excessive response to tones of coincident period. This is known technically as " distortion."

(2) The average amplitude of the motion of the air is minute (about 10^{-7} cm.).

(3) On this account a horn is added to many instruments to obtain increased response. The horn adds more resonances.

Standard Sources. It is important, not only in connection with

sound analysis, to have reliable standards of sound. It is not generally necessary, and not even feasible until we have a reliable absolute measurer of sound, to have a complete series of tones covering the musical scale and producing sound of intensity expressed as so many units. Miller's approximation to a series of tones of equal loudness, consisted of a " stop " of sixty-one organ pipes, voiced by the builder so as to give equal loudness, as judged by a skilled ear. He was thus reduced to a subjective standard in calibrating his instrument, and that standard one of uncertain constancy. For most purposes it is sufficient for an experimenter to have a source whose frequency and intensity can be relied upon not to vary from day to day, and on whose constancy he can base his results ; but it is a standard only for the experimenter, it is in no sense a universal standard. Such is the tuning fork, driven by a constant current, and maintained at constant amplitude, with a small correction for temperature. The reliability of the tuning-fork has led to its constant use as a fixed frequency standard.

When a source of greater intensity is required the " siren " may be used. This instrument, designed by Cagniard de la Tour,[1] consists in principle of a wheel rotating at constant speed, having a number of equidistant holes which pass over the orifice of a tube leading from a reservoir containing air under pressure, and which release a number of puffs of air as the orifice is opened and closed. If these follow each other sufficiently rapidly a tone is produced whose frequency can be calculated if the speed of rotation of the disc is known.

Frequency Standardization. In order to find the exact number of vibrations per second made by a standard, it is necessary to compare the standard with a period which can be accurately measured. This is best a pendulum, and in Rayleigh's method [2] a standard fork is made to drive a phonic wheel (p. 112) carrying a stroboscopic disc, through the slits in which the oscillations of the pendulum can be observed. The number of poles on the phonic wheel and the number of slits on the disc are so adjusted that the interval between successive glimpses is about one-eighth of the period of the pendulum. The operation then proceeds by the method of coincidences, i.e., the time is measured which elapses between successive appearances of the pendulum at any given phase of its swing, viewed through the slits. The clock by which this time, and the period of the pendulum, have been determined must be compared with a chronometer or other reliable time source. König [3] made the tuning-fork control the escapement of a clock, which could be compared with a standard time source.

Commercial frequency meters are available. These usually consist of a telephone excited by a thermionic valve circuit set in oscillation at the frequency of a resonant circuit to which it is coupled and in which the capacity of a variable condenser alters the frequency. The condenser dial is marked directly in cycles per sec. In a " beat-frequency oscillator " two such circuits whose natural frequencies are above the audible pitch limits when separated are so coupled that the difference tone between them produces an audible note in the telephone. One circuit may be fixed while the other is varied to alter the combination tone.

Phonautograph or Phonograph. The first instrument for recording sound waves was Scott's Phonautograph, 1859.[4] The membrane was stretched over the narrow end of an ellipsoidal horn and its movements were recorded through the medium of a lever bent at right angles, one end resting on the membrane, and the other carrying a pencil which drew the wave-form on a drum rotated by hand, the records being obtained in the same way as those from a tuning-fork (cf. p. 77). Edison replaced the lever by a stylus attached to the membrane in such a way that the former pressed with greater or less force against the drum as the membrane vibrated. He found that, when the drum was covered with wax or similar soft material the sounds impinging on the membrane could be reproduced after the " record " had been taken, if the style was run a second time over the minute crests and troughs formed in the wax by the original sound waves. Edison's phonograph was at first intended as a scientific instrument for recording and reproducing sounds, but the modern application of the instrument to music and phonetics is familiar to all.

Its development as a reproducer has run along lines leading to increased but more uniform response over the entire musical scale, by horns of peculiar shape, and by multiple resonances in the housing of the diaphragm, which tend to equalize the response over the whole musical range by overlapping resonance peaks (cf. p. 54).

The Phonodeik. The receiver of this instrument by Miller [5] is a diaphragm of thin glass (000·3 in.) at the end of a resonator horn. A silk fibre or a very thin platinum wire leads from the centre of the diaphragm over a pulley on the end of a minute spindle to a light spring. On the spindle is a mirror, which the movement of the diaphragm causes to rotate. The movement of the mirror deflects a beam of light reflected on to a moving sensitized film (Fig. 74).

The spot of light thus traces a record of the sound wave on the

film, magnifying the motion of the diaphragm 2,500 times, and giving a record $2\frac{1}{2}$ in. wide. A zero line to serve as axis of the curve is also drawn, and time flashes are given from a tuning fork.

An ideal instrument would give the same response to sounds of the same intensity over the whole of the musical scale. As a matter of fact, the diaphragm, and especially the horn, have natural frequencies of their own, and when a sound corresponding to one of these falls upon the instrument, the amplitude of this component is magnified by the resonance of the instrument. It was found impossible to eliminate these; the natural period of the diaphragm can be made high, but not that of the horn, and a horn is indispensable if sounds of the normal intensity in the air are to be recorded. Miller therefore set out to reduce the trace of a sound wave to absolute proportions by first calibrating the instrument with tones of constant loudness over the whole range of frequency. Then when the analysis of the trace has been accomplished, each amplitude is reduced as demanded by

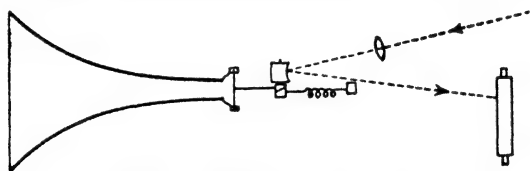



Fig. 74.—The Phonodeik (*Miller*).

this calibration curve. The coefficients of the requisite Fourier series are determined by operating on a magnified trace of the corrected curve by mechanical analysers.

There are available other methods for recording the movement of the diaphragm of such a recording instrument. A telephone receiver may be used and the fluctuations of current caused by the vibration measured on a galvanometer sufficiently damped to respond to rapid current changes or led to a cathode ray oscillograph.[6] In place of the oscillograph, the current may be rectified by a crystal detector, and an ordinary galvanometer employed.[7]

The Cathode Ray Oscillograph. The cathode ray tube (Fig. 75) consists of a vacuum tube one end of which is cylindrical while the other is broadened out to hold a fluorescent screen. At the narrow end a heated filament and associated shields direct a beam of electrons axially along the tube to fall on the screen and there produce a bright spot. Two pairs of plates to which electrostatic fields may be applied let into the side of the tube allow the beam to be deflected either hori-

zontally or vertically. To use this apparatus as an oscillograph one pair of plates is connected to the alternating field (suitably amplified) produced by a microphone. If this field oscillates the spot in a vertical plane we can pass a sensitized film in a horizontal direction in front of the screen, and get a record of the oscillatory current from the microphone. Alternatively by means of a " sweep circuit " connected to the other pair of plates the spot may at the same time be swept slowly from left to right horizontally with a jump back to the start when it has reached the edge of the screen and again the slow sweep to the right, *ad inf.* This apparently complicated motion is simply achieved by the slow charge and sudden discharge of a condenser connected to a source of e.m.f. and suitable resistance. If the period of this relaxation oscillation coincides with the fundamental period in the sound to be recorded, its wave form will be held stationary on the viewing screen as if drawn on paper.

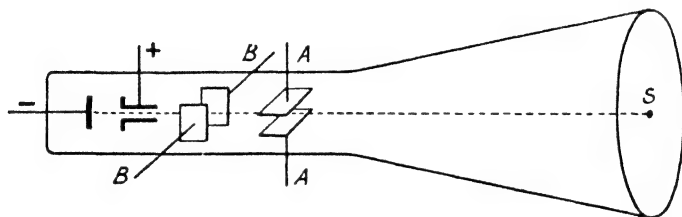



FIG. 75.—The Cathode Ray Oscillograph.

The cathode ray oscillograph in conjunction with a condenser microphone and a distortionless amplifier is probably the most accurate sound-recording device known. Since the recording is done directly by an electron beam there is no lag or distortion due to heavy or damped moving parts such as other oscillographs show. It is particularly useful in recording speech sounds where high overtones are important. Curry [8] has been able to photograph the motion of the fluorescent spot on a film moving at 6 ft. per sec., and the upper limit to the frequency which can be recorded seems to be set by the response of the microphone rather than by the oscillograph itself.

This instrument, in combination with a loud-speaker and a microphone, is well adapted to the measurement of the velocity of sound in air. One method, originally suggested by Wold and Stephenson [9] and recently revived by Patchett [10] and Knowles,[11] can be regarded

as a modern version of Hebb's method (p. 7). The loud-speaker is supplied by a pure-tone valve oscillator some of whose output is fed to one pair of the oscillograph plates. The sound picked up by a microphone at some distance is converted into a potential difference, suitably amplified and applied to the other pair of plates so that the potential differences on the two pairs of plates have about the same amplitude. Since the potential difference from the microphone lags behind that applied to the loud-speaker, the electron spot traces out a Lissajous figure on the screen. From its shape the phase lag is determined (cf. p. 42), and so the time taken for the sound to travel from source to receiver.

The apparatus used by Colwell, Friend and McGraw [12] uses a double beam oscillograph on which each spot is swept synchronously from left to right by means of the pair of plates on the horizontal axis of the tube. There are two pairs of plates on the vertical axis, one for each electron beam, by means of which (with the aid of the sweep circuit) two wave forms may be compared. A glow-discharge valve working in a simple circuit off the alternating current mains causes a series of short pulses of sound at the mains frequency to be emitted by the loud-speaker, part of this potential difference being applied to one electron beam. The other beam receives its potential difference from the sound picked up by the microphone. Both beams are swept horizontally at the same (mains) frequency, so that each wave-form appears as a series of stationary vertical jags in a horizontal straight line, but with one set displaced with reference to the other owing to the phase lag (Fig. 76). The microphone is put at such a distance from the source that the two sets of jags lie over each other; then it is moved away until the two sets are again in phase. This shift of position is then equal to the wave-length of the sound having a frequency equal to the mains, which in Great Britain is usually

Fig. 76.—Emission and Reflection of Pulses (Cathode Ray Screen).

50 cycles/sec. The experiment occupies a rather large laboratory, if done indoors, but the space required can be reduced at the loss of a little accuracy if the emitting circuit is arranged to give pulses at a multiple of the mains frequency.

Band Analysers. The principle of these instruments is that the sound amplified and transformed into an electric signal is passed through a series of filters. The reed analyser of Hartmann-Kempf (p. 110) was the precursor of this device. The width of band of each filter determines the amount of detail in the analysis and also the time taken therein, these two factors being inversely proportional. For the analysis of noise each band often covers an octave but it is better to make the band-width proportional to the mean frequency in each band. This is because in many instances the power diminishes in the upper part of the gamut, and makes such an arrangement more representative. Alternative sub-division is possible having regard to the subjective estimation of pitch (p. 280).

Heterodyne Analysers. This apparatus consists of a variable frequency oscillator whose output is modulated by the incoming electric signal in such a way that summation as well as difference tones are involved in the resultant (p. 60). The summation tone is picked out by a filter and the amplitude of its rectified response shown on a thermionic voltmeter or on an oscillograph screen. The filter often takes the form of a piezo-electric quartz crystal or a nickel rod in magneto-strictive oscillation which will respond when the variable oscillator is tuned to a frequency which, added to the component to be detected, produces the natural frequency of the crystal or rod. Care has to be taken that the amplitudes of these components are not such that combination tones other than the simple sum and difference are involved. Normally the dial of the oscillator is rotated by hand, but for rapid analysis an electric motor may do this in synchronism with the sweep of the oscillograph electron beam.

In analysers giving permanent records mechanical writing on paper may take the place of photographic recording with a cathode ray oscillograph. In the Neumann recorder apparatus a pen is moved in steps proportional to the input over a piece of paper, which is moving at constant speed in a perpendicular direction. The pen, or the spotlight which may replace it when sensitized paper is used, may be made to record either loudness or pitch.

Optical Analysers. The basis of this type is an apparatus due to Dietsch and Fricke. It requires that the wave form be made avail-

able as a black on white (variable *area*) type of film (3 or 4 in. wide), and of such a length that it can be wrapped round a glass cylinder with exact overlapping after one or several complete cycles. Along the axis of the cylinder (*C*, Fig. 77) lies the filament of the special lamp *L*. By means of the constant-speed motor, *E*, the glass cylinder is rotated, and as this happens the lamp shining past the film and the slit *D* casts an image of a narrow section of the film on the photo-electric cell, *P*. The wave form on the film is thus converted back into synchronous electric waves in the circuit containing the photo-electric cell and the analyser.

After the current leaves the amplifier it may, of course, be immediately analysed in the wave analyser aforementioned, but the feature

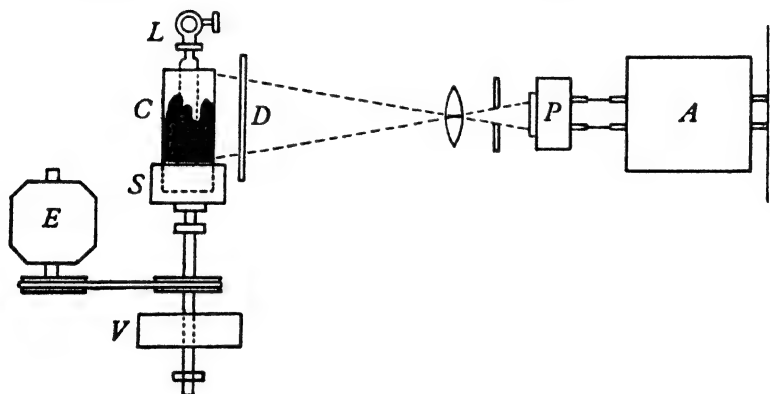



Fig. 77.—Optical Analysis of Sound Film (*Richardson*).

of Dietsch and Fricke's [13] device which is of particular value for inharmonic components was a vibrating reed to the electro-magnet of which this current was fed. This electrical wave forms the input to a coil mounted over a vibrating reed. Varying the speed of rotation of the cylinder causes the reed to respond every time a component in the photo-electric current has the same frequency as that natural to the reed.

In the author's [14] variant of this apparatus, the reed is replaced by a vibration galvanometer of 50 c./sec. A considerable length of film (covering many periods) is wrapped round a narrow perspex drum which can be rotated slowly through a geared electric motor. The speed of the drum at any instant can be recorded stroboscopically. If the drum is now revolved with increasing speed the galvanometer

will respond (at an amplitude which can be made a linear function of the photo-electric current amplitude) whenever the number of the peaks of a particular component pass the slit at 50 c./sec. Acoustic gratings like those referred to on p. 15 have also been used as analysers.

Analysis of Unsteady Sound. An inharmonic unsteady sound such as some noises comprise defies Fourier analysis, but there are two types of sound found in technical acoustics which can be analysed.

The first of these is the pulse. Inasmuch as a pulse resolves on analysis into an infinite Fourier series, analysis by machine seems impossible, but in fact the component intensities decrease as the frequency mounts, so that in practice a limited number of harmonics suffices. This is for a square wave. In echo-sounding (p. 325) we encounter another type of pulse whose envelope approximates to a castellated wave but whose fine structure is in fact a simple tone. Such, provided it lasts several periods, presents no difficulty.

The type of transient often encountered in the stopping and starting sounds of musical instruments is a modified copy of the full quality, perhaps with one or two components of the steady timbre missing, perhaps with additions. This also the optical analyser can deal with step by step, if specimen portions of the film are selected and analysed during the transition.

Resonators. The air columns contained in organ pipes which we have already described can serve as examples of resonators, but in analysis something possessing sharper resonance is required. Helmholtz [15] found that vessels having an internal capacity large compared with the orifice or neck by which communication with the atmosphere was made were more selective as resonators, the criterion being that the condensation in the main reservoir should be practically uniform, and the to-and-fro motion of the air in vibration should be significant only at the neck. The theory will be found on p. 223.

Helmholtz resonators of the cylindrical type can be adjusted in volume and therefore in frequency by arranging a part to slide over the other on the principle of the trombone. The law of variation of frequency with volume—n^2v = a constant—was found experimentally much earlier by Sondhauss.[16]

Before describing the use of resonators, it is necessary to emphasize the fundamental difference between the theoretical "pipe" and the "resonator." The motion in the former is a case of stationary waves, its longest dimension is comparable with the wave-length, and the

displacemnet along the interior rises and falls continuously. The dimensions of the latter are small compared with the wave-length, and the orifice or neck is so narrow that the motion may be considered as confined to its neighbourhood. In practice, the bodies of air we deal with may form a compromise between these two aspects.

Bate [17] describes a simple apparatus for the determination of the velocity of sound at low temperatures, consisting of two cylindrical Helmholtz resonators with necks. The cylinders have the same diameter but differ in length by 8·4 cm. The necks are identical, but can be altered in length by equal amounts. It follows that when the two resonators separately excited by an air-jet have the same frequency, the half wave-length of the vibrations in the two resonators is 8·4 cm., the larger one eliciting its first overtone. Hence the velocity of sound for this wave-length can be calculated. The resonators are put in a low-temperature enclosure. It does not matter if the necks jut out, as long as the gradient of temperature along each is the same.

Hot-wire Microphone. This instrument devised and developed by Tucker and Paris [18] consists of a Helmholtz resonator with neck, with a "grid" formed of a 0·0006 cm. diameter platinum wire bent into the shape shown (Fig. 78*b*), and placed at the entry of the neck into the main body of the resonator. Connection is made from the

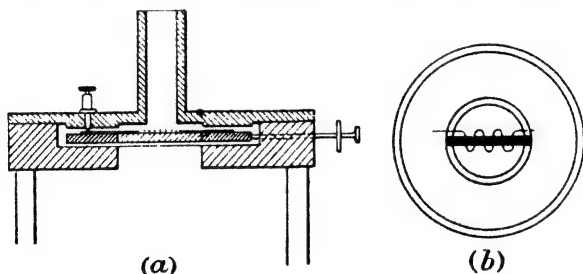



Fig. 78.—Hot-Wire Microphone (*Tucker*).

two ends of this coil of wire to two annular discs of silver foil, one above and one below a piece of mica, which forms the frame of the grid.

The frame is clamped into the "collar" of the resonator (Fig. 78*a*), and a current passed by the terminals through the platinum wire until it is just below red-heat. Under these conditions a very slight motion of the air through the neck is sufficient to change the resistance by a measurable amount. The change in resistance in the alternating draught which takes place in the neck when the resonator

is responding to a tone, partakes both of a steady drop and of a complex periodic change containing the fundamental and harmonics of the frequency of the air motion. It is possible to measure the amplitude of the response, either by means of a vibration galvanometer tuned to the fundamental of the resonator, or else by measuring the average resistance of the hot wire during the motion, by a simple Wheatstone Bridge method. The resistance so measured is the same as would result from a steady draught of velocity equal to the maximum velocity ($2\pi na$) of the S.H.M. in the neck (cf. p. 187). So that if the resistance-velocity curve of the grid in a steady draught has been obtained, this can be used as a calibration curve to obtain from the measured resistance the amplitude of the motion in the neck, which amplitude indicates the extent of the response of the resonator.

For small velocities it is found that the steady drop in resistance is proportional to the square of the air velocity, and therefore to the energy of vibration in the neck of the resonator, or to the intensity of the sound therein. The grid is more sensitive, i.e., shows the greater resistance change to a given amplitude of air motion, as the heating current is increased. Normally, the grid is kept just below red heat. Apart from its use in analysis, the instrument, as its name indicates, serves as a sensitive detector for sounds of the frequency of the resonator.

Paris [19] has adapted the same principle to the double resonator, consisting of two Helmholtz resonators joined by a neck in which the heated grid is placed, protected from casual draughts. The vibrations of the two component resonators are "coupled," and as the analysis of page 53 showed, the two tones to which the system responds are not those due to each resonator acting alone, but lie outside these frequencies, the smaller vessel having its frequency raised, the larger lowered by the presence of the other. [20] Paris placed the grid in a Wheatstone Bridge, the balance of which was disturbed when the double resonator responded to sound, the loss of balance causing a deflection of the galvanometer. Using a siren of an output as far as possible constant, he calibrated the instrument over a range of frequencies, using the galvanometer deflection as an indication of the response of the resonator. A typical curve is shown (Fig. 79).

If the frequencies of the component resonators are equal, the sensitivity of the double resonator can be made much greater than that of the single form, by making the inner resonator of small volume compared with the outer. If the conductivities of the two necks be

denoted by l_1, l_2 the volumes of the resonators by v_1, v_2 :—since $n_1 = n_2$, therefore $\frac{l_1}{l_2} = \frac{v_1}{v_2}$. Other things being equal, the rate of flow will be greater through the neck having the smaller conductivity, and this, according to the above reasoning, will be at the entrance to the smaller vessel. Thus, by making the inner vessel smaller, we get a greater draught in the neck separating the vessels, than in that opening from the atmosphere, and therefore greater sensitivity of the grid in this position.

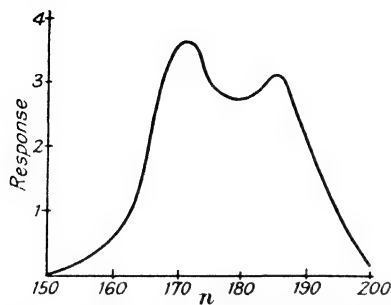



Fig.—79. Response of Double Resonator (*Paris*).

The double hot-wire microphone has recently been used by Tucker [21] to determine the velocity of sound in gases. Sound is introduced by a telephone diaphragm forming the base of one of the cylindrical enclosures, the base of the other cylinder being also closed, and both temperature and the acoustic response of the resonator are indicated by the change in electrical resistance of the wire in the neck used in the cold and hot state respectively. Since the resonant frequency of the system is proportional to the velocity of sound in the gas within it, it is possible to compare the velocities in various gases with that in dry air and at the time of the experiment to keep them in a sealed enclosure. The frequency scale of the source—a maintained oscillator —was checked against a series of standard tuning-forks. Comparative values of velocity over the range of temperature 19° C. to 104° C. were obtained in dry air and also in carbon dioxide and hydrogen at room temperature. Then water vapour at various temperatures was introduced into the air and so the value of γ at various temperatures was derived. Finally, some measurements were made in mixtures of the vapours of ether and acetone with air. It is pointed out that, as compared with tube methods for the velocity of sound, the double Helmholtz resonator is compact enough to enable the use of small quantities of gas and to be readily capable of temperature control in a small enclosure.

The damping in a resonator depends partly on the size of the neck since, in any form of construction, a narrow neck impedes the air motion, and partly on the material of which the walls of the reservoir

are made. If the walls are lined with absorbent material the resonance peaks are broader. From the shape of the peak, the damping coefficient of the resonator and hence the absorbent properties of such a lining can be deduced. This forms another use for the ubiquitous hot-wire microphone.[22]

Rayleigh Disc and Phonometer. Rayleigh [23] observed that a light disc suspended in a sounding cylindrical resonator tended to set itself across the tube, i.e., at right angles to the direction of the alternating air current, and further that, if the disc was suspended by a torsion thread so as to lie at an angle to the opening when the resonator was unexcited, the extent of the turning towards the square-on position was proportional to the energy in the motion. Rayleigh's

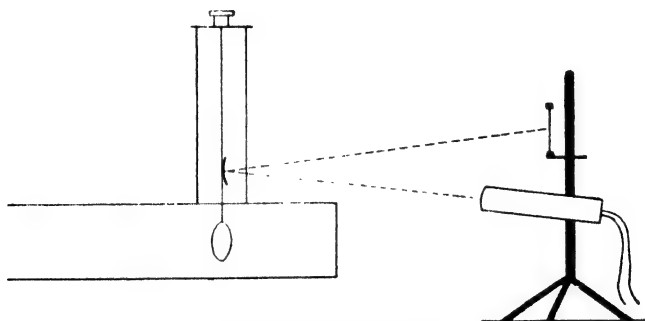



Fig. 80.—Rayleigh Disc.

original disc was a small galvanometer mirror with attached magnets suspended by a fibre in a magnetic field, so that the deflection could be registered by means of a beam of light reflected from the disc itself through the side of a resonator made of glass. The instrument is very sensitive, and has been used in a number of researches. A common form of the instrument is shown in Fig. 80.

The disc is of mica 1 cm. in radius, suspended by a quartz fibre 10 cm. long and lies, when undeflected, at an angle of 45° to the axis of the tube resonator (in the form of a glass tube 2·5 cm. diameter) in the centre of which it hangs.[24] It is found best to have the radius of the disc about half that of the tube. To indicate the movement of the disc, a mirror as used on galvanometers is attached to the fibre outside the resonator. The hole by which the fibre enters the latter must, of course, be as small as possible consistent with free movement of the fibre.

König[25] showed that the turning moment on the disc, M, is proportional to the mean square of the velocity in the alternating air current, and this again (at constant frequency) is proportional to the intensity at the point in question. If C is the constant of the fibre, i.e., the torque per unit twist, an angular deflection ϕ will be produced such that $C\phi = M$.

When ϕ is small enough for the movement of the spot of light on the scale to be taken as its measure, the deflection on the scale will be proportional to $\overline{V^2}$, i.e., to the intensity. The instrument is suited only to the measurement of intensities of sounds of constant frequency and quality, and should be employed at resonance for greater sensitivity.

The formula for the moment on the disc as developed by König is :—

$$M = \frac{4}{3}\rho r^3 \overline{V^2} \sin 2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

r being the radius, ρ the density of the air, and θ the angle of repose of the disc relative to the stream, and is a maximum when θ equals 45°. The formula has been experimentally verified by Zernov,[26] who moved a box (surrounding the disc) and the air therein on a tuning fork, thus producing known velocity amplitudes. It follows from his work that it is possible to use the disc to measure the absolute velocity or displacement amplitude of the S.H.M. of the air surrounding it, if the constants of the disc and fibre are known.

In the simple theory of the Rayleigh and Altberg discs, two by no means negligible factors were ignored, viz., (1) the diffraction of the radiation by a small disc and its effect on the sound field, (2) the fact that the disc tends to swing with the field. The necessary corrections have been determined theoretically by King,[27] while Wood [28] has independently examined both theoretically and by experiment the magnitude of the second factor. The formula for the moment on the disc (67) must be multiplied by the inertia factor $(1-\beta)^2$ where β is the ratio of the velocity amplitude taken up by the disc to that of the air ($\bar{V}$) which excites it into vibration. Thus the relative velocity of fluid and disc is $\bar{V}(1-\beta)$, and this term replaces $\bar{V}$ in (67). The term $\frac{4}{3}\rho r^3$ is the equivalent fluid load on a thin disc set at 45° to the stream. If M is the mass of the disc, m that of the fluid displaced by the disc (in the Archimedean sense), the ratio of velocities

$$\beta = \frac{m + \frac{4}{3}\rho r^3}{M + \frac{4}{3}\rho r^3}.$$

In the experiments, Wood used a family of discs of the same radius but different mass in water at a point in the sound field of unvarying intensity, and verified the applicability of the correcting term. The possibility of flexural vibrations is also considered, but it is shown that a disc 2 cm. in diameter and 2 mm. thick will be unaffected by resonance in the audible region.

There are still three further assumptions in König's theory, viz., (1) that the fluid is incompressible, (2) that viscous forces are negligible and (3) that the fluid does not " spill " over the edges of the disc, which would, in fact, be a consequence of viscosity in the fluid. The first will not matter except when the wave-length of sound is comparable with the size of the disc, but Merrington and Oatley [29] have investigated the " viscous effect " experimentally. After making measurements of the torque on a small disc in air, repeating carefully the work of Zernov (*vide supra*), they test their results against König's formula, with the King correction, and find discrepancies up to ten per cent., which they attribute to the second and third factors. One cause of this was, in fact, found to lie in the departure of the aerial motion from the streamline form which it would take up in the absence of viscosity and inertia. When smoke was let into the sounding resonator, vortices rather like those in Æolian tones (p. 147) were discovered at either edge of the disc, rotating in opposite directions.

The critical velocity of the departure from streamline flow could, however, not be observed, since the vortices were present down to the lowest frequency of vibration that could be attained. They suggest that, over the range 10 to 50 cycles/sec. the factor $\frac{4}{3}$ which occurs in the numerator and denominator of King's equation for β (*above*) be changed to an empirical factor 1·47 for better agreement with experiment, though it is possible that this factor itself may vary with amplitude and frequency (i.e., with the intensity) at higher frequencies than those they used.

Striæ in Kundt's Tube. König [30] found that a suspended system consisting of two equal spheres showed a similar tendency to set itself with the line joining the spheres normal to the stream. This behaviour led him to suggest an explanation of the formation of striæ in Kundt's dust figures, wherein the little spheres of dust are found in rows across the tube, i.e., normal to the stationary vibration. He found, for example, that two small spheres of cork suspended by a fibre in such a tube at an antinode of stationary vibrations adhere together if they happen to lie with the line joining their centres across the tube, but

repel each other if turned so as to lie along the tube. By similar methods to those employed for the theory of the Rayleigh disc König showed that the force of repulsion along the axis in such a case was given by :—

$$F = \frac{3}{2}\frac{\pi\rho r^6}{l^4}\overline{V}^2 \cos\theta(3 - 5\cos^2\theta) \quad . \quad . \quad . \quad . \quad (68)$$

l being the distance between the spheres making an angle θ with the axis of the tube, and the other symbols having the same meaning as before (cf. 67). The particles ultimately reach equilibrium when the force is zero, i.e., they pile themselves in rows across the tube. The formula has been verified for a pair by Thomas,[31] using a device similar to that of Zernov. The force of repulsion which encourages this arrangement depends on $\overline{V}$ which is the mean velocity in an alternating current and therefore decreases in passing from an antinode to a node, as Robinson [32] pointed out, with the consequence that the striation gets less pronounced, and the successive striæ nearer together towards the node. Also the force tending to form the striæ, in accordance with (68) will increase with the intensity of the sound, since this produces an overall increase in $\overline{V}$. It may be noted in passing that an electric spark discharge at the mouth of the tube will fashion dust figures similar to those of the more usual rubbed rod. These are generally ascribed to alternations of definite frequency in the spark discharge.

Recently, investigators have used as source a telephone diaphragm driven by a valve oscillator (p. 134) in place of the rod, enabling much greater power to be used. Some observations imperfectly noted by Dvorak [33] have been clarified by Andrade.[34] There is first a tendency of the dust to form a disc at each antinode which is a distinct feature with the maintained oscillation, but not usually observed under a sporadic rubbing of a rod. Of still greater interest are the general circulations of particles from node to antinode, of which Rayleigh [35] had given a theoretical treatment, but which were first demonstrated in an experiment by Andrade. Theory indicates currents of the type shown in Fig. 81*b* (bottom left) running along the wall from loop to node, in to centre, and back along the axis of the tube. Andrade has published some very beautiful photographs both of these circulations and of the antinodal discs. The upper drawing (*a*) shows two separate half circulations, that on the left a prediction by Rayleigh, that on the right traced from such photographs, the clarity of which is in the main due to freedom from harmonics in the oscillator which drove the tube and careful water-jacketing of the tube to minimize convection

currents. The circulations were best set in evidence by tobacco smoke. If an obstacle such as a ball bearing was placed on the axis of the tube, secondary oscillations were set up in its vicinity like large vortices (Fig. 81*c*, bottom right). Andrade also explains the force which sets individual particles into rows across the tube (*vide supra*) in terms of these vortices. He observed that when two particles were close together their individual vortex systems coalesced and that while this was happening they set themselves into line perpendicular to the axis of aerial vibration, with a combined vortex system around the pair like that shown for the single one in the figure. This work suggests that (68) does not truly apply to striæ formation.

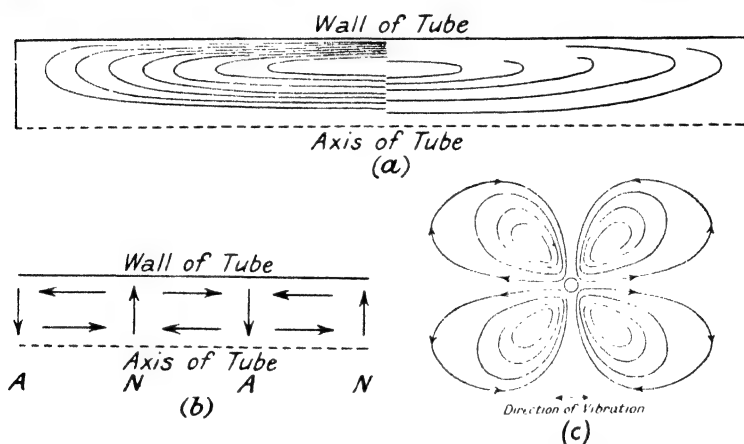



Fig. 81.—Circulations in Kundt's Tube (*Andrade*).

Robinson and Stephens [36] have discovered an interesting variant on the dust tube. In place of particles, they introduce a series of soap films, formed across the tube at intervals. Those near an antinode burst, while the remainder settle in the nodes when the source is set in vibration.

" Second Viscosity." The coefficient of viscosity which enters into all equations of damped vibrations in a fluid relates to shear vibrations only. Stokes [37] a century ago pointed out that it might be necessary to consider the possibility of energy being expended in dilating the fluid during the passage of the sound wave. At the time he decided to ignore such additional damping in an ordinary fluid but recently Eckart [38] has postulated the existence of this dilatational

or "second" viscosity in certain liquids at sufficiently high frequency. He considers that this existence might be proved if one examined the flow in a liquid in front of a piezo-electric quartz oscillator in the form of a disc, emitting high-frequency waves into a liquid. Liebermann [39] has carried out the experiment and demonstrated the "streaming" of a liquid containing aluminium flakes in front of a quartz disc in a tube; the flow forms two circulations, exactly like one half of Andrade's photograph (Fig. 81c) and Liebermann opines that mensuration of these velocities enables him to deduce the value of the "second viscosity" for this liquid. In view, however, of the fact that one can obtain Andrade's vortices in a gas at frequencies of only 2 or 3 per sec., one must hold that the existence of "second viscosity" at high frequencies is not yet proven.

Radiation Pressure. Three attempts have been made to produce absolute instruments. Raps [40] utilized the optical interference method previously described (see p. 185). One of the interfering beams of light passed through the atmosphere in the neighbourhood of an intense sound source, the other passed through a closed vessel. Measurable oscillations of the bands were obtained, but he does not state what precautions were taken to ensure that the sound did not pass through the walls of the control vessel.

Altberg [41] measured the pressure due to the sound. Dvorak [42] found that when sound waves in air impinged upon a solid wall they exerted a steady pressure upon it. Such a pressure is common to all forms of radiation, and is to be distinguished from the oscillatory change of pressure observed at a node of stationary waves. Consider a train of plane waves incident normally in a column of unit cross-section on a wall with velocity c, the sound energy density being Γ (ergs per c.c.), and the wall moved towards the incident sound with velocity v (cm./sec.). In one second $c + v$ waves hit the wall, but in the reflected wave-train these are compressed into a length $c - v$. Consequently, the energy density in the reflected train is greater by an amount $\delta\Gamma$ where

$$1 + \frac{\delta\Gamma}{\Gamma} = \frac{c+v}{c-v} = 1 + \frac{2v}{c}, \text{ approximately.}$$

The total increase of energy in a column of length c is $c\delta\Gamma = 2v\Gamma$, and this must equal the work done by the wall in compressing the waves, i.e., Pv; so $P = 2\Gamma$.

Lebedew [43] has extended his researches in light-pressure to this

discovery of Altberg, and has verified the law which ascribes this pressure to the sound energy in unit volume arriving at the wall per second, i.e., to $\frac{I}{c}$, or $\frac{2I}{c}$ when the waves reflected from the wall are also counted, I being the energy per second per unit area, and c the velocity of sound. On account of this radiation pressure a resonator experiences a repulsive force away from a tuning fork or source of sound placed at its mouth, owing to the resultant pressure which acts at the rear wall of the resonator. But to return to Altberg's application of this principle to sound intensity measurements. A hole is bored in the wall on which the sound impinges, and is nearly closed by a loose plug which is suspended from one arm of a very delicate torsion balance, serving to measure the force on the plug in the presence of the source of sound. The constants of the torsion fibre having been determined, it is possible to measure the force on the plug by the deflection of the torsion balance, as exhibited by a spot of light. To get a readable deflection intense sources of sound were necessary. Altberg used the longitudinal vibrations of a glass rod excited by mechanical rubbing; the sound was unendurable unless the ears were stopped. The average pressure on the plug was estimated to be 0·24 dynes per sq. cm., whence $I = 4{,}100$ ergs. per sec. per sq. cm. This is the energy that crosses unit area per second and is known as the strength of the sound (per unit area). For simple harmonic waves the kinetic energy in unit volume at any instant, $\Gamma = \frac{1}{2}\rho\left[\frac{d\xi}{dt}\right]^2$, $\frac{d\xi}{dt}$ being the velocity in the vibration. Integrating over the complete period, we find the energy in unit volume given by $\frac{1}{2}\rho(2\pi na)^2$. Hence the energy crossing unit area per second is :—

$$\Gamma c = I = \tfrac{1}{2}\rho(2\pi na)^2 c = \tfrac{1}{2}\rho\left[\frac{d\xi}{dt}\right]^2_{max} c \quad . \quad . \quad . \quad . \quad . \quad (69)$$

so that the pressure measurement gives a value of a if n be known.

Barus [44] employs a resonator with capillary neck, which he calls a pin-hole resonator, whose response he measures on this principle. A tube leads from the body of the resonator to a mercury surface. This surface is depressed a minute distance when the resonator sounds, requiring optical interference methods for the determination of the change of level of the surface due to the pressure on it.

The Null Method of Absolute Measurement. Resonant action of the diaphragm detracts from the reliability of measurements of

sound intensity made by an instrument involving such a system. In order to avoid this trouble Gerlach [45] has adopted the ingenious idea of compensating the forces on the diaphragm due to the impinging sound waves, by measurable electrodynamic forces, so that the diaphragm remains at rest under the joint action of the opposing mechanical and electrical forces. The value of the latter forces, in mechanical units, gives the value of the fluid forces acting on the diaphragm, and hence the intensity of the impinging sound. Since the membrane is prevented from vibrating, it makes no difference whether the frequency is equal to, or far removed from, the natural frequency of the system, indeed, the inventor claims greater reliability for the instrument at resonance, since any slight out-of-balance between the opposing forces will be rapidly magnified by resonance into a large vibration, making the instrument very sensitive to this frequency. The principle on which the compensation is accomplished is to pass an alternating current of the same frequency across the diaphragm which is placed in a magnetic field. Both the frequency and the phase of the current must be adjusted until complete compensation is reached, and no movement of the diaphragm can be detected. This adjustment is made on the alternator which gives the current, and a listening tube behind the diaphragm leading to the ears in stethoscopic fashion serves to test the condition of compensation by the absence of sound from the membrane; the original sound waves must of course be prevented from reaching the hearing system. At present the apparatus is confined to S.H. sound waves and S.H. alternating currents.

The diaphragm consists of a thin rectangular band of aluminium held by two clamps at its ends, serving as terminals for the A.C. A field across and in the plane of the diaphragm is produced by a permanent magnet. The sound waves are allowed to affect only the central portion of the band where the magnetic field is uniform, the rest being kept fixed by rubber clamps. The force per unit area of the exposed part of the diaphragm, which balances that due to the sound, is $\frac{Bi}{b}$ (in c.g.s. units) where B is the induction in the aluminium due to the magnet, i the instantaneous current strength and b the width of the conducting band, assuming that the lines of magnetic force are perpendicular to the current lines. As long as the strength of the magnet is maintained, B and b are constants for the instrument which can be determined once for all; then the "root mean square"

value of the compensating A.C. as given by an ammeter determines the mean pressure per unit area on the membrane, according to the above formula, which therefore gives the mean pressure due to the sound waves. The apparatus is used by the Siemens firm for testing the acoustical output of telephone transmitters and loud-speakers; but unfortunately no figures are yet available to demonstrate the sensitivity or facility of working of this instrument, so that the method must be regarded as in the tentative stage.

References

1. *Ann. Chim. Phys.*, **12**, 167, 1819.
2. *Phil. Trans. Roy Soc.*, **174**, 316, 1883.
3. *Ann. Physik*, **9**, 394, 1880.
4. *Cosmos*, **14**, 314, 1859.
5. *Engineering*, **94**, 550, 1912. *See also* Anderson, *Proc. Opt. Soc. Amer.*, **11**, 31, 1925.
6. Watson, *Phys. Rev.*, **30**, 471, 1910.
7. Pierce, *Proc. Amer. Acad. Sci.*, **43**, 375, 1928.
8. *J. Sci. Instrum.*, **11**, 162, 1934.
9. *Phys. Rev.*, **21**, 706, 1923.
10. *Proc. Phys. Soc.*, **55**, 324, 1943.
11. *J. Sci. Instrum.*, **20**, 164, 1943.
12. *J. Frank. Inst.*, **225**, 579, 1938. *See also* Lenihan, *Nature*, **152**, 662, 1943; *Acustica*, **2**, 205, 1952; Biquard and Ahier, *Cahiers de Phys.*, **15**, 21, 1943.
13. *Elekt. Nach. Tech.*, **9**, 9, 1932.
14. *Proc. Music. Assoc.*, **66**, 53, 1940.
15. *Sensations of Tone*, p. 373.
16. *Ann. Physik*, **81**, 235, 1850.
17. *J. Sci. Instrum.*, **17**, 68, 1940.
18. *Phil. Trans. Roy. Soc.*, **221**, 389, 1921.
19. *Proc. Roy. Soc.*, *A*, **101**, 391, 1922; *Phil. Mag.*, **48**, 769, 1924, and **2**, 751, 1926; *Science Progress*, **20**, 68, 1925.
20. Rayleigh, *Phil. Trans. Roy. Soc.*, **161**, 77, 1870, and *Phil. Mag.*, **36**, 231, 1918.
21. *Phil. Mag.*, **34**, 217, 1943.
22. Tucker, *ibid.*, **36**, 473, 1945.
23. *Ibid.*, **14**, 186, 1882.
24. Taylor, *Phys. Rev.*, **2**, 270, 1913.
25. *Ann. Physik*, **43**, 43, 1891.
26. *Ibid.*, **26**, 79, 1908.
27. *Proc. Roy. Soc.*, *A*, **153**, 1 and 17, 1935.
28. *Proc. Phys. Soc.*, **47**, 779, 1935. *See also* King, *Proc. Roy. Soc.*, *A*, **147**, 212, 1934, and **153**, 1, 1935; Lehmann, *Zeits. tech. Phys.*, **18**, 309, 1937.
29. *Proc. Roy. Soc.*, *A*, **171**, 505, 1939.
30. *Ann. Physik*, **42**, 343 and 549, 1891.
31. *Described by* König, *Phys. Zeits.*, **12**, 991, 1911. *See also Comptes Rendus*, **152**, 1160, 1911.
32. *Phys. Zeits.*, **9**, 807, 1908, and **12**, 439, 1911; *Phil. Mag.*, **18**, 180, 1909 and **19**, 476, 1910.

33. *Ann. Physik*, **153**, 102, 1874 and **157**, 42, 1876.
34. *Proc. Roy. Soc.*, *A*, **134**, 445, 1931 ; *Phil. Trans.*, **230**, 413, 1932.
35. *Phil. Trans.*, **175**, 1, 1883. *See also* SCHLICHTING, *Phys. Zeits.*, **33**, 329, 1932 ; SCHUSTER and MATZ, *Akust. Zeits.*, **5**, 349, 1940 ; WEST, *Proc. Phys. Soc.*, **64B**, 483, 1951.
36. *Phil. Mag.*, **17**, 27, 1934 ; HOWATSON, *Phil. Mag.*, **36**, 20, 1945.
37. *Trans. Camb. Phil. Soc.*, 1845.
38. *Phys. Rev.*, **73**, 68, 1948.
39. *Ibid.*, **75**, 1415, 1949.
40. *Ann. Physik*, **50**, 193, 1891.
41. *Ibid.*, **11**, 405, 1903.
42. *Akad. Wiss. Wien, Ber.*, **72**, 213, 1875, and **84**, 710, 1882.
43. *Ann. Physik*, **59**, 116, 1896, and **62**, 158, 1897.
44. *Science*, **53**, 489, 1921 ; *Proc. Nat. Acad. Sci.*, **7**, 207, 1921 ; **8**, 66 and 163, 1922 ; **12**, 137 and 223, 1926. *See also* CARRIÈRE, *J. Physique*, **4**, 413, 1923.
45. *Wiss. Veröff. Siemens-Konzern*, **3**, 139, 1923. *See also* SMITH, *Proc. Phys. Soc.*, **41**, 496, 1929.

CHAPTER NINE

ACOUSTIC IMPEDANCE

Acoustic Impedance. When a steady force is applied to a body of gas in a conduit, tending to move it along the conduit, there exists a definite relation between the applied pressure and the current produced, called the resistance; this resistance is due to viscosity. When the applied pressure is alternating, viscosity is not the only force opposing the movement of the gas; there is also a factor dependent on the frequency. The ratio between the applied pressure and the velocity produced by it, in the more general case, is known as the acoustic impedance, by analogy with the electrical impedance, which is the applied alternating electromotive force divided by the current produced by it. Taking the ideal case in which the displacement ξ of the gas is uniform over any one section (area S) and viscous resistance may be neglected, if the applied pressure is $P \sin \omega t$, the velocity produced $\frac{d\xi}{dt}$ can be written in the form $\frac{1}{S} \cdot \frac{dX}{dt}$, where X is of the dimensions L^3, and is a species of volume displacement. Then actually the impedance (Z) is defined by the relation

$$\frac{dX}{dt} = \frac{P \sin \omega t}{Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

$\left[\text{the corresponding electrical equation is } \frac{dQ}{dt}(= I) = \frac{E \sin \omega t}{Z}\right].$

Suppose we have a cylindrical volume of gas, area of cross-section A, mass m, elasticity k, free from viscous forces, and subjected to an applied alternating force $F \sin \omega t$. The equation of motion will be

$$m\frac{d^2\xi}{dt^2} + k\xi = F \sin \omega t \quad \text{(cf. 22)}$$

Further let the instantaneous displacement be uniform over any one section of the gas, though varying from section to section, so that we can put $\xi S = X$ and $F = PS$, where P is the maximum value of the applied *pressure*. Then :—

$$\frac{m}{S^2}\frac{d^2X}{dt^2} + \frac{k}{S^2}X = P \sin \omega t.$$

Case 1. Let $k = 0$ (restoring force negligible in comparison with inertia),

$$\frac{m}{S^2}\frac{d^2X}{dt^2} = P \sin \omega t$$

Integrate and :—

$$\frac{m}{S^2}\frac{dX}{dt} = -\frac{P}{\omega}\cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

Comparing with (70), we find $Z = \frac{\omega m}{S^2}$. $\frac{m}{S^2}$ is called the inertance (L) of the body of gas.

Case 2. Let $m = 0$ (mass negligible compared with elastic force),

$$\frac{k}{S^2} . X = P \sin \omega t.$$

Differentiate and :—

$$\frac{k}{S^2}\frac{dX}{dt} = \omega P \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

Therefore $Z = \frac{k}{\omega S^2} = \frac{1}{\omega C}$, where C is called the capacitance.

Case 3. Both m and k finite. The solution is :—

$$\frac{dX}{dt} = \frac{P \cos \omega t}{\frac{k}{\omega S^2} - \frac{\omega m}{S^2}} = \frac{P \cos \omega t}{\frac{1}{\omega C} - \omega L} \quad \text{(cf. 25)}$$

$Z = \frac{1}{\omega C} - \omega L$ precisely as in the electrical case, save that the electrical analogue of L is called the inductance. The minus sign indicates that the effects of inertance and capacitance oppose each other. This expression will be a maximum when $\frac{1}{\omega C} - \omega L$ is a minimum, i.e., when $\omega^2 = \frac{1}{LC}$. This is the value of ω which will excite the system to resonance ; in fact the natural frequency of the system is given by :—

$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

The reader who is familiar with alternating current theory will have recognized by this time that we have been developing equations similar to those used in that branch of electrical theory. Equation

(73) with L and C given their electrical significance, gives the natural frequency of electrical circuits which we have often employed in connection with the maintenance of vibrations.

Helmholtz Resonator as Inertance and Capacitance in Series.

Mass of air in neck $= \rho l A$.

Hence $L = \frac{m}{A^2} = \frac{\rho l}{A}$, where l is to include the end-correction.

Capacitance of air in the cylindrical body is given by $\frac{1}{C} = \frac{k}{S^2}$ where k is the restoring force for unit volume-displacement, therefore $k = S\delta p$. Also, the change in volume, produced by such displacement $\delta v = S \times 1$.

Therefore
$$\frac{1}{C} = \frac{k}{S^2} = \frac{\delta p}{S} = \frac{\delta p}{\delta v} = \frac{\gamma p}{v} = \frac{c^2 \rho}{v}$$

And the natural frequency

$$n = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi}\sqrt{\frac{A}{\rho l} \cdot \frac{c^2\rho}{v}} = \frac{c}{2\pi}\sqrt{\frac{A}{lv}} = \frac{c}{2\pi}\sqrt{\frac{\kappa}{v}}$$

κ being called the "conductivity" of the orifice.

This theory assumes all the inertance to be concentrated in the neck, and the capacitance in the body of the vessel. Helmholtz single and double resonators consist therefore, theoretically at least, of inertances and capacitances in series.

A nondescript body of air such as that in a tube possesses in general both capacity and mass. Such a body must be regarded as an inertance and a capacitance in parallel. The reciprocal of the impedance due to such is given by the sum of the reciprocals of its constituents, or

$$\frac{1}{Z} = C\omega - \frac{1}{L\omega} = \frac{LC\omega^2 - 1}{L\omega}.$$

Conduits with Branches. We will now, following Stewart, consider a number of equal impedances, (Z_1) in a conduit, separated by branches containing other equal impedances, (Z_2) (Fig. 82). The tubes must be so short that no appreciable phase differences exist, at least over any one section. It will be shown that any simple harmonic vibration impressed on one end of the network will fail to "get through" unless the ratio of the impedance values for this particular frequency, Z_1/Z_2, lies between 0 and -4.

Let the value of the applied alternating pressure be $P_A \sin \omega t$ at

A, $P_B \sin \omega t$ at B. Further let $\dot{X}$ be the velocity or rate of change of volume-displacement produced by the applied pressure, having amplitude $\dot{X}_{j-1}$ over EA, $\dot{X}_j$ over AB and $\dot{X}_{j+1}$ over BF. Then since

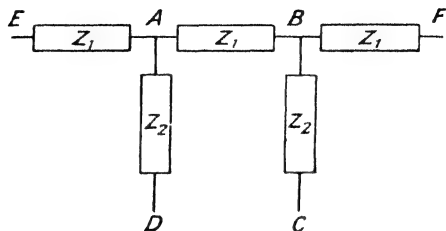



FIG. 82.—Conduit with Branches.

the algebraic sum of the currents meeting at any junction must be zero (cf. "Kirchhoff's law" in electricity), the amplitude over AD must be $\dot{X}_{j-1} - \dot{X}_j$ and over BC, $\dot{X}_j - \dot{X}_{j+1}$, ($C$ and D are terminals of constant pressure).

Then the alternating pressure-difference over :—

$$AB = (P_A - P_B) \sin \omega t = Z_1 \dot{X}_j \text{ by (70)},$$
$$AD = P_A \sin \omega t = Z_2(\dot{X}_{j-1} - \dot{X}_j)$$
$$BC = P_B \sin \omega t = Z_2(\dot{X}_j - \dot{X}_{j+1})$$

Therefore $Z_1 \dot{X}_j = Z_2(\dot{X}_{j-1} - \dot{X}_j) - Z_2(\dot{X}_j - \dot{X}_{j+1})$

or
$$\frac{\dot{X}_{j+1}}{\dot{X}_j} + \frac{\dot{X}_{j-1}}{\dot{X}_j} = \left(\frac{Z_1}{Z_2} + 2\right) \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Owing to the impedance, there will be a fall in velocity amplitude down the conduit, and considerations of the symmetry of the circuit point to there being the same relative loss over each section EA, AB, etc., so that we may write :—

$$\frac{\dot{X}_j}{\dot{X}_{j-1}} = \frac{\dot{X}_{j+1}}{\dot{X}_j} = \quad . \quad . \quad . \quad = e^{\alpha}, \text{ (say)}$$

where α represents the rate of decay of flow amplitude with distance down the main conduit. Substituting in (74), we find :—

$$e^{\alpha} + e^{-\alpha} = \frac{Z_1}{Z_2} + 2$$

or
$$\cosh \alpha = 1 + \frac{1}{2}\frac{Z_1}{Z_2}.$$

When α works out as an imaginary quantity, this means that there

is no attenuation of amplitude. This is so when cosh α lies between $+1$ and -1, or :—

$$0 > \frac{Z_1}{Z_2} > -4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

In words, if ω is such that the impedance values satisfy this condition, a tone of this frequency will traverse the conduit with undiminished amplitude (friction disregarded) ; otherwise, if the conduit is long enough, its amplitude will be reduced to zero in transit.

Acoustic Filters. The effective impedance being a function of the inertance, capacitance and *frequency*, the statement expressed symbolically in (75) means that if a conglomeration of tones are led into such a conduit, all those having frequencies which do not satisfy this condition will be rapidly attenuated, and only those covering a certain range and satisfying (75) will get through. The system therefore acts as a sound filter. Analogous electric circuits have been employed in telephony for the past two decades, but Stewart [1] was the first to envisage the possibility of, and to construct, acoustic filters. Three types are possible:

1. High-pass filters.

Z_1 consists of capacitance, $\frac{1}{\omega C_1}$, only.

Z_2 consists of inertance, ωL_2, only.

If $\frac{Z_1}{Z_2} = 0$, $\omega = \infty$; if $\frac{Z_1}{Z_2} = -4$, $\omega = \frac{1}{2\sqrt{L_2C_1}}$, so that the filter passes all frequencies between $\frac{1}{4\pi}\sqrt{\frac{1}{L_2C_1}}$, and infinity. In practice this consists of a wide tube with holes, which may have short necks (L) surrounding them.

2. Low-pass filters.

Z_1 an inertance, and Z_2 a capacitance merely.

If $\frac{Z_1}{Z_2} = 0$, $\omega = 0$; if $\frac{Z_1}{Z_2} = -4$, $\omega = \frac{2}{\sqrt{L_1C_2}}$, so that the filter passes all frequencies between 0 and $\frac{1}{\pi}\sqrt{\frac{1}{L_1C_2}}$.

For this filter, a tube having branch cavities of considerable size (C) set close together, will serve.

3. Medium-pass or band filters.

These like the high-pass filters have capacitance C_1 in the main, but both capacitance C_2 and inertance L_2 in the branch lines. Their limits are :—

$$\omega = \frac{1}{\sqrt{(C_2 + 4C_1)L_2}} \text{ and } \omega = \frac{1}{\sqrt{L_2C_2}}.$$

The main tube of such a filter has side holes with necks, but the necks are broken at the centre, and communicate with closed vessels which surround them.

Besides Stewart, Canac [2] has been successful in constructing such filters. A test of the filtration possibilities of a high-pass filter is shown in Fig. 83 in which the intensity of sound T from a telephone transmitter, excited at various frequencies, after passing through the filter is shown. Sections of the filters are shown as insets to Fig. 85.

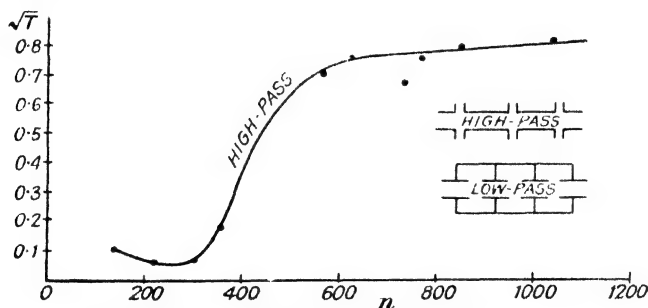



Fig. 83.—Energy transmitted by High-Pass Filter.

These filters should be very useful in sound experiments for purifying a note, and could also be employed commercially. For instance, a low-pass filter in the " tone-arm " of a gramophone should be useful in filtering out the " scratch " of the needle.

Schuster and Kipnis [3] have also examined the exhaust silencers of motor vehicles from the aspect of filters. The ordinary baffle silencer consisting of a long tube divided off by baffles pierced with coaxial holes is essentially a low-pass filter as Davis [4] points out. As the most considerable engine noises are coperiodic with the revolutions, or a small multiple thereof, it is not very efficacious.

Both Hurst [5] and Erwin Meyer [6] have independently treated the problem of transmission through a manifold partition, and each has brought out the filter characteristics of this type of sound insulator. Hurst develops from first principles the theory of transmission through

a series of plane infinite rigid partitions with constant separation, and shows that as the frequency of the oncoming sound is raised the system exhibits a series of alternate attenuation and transmission bands, the high-pitch terminals of the latter having heads like band spectra corresponding to the condition $l = j\lambda/2$, where l is the spacing and j an integer. The latter author is more concerned with the frequency limits and the actual value of the attenuation which such a filter can be expected to exhibit in the lower part of the gamut, i.e., when the spacing l is small compared to the wave-length, the obverse condition to that postulated by the Canadian scientist. With this proviso, the system will form a high-pass filter of limiting pulsatance $\omega = 2c.(lm/\rho)^{-\frac{1}{2}}$, m being the mass per unit area of the partition and ρ the density of the air. Experiments with 5, 10, 15 element model systems made of cellophane or cardboard confirmed this view, at least in its general features, small discrepancies in the expected attenuation being ascribed to resonant drumming of the air pockets. This could, of course, be reduced by straw or similar packing.

A number of mechanical models have been built to demonstrate the properties of acoustic (and electric) filters. Most of these employ rods for the main line loaded by collars to act as branches. Lindsay and his collaborators [7] have examined the theory of a number of these. One constructed by West [8] is useful for demonstration purposes. It consists of a series of equal masses m suspended on springs of restoring force k per unit displacement, distance a apart. The line consists of an elastic cord under tension F and threaded through holes in the masses. The input is furnished by an electric motor coupled to one end of the cord by a rocker arm which contributes a transverse simple harmonic motion to the system, which then acts as a band-pass filter whose limits are (a) the period of a single mass on its spring, and (b) the period which has a half wave-length equal to the distance between consecutive masses. The one limit corresponds then to resonance in the branch, the other to resonance in the line element of an acoustic (or electric) filter. Between these limits the length of the wave transmitted progressively changes, the relation between the frequency and wave-length being

$$n = (2\pi)^{-1}.\{(4\mathrm{F}/ma)\sin^2 \pi a/\lambda. + k^2/m\}^{\frac{1}{2}},$$

which checks well with experiment. Considerable dispersion is shown by the filter, the wave velocity varying from 265 to 1,080 cm./sec. within the range of the experiments.

Impedance of Pipes. Up to the present we have considered the motion in the system to be wholly in phase. If we have to deal with conduits or sections of filters of length comparable to the wave-length of the transmitted sound we must, of course, use the wave equations. The electrical analogue will now be the cable, without attenuation, if we neglect viscous resistance. It will also be more convenient in dealing with pipes to write the simple harmonic variation with time in the exponential form, viz., $e^{i\omega t}$ instead of $\sin \omega t$ or $\cos \omega t$. The particle velocity at any section along the pipe at a distance x from the end then takes the form

$$\dot{\xi} = \left(A \sin \frac{\omega}{c}x + B \cos \frac{\omega}{c}x\right)e^{i\omega t}$$

Whence $$\xi = (i\omega)^{-1}\left(A \sin \frac{\omega}{c}x + B \cos \frac{\omega}{c}x\right)e^{i\omega t}$$

But $$-\frac{\partial(\delta p)}{\partial x} = \gamma p \frac{\partial^2 \xi}{\partial x^2} \quad \text{(cf. p. 3)}$$

Whence $$\delta p = -\rho c^2 \frac{\partial \xi}{\partial x} = i\rho c\left(\mathrm{A} \cos \frac{\omega}{c}x - \mathrm{B} \sin \frac{\omega}{c}x\right)e^{i\omega t}$$

$$Z_l = \frac{\delta p_l}{\dot{X}_l} = \frac{i\rho c}{S}\left[\frac{A \cos \frac{\omega}{c}l - B \sin \frac{\omega}{c}l}{A \sin \frac{\omega}{c}l + B \cos \frac{\omega}{c}l}\right] \quad . \quad . \quad . \quad . \quad (76)$$

But when $$x = 0\,; \quad Z_0 = \frac{i\rho c}{S}\left[\frac{A}{B}\right]$$

Therefore $$Z_l = \frac{i\rho c}{S}\left[\frac{Z_0 \cos \frac{\omega}{c}l - \frac{i\rho c}{S} \sin \frac{\omega}{c}l}{Z_0 \sin \frac{\omega}{c}l + \frac{i\rho c}{S} \cos \frac{\omega}{c}l}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

or $$Z_l = \frac{Z_0 - \frac{i\rho c}{S} \tan \frac{\omega}{c}l}{\frac{S}{i\rho c}Z_0 \tan \frac{\omega}{c}l + 1} \quad . \quad . \quad . \quad . \quad . \quad (78)$$

The impedance of an orifice or neck Z' follows if we make Z_l zero in this formula and l so short that $\tan \frac{\omega}{c}l = \frac{\omega}{c}l$.

$$Z' = \frac{i\rho c}{S} \cdot \frac{\omega}{c}l = \frac{i\rho\omega}{\kappa}$$

where $\kappa \; (= S/l)$ is its "conductivity."

If the pipe is stopped at $x = l$, $Z_l = \infty$ and then

$$Z_0 = -\frac{i\rho c}{S}\cot\frac{\omega}{c}l \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

If, on the other hand, it is completely open at $x = 0$, as well as at $x = l$,

$$Z_0 = +\frac{i\rho c}{S}\tan\frac{\omega}{c}l \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

These expressions equated to zero give the resonant frequencies corresponding to those on p. 166, obtained by the "classical" means.

We consider a partially stopped pipe in resonance as being the sum of two impedances, pipe + orifice in series, and find the resonant frequency by equating the sum to zero. Thus for a pipe partially stopped at one end ($x = 0$) and completely stopped at the other ($x = l$)

$$Z_l = \frac{i\rho\omega}{\kappa} - \frac{i\rho c}{S}\cot\frac{\omega}{c}l.$$

or, putting $k = \omega/c$, the "wave-number," resonance will occur when

$$\tan kl = \kappa/kS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

This formula can be solved graphically by plotting $\tan kl$ and κ/kS against $1/kl$ and finding where the curves intersect.[9] Note that when κ is big this formula reduces to $\cot kl = 0$ for the stopped pipe (cf. 79), while when it is small enough for $\tan kl$ to be put equal to kl, it reduces to the Helmholtz resonator formula, i.e.,

$$k^2 = \kappa/lS = \kappa/v.$$

Fig. 84 shows the theoretical values (dotted lines) compared with

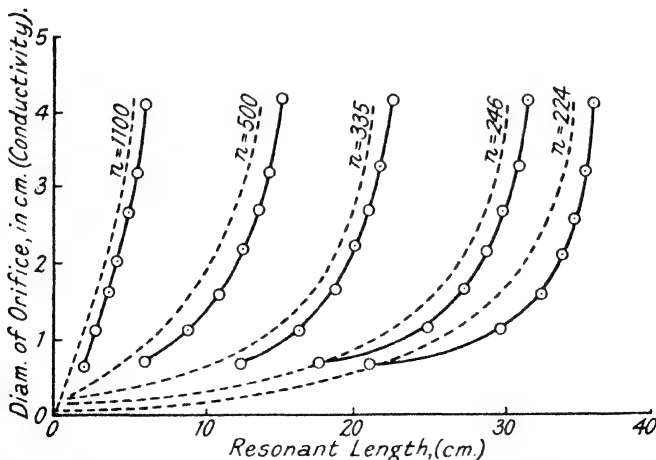



Fig. 84.—Resonance of Partially Stopped Pipe.

some actual values obtained by tuning resonant lengths of a pipe to a tuning fork while the upper end of the pipe was covered with a metal plate having holes of various sizes drilled in it. In this case $\kappa =$ the diameter of the orifice. The formula has also been verified by Mary Browning,[10] who emphasizes that the volume of a cylindrical Helmholtz resonator to resonate with a particular frequency is not fixed but depends on the cross-section, S. She found that the resonant frequency was proportional to $1/v^j$, wherein j varied with S between 0·5 and 0·8. If the frequency is high and the diameter of the neck small, j approaches the value $\frac{1}{2}$, in agreement with the simple Helmholtz formula.

Pipes with Side Holes. A rather similar case in which the resonant frequencies can be calculated by putting the net impedance equal to zero is that of a pipe with a hole in its side. Thus for a pipe with a hole distant L from one end and l from the other, the impedance can be considered as made up of an orifice (impedance Z_0) added to two pipes of lengths L and l (Z_L and Z_l) *in parallel.* So that the resonant frequency is given by

$$Z_0 + \frac{1}{1/Z_L + 1/Z_l} = 0$$

or

$$\frac{1}{Z_0} = -\frac{1}{Z_L} - \frac{1}{Z_l}.$$

If the complete pipe is open at each end

$$-\frac{\kappa}{i\rho\omega} = \frac{S}{i\rho c}\left[\cot\frac{\omega}{c}L + \cot\frac{\omega}{c}l\right], \text{ from (80)}$$

or

$$\frac{\kappa}{kS} = -\cot kL - \cot kl \quad . \quad . \quad . \quad . \quad . \quad (82)$$

This corresponds to the flute with one side hole open; the resonant frequency is not the same as that of a pipe terminating at the first open hole, as an approximate theory would indicate. For a stopped pipe with one side hole open the corresponding formula is

$$\frac{\kappa}{kS} = \tan kL - \cot kl \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

This corresponds to the clarinet with one side hole open. The case of the oboe (closed cone) can be dealt with by the same formula (82) as the flute. The formulæ (82) and (83) give good agreement with measurements on actual wind instruments.[11]

If the instrument has a number of side holes open, as is often the case in playing wood-wind instruments, the equations become a rather awkward series of continued fractions but, as the amplitude in the orifices lower down the tube is small, the contribution of those beyond the first two or three open ones may be neglected. The system will in fact tend to act as a filter when a large number of side holes are opened.

Compound Pipes. Cases in which two pipes or a pipe and resonator are joined in *series* or *parallel* have been treated theoretically by Irons. Several of these cases are important in practice as they are used as organ stops, e.g., the Boys' resonator which consists of a Helmholtz resonator on the end of a pipe—the resonator may alternatively be let into the side of the pipe—and the *flûte-à-cheminée*, in which a narrow pipe is a continuation or re-entrant upon a wider one. In the latter case, the resonant frequencies are given by a formula originally due to Aldis,[12] which Bate [13] has verified experimentally, and which we may write as the sum of two open-pipe impedances in series, i.e.,

$$i\rho c\left[\frac{1}{\pi R^2}\tan kL + \frac{1}{\pi r^2}\tan kl\right] = 0$$

L being the length and R the radius of the wide pipe, l and r those of the narrow pipe.

Solutions of the equation just given show that the natural frequencies are not in harmonic relationship, so that the overtones will not be in resonance when the fundamental is. Oberst [14] has used this fact to construct a pipe source of pure tone capable of working at considerable intensity. He fed the tone from a siren into the mouth of the wide pipe and measured the pressure amplitude at the mouth of the narrow end by the pressure of radiation recorded in a liquid manometer (cf. p. 217). At a frequency of 180 cycles/sec. a displacement amplitude of 2 cm. could be produced at the junction between the two pipes, one of which was 107 cm. long and 12 cm. diameter, the other 45 cm. long and 2·5 cm. diameter. The junction can be closed by a thin rubber membrane to prevent air currents from the siren leaking into the narrow tube, without detriment to the sounding qualities.

Taber Jones [15] has treated the theory of the Haskell organ pipe, an open pipe into which a shorter closed pipe is inserted (Fig. 85) excited in the usual edge-tone method of flue pipes. In terms of the symbols

shown on the figure, he proves that the wave-number k of the natural frequency is given by

$$q_1 \cot ka + q_2 \cot kb = (q_1 + q_2) \tan kl \quad . \quad . \quad . \quad (84)$$

The system of acoustic impedances is that of a stopped pipe with two open pipes in series branched upon the open end, and serves to produce bass notes in a comparatively restricted space, more so, in fact, than if a single stopped pipe were used. To make this formula agree with the experimentally observed frequencies, it is of course necessary to make certain assumptions with regard to end corrections. The same formula will fit the case of a simple pipe into which a coaxial solid rod is pushed, q_2 then being equal to the difference in cross-section of the rod and the pipe.

A rather more complicated case is that of a wide pipe with a constriction somewhere inside it. This we may regard as two pipes connected in series by an orifice. Irons [16] has treated this case both

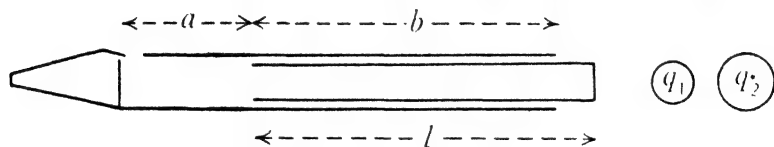



FIG. 85.—Haskell Organ Pipe.

theoretically and experimentally, using the method of Kundt's dust figures to determine wave-lengths. The tube being virtually separated into two stopped pipes of length L and l by the constriction, the resonant frequencies will be given by

$$\frac{kS}{\kappa} - \cot kL - \cot kl = 0$$

Moreover the formulæ for compound pipes can be readily extended to meet cases where the two lengths consist of media of different properties by inserting, e.g., $\rho'c'$ in place of ρc for the " characteristic impedance " (cf. p. 233) in the second medium, a case worked out on classical lines by Lees.[17] Further, if ρ, ρ' and c are known, the determination of the natural frequency of such a compound system will enable c', the velocity of sound in the second material to be measured. In this way it is possible to measure the velocity of sound in soft and brittle solids, by making a compound rod with a length of steel or ash joined to the specimen end to end, a method first used by Stefan.[18]

Characteristic Impedance. Consider the case of plane waves

incident normally on a boundary between two media. Let the equation of the incident wave, moving from left to right, be

$$\xi_1 = f(ct - x) ; \quad \dot{\xi}_1 = cf'(ct - x)$$

Then
$$s_1 = -\frac{d\xi_1}{dx} = f'(ct - x)$$

so that $\dot{\xi}_1 = cs_1$, where s_1 is the condensation. For the reflected wave,

$$\xi_2 = f(ct + x)$$

and
$$\dot{\xi}_2 = -cs_2$$

while for the transmitted wave, $\xi' = f(c't - x)$

$$\dot{\xi}' = c's'$$

Since the net particle velocity normal to the boundary must be the same on each side,

$$\dot{\xi}_1 + \dot{\xi}_2 = \dot{\xi}'$$

or
$$cs_1 - cs_2 = c's'.$$

Since the normal pressure at the boundary must be the same in each medium

$$es_1 + es_2 = e's'.$$

(Note that the elasticity = applied pressure/resulting condensation.) Eliminating s' between these two equations, we obtain

$$\frac{s_2}{s_1} = \frac{e'c - ec'}{e'c + ec'} = \frac{\rho'c' - \rho c}{\rho'c' + \rho c} \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

since elasticity = $(\text{velocity})^2 \times$ density.

This ratio expresses the relative pressure amplitudes in the reflected and incident sound waves. There will in fact be no reflection if $\rho'c' = \rho c$. This product of the density and velocity of sound is a characteristic of the medium and the type of wave and is called the "specific impedance." We have already met it in connection with the pressure exerted on a solid boundary by the sound (p. 217) where it forms part of the energy density or intensity crossing unit area per second. In connection with sound transmission it is more often known as the "characteristic impedance of the medium." Considering a plane wave crossing unit area of the medium,

$$z = \frac{\delta p}{\dot{X}} = \frac{es}{cs} = \rho c$$

This impedance, being real and independent of frequency, is a pure

resistance, but this is not true of spherical waves, to which we must turn our attention for a while.

In the development of our fundamental wave equation, we introduce a quantity ϕ whose derivative with respect to any direction is to represent the particle velocity in that direction, e.g., in the present case $\frac{\partial \phi}{\partial r}$ represents the radial velocity. It can then be shown that $\dot{\phi} = -c^2 s$.* For a spherical wave we write, satisfying (63), p. 169,

$$rs = e^{ik(ct-r)}, \text{ where as before } k = \omega/c$$

$$\phi = -c^2 \int s dt = -\frac{c}{ikr}.(rs)$$

$$\frac{\partial \phi}{\partial r} = \frac{c}{ikr}\left(\frac{1}{r} + ik\right).(rs)$$

Also (cf. p. 228) $\quad \delta p = \rho c^2 s = \frac{\rho c^2}{r}(rs).$

Dividing these last expressions, we obtain the characteristic impedance to spherical radiation, viz.,

$$z = \rho c\left(\frac{ik}{1/r + ik}\right) = \rho c\left(\frac{1}{1 - i/kr}\right) \quad . \quad . \quad . \quad . \quad (86)$$

or in a form which allows one to separate the real and imaginary parts

$$z = \rho c\left(\frac{ikr}{1 + ikr}\right) = \rho c\left(\frac{k^2 r^2 + ikr}{1 + k^2 r^2}\right) \quad . \quad . \quad . \quad . \quad (87)$$

The first term in the numerator is a resistance, the second a reactance. If r is very large, (86) reduces to ρc as for plane waves. If, on the other hand, we are considering a place at a small distance (compared with the wave-length) from the source, so that kr is small, (87) shows that the impedance is approximately

$$\rho c k^2 r^2 = 4\pi^2 n^2 \rho r^2 / c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

which is sometimes termed the "radiation resistance."

In giving the impedance theory of pipes we should properly have included this radiation impedance as a consequence of the change over from plane to spherical waves at the mouth. Its influence on the natural frequencies of the system is small however, and the effect is bound up with the end-correction of the tube.

Our equation for a partially stopped pipe (81) can be regarded as

* Cf. Lamb, *Dynamical Theory of Sound*, p. 205.

giving the end-correction a at the partially stopped end (flanged) in the form

$$\tan ka = \frac{kS}{\kappa} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

If the right-hand side of this equation is small $a = S/\kappa$, but if the right-hand side is large we derive an end-correction

$$a = \frac{\lambda}{4}\left[1 - \frac{\kappa\lambda}{\pi^2 S}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

Bate's [19] experiments on this matter led him to:

$$a = \frac{S}{2\sigma}\sqrt{\frac{\pi}{S'}}$$

S' standing for the area of the opening and σ a factor depending on its shape (1 to 1·5). This is a special case of equation (90) to which it resolves if κ is set equal to the diameter and $\sigma = 1$.

Tyte [20] refers to an earlier paper in which he has calculated the end-correction for a conically flanged cylindrical pipe by analogy with corresponding problems in a.c. technology. He now shows that the experimental results of Bate and Wilson fit this theory well and has extended the experiments to deal with cones of smaller apical angle, for which the cone is in the nature of an extension of the cylinder rather than a flange to it. On the basis of this agreement he goes on to treat the classic case of the orifice in a diaphragm forming the open end of a pipe and suggests that the end-correction may be taken as inversely proportional to the area of the orifice. He tries out this idea on the experiments of Irons,[21] Bate and Robinson.[22] The suggested relation between end-correction and reciprocal area holds whether the pipes are flanged or not.

The question of whether a flare at the end of a cylindrical tube amounts to an end-correction on the tube or makes the system a compound pipe has been examined experimentally by Bate and Wilson.[23]

A more important aspect of this matter appears in the calculation of the ratio of the transmitted to the reflected energy. Not only will the considerations leading to equation (85) apply to a material change in the medium, but in addition whenever there is a sudden change in the *total* acoustic impedance there will be reflection. Such change may occur at a constriction in virtue of a change in S, or at the end of a pipe in virtue of the change from plane to spherical waves. Energy

will be radiated most efficiently, i.e., there will be least reflection, if the impedances of the tube and of the open air at the end are matched; this cannot be done with a cylindrical pipe.

In this connection it is interesting to note that Jordan and Everitt [24] have used a vertical pipe pierced with fine side holes mounted over a loud-speaker unit as a model of an antenna, plotting the acoustic field round the source by means of a microphone. By using the analogy between the acoustic and the electro-magnetic fields, they were able to predict the behaviour of a designed antenna, in advance of construction of the actual antenna. Everything in the full-size field can be imitated in this way, including mutual action of several antennæ, except, of course, polarization effects.

In theory, the acoustic analogue of the antenna is a vertical stretched string driven by attachment to some sort of vibrator at the lower end and fixed at the upper end, but the string, owing to the very small surface it presents to the ambient fluid, is a very poor radiator. Hence the preference for the pipe analogue and—to let it radiate in the same fashion as an antenna—the provision of a stopped upper end and a series of fine holes in the side.

The Horn as Radiator. Consider a tube with cross-section increasing along its length having a diaphragm vibrating in piston fashion at its narrow end, and terminating at the other in a wide flare. The piston will work most efficiently into the horn if the acoustic impedance of the air at the throat is matched to its mechanical impedance. This can usually be done by constructing an air cavity in the throat, one side of which is closed by the piston, while the other leads through a narrow aperture into the body of the horn. The inertance and capacitance of the throat can be adjusted by calculation (cf. the Helmholtz resonator) to equal the mass and stiffness respectively of the diaphragm. Incidentally, the throat acts as an acoustic transformer, the product of pressure and velocity amplitudes on either side of the constriction being approximately the same. The waves then pass along the widening tube and are let into the atmosphere from an opening whose diameter is large. They therefore start out from this place as spherical waves of considerable radius and, provided the wave-length is not too large, the acoustic impedance is nearly the same as for plane waves. Radiation then takes place efficiently, there is little reflection back into the horn except at low frequencies, and moreover at wave-lengths small compared with the aperture there will be a directive effect. In spite of the tendency of

the waves to spread, interference will ensure that most of the energy is sent out as a beam parallel to the axis of the horn, as in the corresponding optical case.

Looked at from another point of view, for spherical waves of small radius and therefore diverging from a narrow aperture, the particle velocity and pressure are nearly 90° out of phase, and therefore the impedance of the spherical waves is nearly all reactance—a function of the frequency—whereas that of the plane waves inside the tube is a pure resistance independent of frequency. Even if their absolute values are equal they are not thereby matched. As the frequency is raised and the aperture increased in diameter, these two factors come more nearly into phase, until the impedance of the medium outside is a pure resistance like that inside.

A cylindrical or *slightly* conical pipe is essentially then a resonator. Its function is to have marked natural frequencies in virtue of the large amount of reflection at the open end. Orchestral wind instruments are of this type ; as radiators they are inefficient except at high frequencies, in spite of the flare at the end of most of them, which is too " sudden " to be of use acoustically. At the other extreme, the loud-speaker horn is designed with a gradual flare to be an efficient radiator without markedly detached resonances.

Horns of types other than the cone were first studied theoretically by Webster,[25] and latterly from both practical and theoretical aspects by the staff of the Bell Telephone Laboratories. It is found that the exponential horn, one whose section at a distance x from the throat of area S_0 is given by $S = S_0e^{mx}$, is the best, in that the impedance at the open end becomes a pure resistance at much lower frequencies than that of the conical horn—$S = S_0(1 + mx)$—of the same initial and final cross-sections. It therefore radiates low frequency sounds more efficiently.

We have given the theory of the straight conical pipe on p. 168. A more general formula applicable to a column of expanding cross-section may be written thus :

$$S\frac{\partial s}{\partial t} + \frac{\partial}{\partial x}(S\dot{\xi}) = 0$$

where S is the cross-section at x, s the condensation.

Now $$\dot{\phi} = c^2 s \quad \text{and} \quad \dot{\xi} = -\frac{\partial \phi}{\partial x}$$

so $$\ddot{\phi} = \frac{c^2}{S}\frac{\partial}{\partial x}\left(S\frac{\partial \phi}{\partial x}\right) = c^2\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial x}\cdot\frac{\partial}{\partial x}(\log S)\right] \quad . \quad . \quad (91)$$

We may apply this formula to a cone of solid angle Ω if we set $S = \Omega x^2$ to get $\phi = \frac{A}{\Omega x} \sin(\omega t - kx)$.

Next we apply it to the exponential horn $\log(S/S_0) = mx$, and (91) becomes

$$\frac{\partial^2\phi}{\partial x^2} + m\frac{\partial\phi}{\partial x} + \frac{\omega^2}{c^2}\phi = 0$$

of which the solution is

$$\phi = Ae^{-(\alpha + i\beta)x}$$

with $\alpha = \frac{m}{2} = \frac{\omega_0}{c}$, say;

$$\beta^2 = \frac{\omega^2}{c^2} - \frac{m^2}{4} = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_0^2}{\omega^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (92)$$

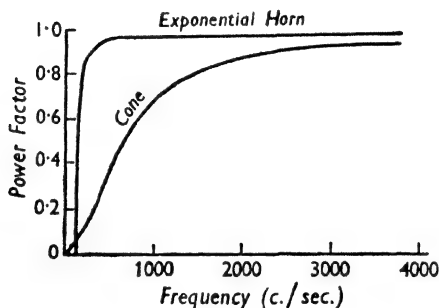



Fig. 86.—Comparative Efficiency of Exponential Horn and Cone (*Crandall*).

The pulsatance $\omega_0 = mc/2$ which we have here introduced is called the "cut-off" pulsatance for when $\omega = \omega_0$, $\beta = 0$ and transmission fails.

The power factor is in fact $\sqrt{(1 - \omega^2/\omega_0^2)}$ and rapidly approaches 1 as ω is raised above ω_0, more rapidly indeed than does that of a cone of same length (x), of which the power factor is $\frac{k^2x}{1 + k^2x^2}$ as can be seen by picking out the real part of the impedance of a spherical wave from equation (87) p. 423. The two functions are plotted on Fig. 86.

Length for length, therefore, the exponential horn is more efficient as a radiator at frequencies just above the cut-off than is the cone.

The Annular Effect. It has been assumed up to the present that the motion across the section of a pipe or orifice is uniform. It

is obvious that in the immediate neighbourhood of the solid confines of a conduit there will be a drop in the displacement amplitude to zero, but before considering this effect of viscous resistance, there is another effect, more important in practice, which invalidates this assumption. It was discovered by the author in experiments with a hot wire traversed across an orifice or conduit in which aerial oscillations were taking place, that the velocity amplitude was much *greater* in the outer annuli just before the wall was reached than at the centre

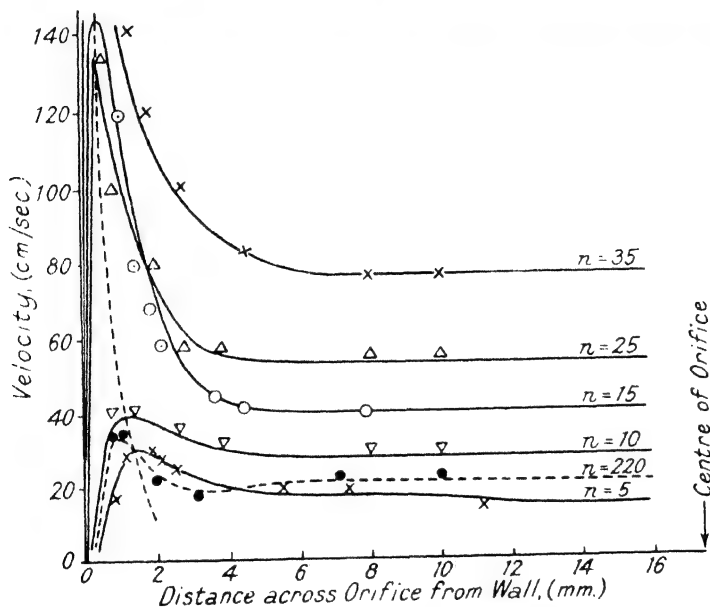



Fig. 87.—The Annular Effect.

of the tube. There is in fact a peak of alternating velocity whose magnitude (relative to the central velocity) increases with frequency, but whose distance from the wall decreases as this factor goes up. This circumstance, which gives the distribution of velocity across the tube or orifice quite a different trend to that of one-way air flow, immediately calls to mind the skin effect in electrical technology, shown by high-frequency alternating currents flowing in a magnetic conductor. Fig. 87 shows some typical results. This "annular effect" can be explained by considering the equations of motion of the air in the tube, and is due to a combination of inertia and viscosity. It appears from the theory that the distance (δ) of the peaks

from the walls of the tube is given by the formula $\delta\sqrt{n} =$ a constant.[26]

This law, that δ is proportional to $n^{-\frac{1}{2}}$ is borne out in practice; the position of the peaks is shown by the dotted line on Fig. 88. The existence of this annular effect has been confirmed by Carrière [27] who noted the distances traversed by smoke particles executing S.H.M. in a pipe. The amplitude was a maximum at points a short distance from the walls of the pipe.

Imperfect Reflectors. If, in equation (78) for the pipe, we set Z_0 equal to the impedance of an imperfect reflector, and can determine the impedance Z_l looking in at the other end, we can, with the aid of equation (85), determine the reflection coefficient of the specimen, for the first equation with Z_l and ω known will give us the ratio of ρc to Z_0, i.e., the ratio of the characteristic impedances of the wide pipe and the pervious material respectively, while this ratio, if substituted in the second equation, will give us s_2/s_1, the amplitude reflection coefficient of the medium. This method was first suggested and used by Wente and Bedell [28] to measure the absorption coefficient of a specimen to sound. It has also been applied by Penman and Richardson [29] to test the velocity of sound in narrow tubes, the specimen in this case being a bundle of glass tubes of diameter 0·2 mm. forming the imperfect stop at one end of a pipe $1\frac{1}{2}$ in. wide, driven at the other end by a telephone, maintained with constant frequency.

Small scale methods are based on observations of Tuma [30] that when stationary waves are formed between a source of sound and an imperfectly reflecting wall, pseudo nodes and antinodes are formed, having respectively less and greater pressure amplitudes than those indicated by the simple theory of stationary waves, which supposes the waves to be reflected without loss of amplitude. A similar remark applies to the displacement amplitudes. The idea was developed by Taylor [31] using a pipe resonator having the thick wooden stop at the end covered with the material in question. This resonator was excited by a suitably tuned organ pipe. He determined the relative displacement amplitudes at the pseudo nodes and antinodes by means of a Rayleigh disc resonator to which the point of observation was connected by a long search tube. This method is open to criticism by virtue of the stationary vibrations which would tend to form in the search tube, causing deflections of the disc dependent on its position relative to these.

Weisbach [32] repeated the method, using a microphone in the pipe

resonator itself in place of the Rayleigh disc. The tube was made wide compared with the microphone and the amplitude of response measured on an Einthoven string galvanometer. The writer [33] has used the hot-wire grid (p. 187) in such a resonator to determine the displacement amplitude at a given point, (1) with a stop of thick teak to give as nearly as possible perfect reflection, and (2) with this stop covered by a layer of the material to be investigated. The reflection coefficient can be calculated from the ratio of the amplitudes in the two cases at any point except at the nodes where the ratio is indeterminate. Referring to p. 41 we see that the ratio of the amplitudes in the incident and reflected waves which make up the stationary vibration is calculable, assuming that the phase change on reflection is unaffected by the substitution. The ratio b^2/a^2 is the reflection coefficient.

Keibs [34] has a simple method of measuring the impedance, and hence the absorption coefficient, of a reflector at the end of a tube of length l with attenuation constant α and propagation constant $\beta = \omega/c$, provided βl is not less than $3 \cot (\alpha + i\beta)l$. He measures the pressure at the sending end when the other is closed successively by a hard stop and by the unknown impedance z. The ratio of these two pressures

$$\frac{p_1}{p_2} = \frac{z}{z + \cot (\alpha + i\beta)l} = \frac{z}{z + 1}, \text{ approximately.}$$

He uses this simple formula to measure the impedance of the ear drum placed at the end of a short tube, a microphone at the other end measuring pressures, while just behind it is a thermophone wire as source.

Practical Measurement of Impedance.

1. *Direct Method* (*Richardson* [35]). The velocity distribution across the orifice is measured by the hot wire method, and the pressure gradient through it by means of the manometric capsule described on p. 184. Thus, in the case of a sounding Helmholtz resonator, if this membrane is inserted into the back of the reservoir and the velocity is integrated across the mouth by means of the hot wire, the ratio of the two gives the apparent impedance.

2. *Acoustic Method* (*Stewart* [36]). Two telephone diaphragms are placed at one end of each of two conduits, and supplied with alternating currents of the same frequency. At their other ends are attachments leading to the ears as in the stethoscope. One of these is fixed in position, but the distance of the other attachment from the

source can be varied by telescoping two parts of the tube over one another. The positions are adjusted until no sound is heard in the stethoscope; this means that the vibrations are arriving at the ear out of phase. The system whose impedance is to be measured is now inserted as a branch on one conduit. Both the currents in the telephone and the position of the sliding attachment have to be changed in order to restore the silence condition. From the first the change of pressure amplitude, and from the second the change of phase due to the insertion of the branch can be calculated. This method, though less direct, therefore gives both magnitude and phase factor of the impedance, whereas the first gives the absolute value only.

3. *Pressure Method* (*Schuster* [37] *and Robinson* [38]). This method involves the equating of the pressure at some point in an impedance which is supposed known, or taken as standard—e.g., a wide pipe—with the pressure on one side of the impedance to be measured, both this and the standard being driven at the other end by the same telephone diaphragm. In Robinson's apparatus the position of the point of attachment of the stethoscope in the wide pipe can be varied by a sliding piece, or the length of the wide pipe itself can be varied until equality of pressure is attained. The apparatus is then like an acoustic Wheatstone Bridge in that a variation in one arm is made until the stethoscope which takes the place of the galvanometer indicates equality of pressure across the junctions. Robinson has measured the impedance of orifices and constrictions in pipes by this method (cf. p. 245).

4. *Electrical Method* (*Flanders* [39]). This is an impedance meter in which the known impedance Z_0 is a tube one-eighth of a wave-length long, which is joined, in turn with the unknown impedance, to a tube of fixed length, the R.M.S. pressure at the junction being read on a condenser microphone. The apparatus (Fig. 88) has the advantage that the ultimate readings are made on a potentiometer. Sound from the loud-speaker L passes along the fixed tube to the junction J, where a side branch leads to the microphone. The electric amplitude of transmitter and receiver after amplification are compared on the potentiometer, consisting of a variable resistance and a variable mutual inductance in series. The voltage across the output amplifier is balanced by a null method against that across the resistance and secondary of the inductance. By analogy, the corresponding electric case comprises an electromotive force E in series with an impedance T up to the junction (more precisely, " the impedance at the junction

looking towards the same when on open circuit," the latter *proviso* implying a rigid stop at J in the acoustic case) and an attached impedance beyond J. The pressure e at J is, like the "potential difference

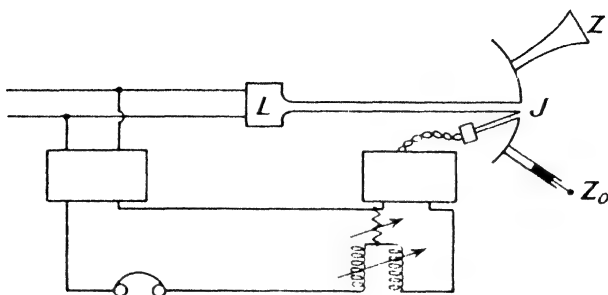



FIG. 88.—Impedance Meter (*Flanders*).

between the terminals" at J and the other end of the cell, equal to $\frac{E}{T+Z}.Z.$

We first attach the "known" impedance Z_0, next a rigid stop, and finally the "unknown" impedance Z and have :—

$$Z = Z_0,\ e_1 = \frac{EZ_0}{T + Z_0}$$
$$Z = \infty,\ e_2 = E$$
$$Z = Z,\quad e_3 = \frac{EZ}{T + Z}$$

(The method will recall to the mind of the reader the well-known experiment of the elementary physics course: "to find the internal resistance of a cell.")

Eliminating E and T, we have :—

$$Z/Z_0 = (e_2/e_1 - 1)/(e_2/e_3 - 1)$$

The pressures e are proportional to the respective currents through the resistance and primary. The drop in voltage across the resistance and secondary is equal and opposite in phase to E, when no current passes through the headphones. If then z is the electrical impedance value of the resistance and inductance (which can be measured by the usual electrical methods), ez is constant, and the equation becomes :—

$$\frac{Z}{Z_0} = \frac{z_1/z_2 - 1}{z_3/z_2 - 1} = \frac{z_1 - z_2}{z_3 - z_2}$$

5. *Bridge Methods* (*Robinson*[40]).—Robinson constructed an acoustic impedance bridge, by analogy with the apparatus familiar in A.C. electrical work. To take the place of the ratio arms of a Wheatstone Bridge, reactances were introduced consisting of wide tubes of length l and area S, for which the impedance was taken to be $-\frac{i\rho c}{S}.\cot 2\pi c/\lambda$, ($\rho$ being density of air, c velocity, and λ wavelength of sound used). The source of sound was a loud-speaker (outside the room) connected to one corner of the diamond network, the opposite corner being a dead-end or earthed into the atmosphere. The arrangement is shown in Fig. 89. The loud-speaker feeds to a junction box J, then follow

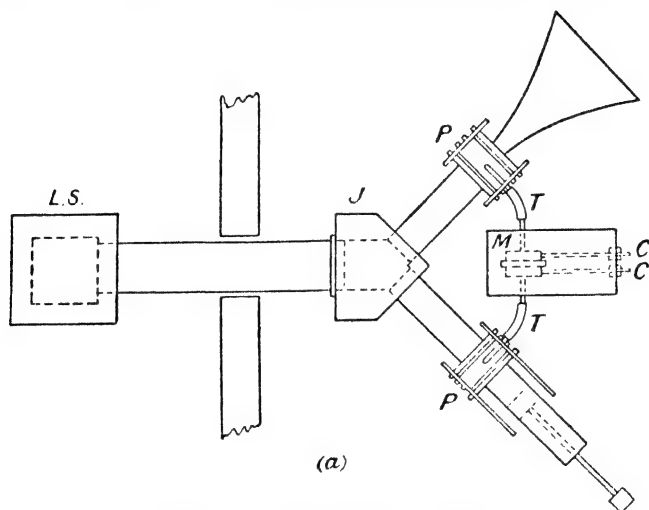



FIG. 89.—Acoustic Bridge (*Robinson*).

the ratio arms, and beyond the connecting pieces P, P, the unknown impedance (shown as a horn) and the known impedance (a tube of adjustable length). Rubber tubes T, T lead to the galvanometer, a microphone M, with compensating tubes and pistons C, C permitting an adjustment of the acoustic impedance on either side of the diaphragm. The apparatus was first used to measure standard forms of impedances such as the Helmholtz resonator. Difficulty in deciding what end-correction to ascribe to an orifice which does not debouch into the open air, but into a coaxial pipe of different section, led eventually to the use of a second method to determine such corrections. Even when the orifices were used alone, without the reservoir, higher

values for the conductivity were obtained than the theoretical value, which for a circular hole in a thin plate is equal to the diameter.

The second apparatus is shown in Fig. 90. It is designed for the measurement of the " conductivities " of orifices. The loud-speaker source is at one end of a tube, divided by the listening-tube at C into two parts, l_1 to the left, l_2 to the right. The terminating impedance on the right (Z_2) is either the orifice or a rigid stop. With the stop *in situ* so that $l_2 = m\lambda/4$, where m is an odd integer, the impedance to the right is zero, and the sound in the microphone a minimum. This sound may be further reduced to zero by the adjustment of the impedance to the left, which is effected by altering the position of the piston in the side tube B. The unknown impedance is then introduced in place of A and l_2 adjusted to restore silence. The effective impedance to the right is again zero, and the value of the unknown impedance may be calculated by equating

$$Z_1 = \frac{i \cos kl_2 . Z_2 - \frac{\rho c}{S} \sin kl_2}{-\frac{S}{\rho c} \sin kl_2 . Z_2 - i \cos kl_2}$$

to zero, from which $Z_2 = \frac{i\rho c}{S} \tan kl_2$. For the unknown impedance Robinson substituted circular orifices in metal plates, either forming constrictions in a stopped tube or debouching directly on the atmosphere.

It was found that the conductivity of a hole of diameter d in a tube of diameter D could be represented by

$$C = \frac{0{\cdot}787d}{(1 - d/D)^{1{\cdot}895}}$$

if it formed a constriction in a tube and $C = d(1 + d/D)^{1{\cdot}19}$ if it were a terminal. A formula of the latter type was first suggested by Bate [41] on the basis of experiments on the natural frequencies of pipes terminated by circular orifices of various diameters (cf. p. 235).

The conductivity at the junction of two tubes of different diameter d and D could be represented by approximately twice the conductivity of a thin orifice of the same diameter as the smaller tube, or more accurately by $C = \frac{2{\cdot}0\, d}{(1 - d/D)^{1{\cdot}45}}$. These relations were verified with the first impedance bridge.

Finally, a simplified form of the bridge was built, consisting of

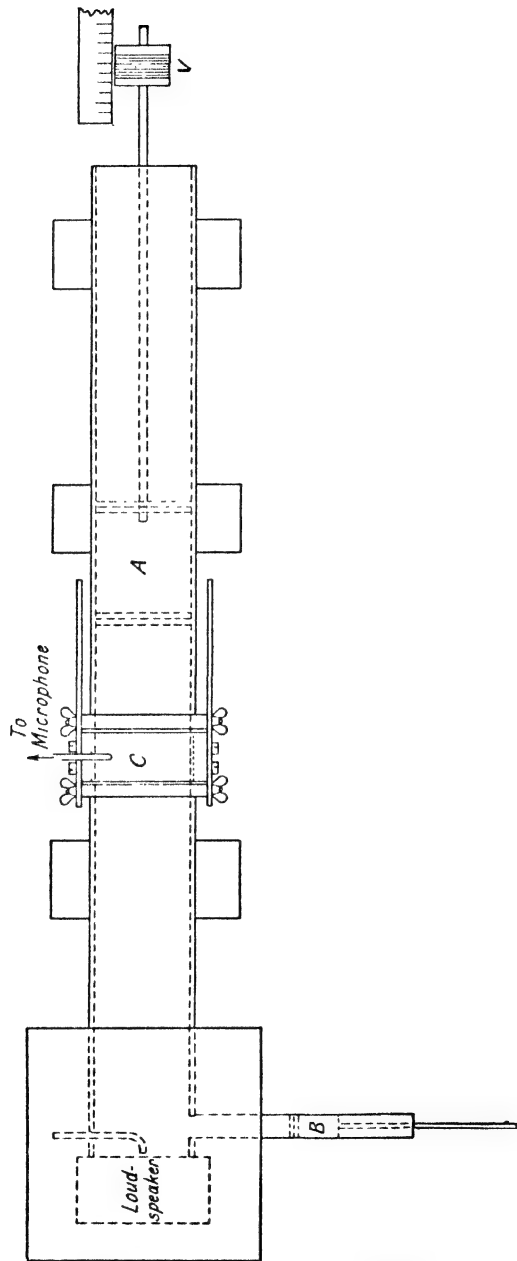



FIG. 90.—Acoustic Bridge (*Robinson*).

a single tube 10 in. long, with a single telephone source on a branch at the centre. Two pressure tubes were brought to the axis of the main tube, one either side of the T junction, 1 in. from the ends of the main tube. The differential microphone was then attached to the two pressure tubes and unknown and known impedances joined to the ends of the main tube. The bridge has proved to be convenient and accurate for determining acoustical reactances, and work on resistances of the slot type is now in progress.

Variable Impedances. It is sometimes desirable to have an impedance at the end of a pipe or as a baffle in a wall whose value can be varied by known amounts. One such, due to Jordan,[42] consists of two short lengths of tube which telescope into each other. Both are perforated by a number of holes passing right through. One end passes into the pipe of which the system is to be the terminal, the other is terminated by a sliding plug. The whole is surrounded by absorbent wool in an outer case. Rotating one tube relative to the other decreases the effective size of the holes (as in a familiar type of pepper sprinkler). Pulling the outer case relatively to the inner moves the piston in the tube. Thus the first operation adjusts resistance and inductance simultaneously. To overcome this change of inductance when a pure resistance change is desired, it is " tuned out " with an equal and opposite capacitive reactance introduced by the movement of the piston.

The impedance constructed by Bolt and Brown [43] is on a larger scale and intended to form part of a boundary wall. The reaction element is provided by boards perforated by small channels, the area of all of which can be simultaneously adjusted, while the capacitance comes from cavities behind these holes. The depth of the cavities is adjusted by a sliding piston. A series resistance is provided in the form of a layer of porous material between the perforations and the cavities.

References

1. *Phys. Rev.*, **20**, 528, 1922; **22**, 502, 1923; **23**, 520, 1924; **25**, 90, 1925; **26**, 688, 1925; **27**, 487 and 494, 1926; **28**, 1088, 1926. Stewart and Sharp, *J. Opt. Soc. Amer.*, **19**, 17, 1929; Schuster, *Ann. Physik*, **4**, 513, 1930.
2. *J. Physique*, **7**, 161, 1926.
3. *Ann. Physik*, **14**, 123, 1932.
4. *Phil. Mag.*, **16**, 787, 1933.
5. *Canad. J. Res.*, **12**, 398, 1935.
6. *Elekt. Nach. Tech.*, **12**, 393, 1935.

7. *J. Acoust. Soc.*, **4**, 155, 1932; **5**, 196, 1934; **8**, 42, 1936; **10**, 41, 1938.
8. *Proc. Phys. Soc.*, **46**, 186, 1934.
9. RICHARDSON, *ibid.*, **40**, 206, 1928.
10. *Phil. Mag.*, **33**, 551, 1943.
11. RICHARDSON, *Acoustics of Orchestral Instruments*, Appendix; IRONS, *Phil. Mag.*, **10**, 945, 1930, and **11**, 535, 1931.
12. *Nature*, **114**, 309, 1924.
13. *Phil. Mag.*, **16**, 562, 1933.
14. *Akust. Zeits.*, **5**, 27, 1940.
15. *J. Acoust. Soc.*, **8**, 196, 1937; **10**, 167, 1939; **11**, 122, 1939.
16. *Phil. Mag.*, **7**, 523 and 873, 1929.
17. *Proc. Phys. Soc.*, **41**, 204, 1929.
18. *Akad. Wiss. Wien, Ber.*, **57**, 697, 1868.
19. *Phil. Mag.*, **24**, 453, 1939.
20. *Ibid.*, **30**, 173, 1940.
21. *Ibid.*, **5**, 580, 1928.
22. *Proc. Phys. Soc.*, **46**, 772, 1934.
23. *Phil. Mag.*, **26**, 752, 1938.
24. *Proc. Inst. Radio Eng.*, **29**, 186, 1941.
25. *Proc. Nat. Acad. Sci.*, **5**, 275, 1919.
26. RICHARDSON, *Proc. Phys. Soc.*, **40**, 206, 1928; RICHARDSON and TYLER, *ibid.*, **42**, 1, 1929; *Phys. Zeits.*, **32**, 509, 1931; SEXL, *Zeits. Phys.*, **61**, 349, 1930.
27. *J. Physique*, **9**, 187, 1928.
28. *Bell. Syst. Tech. J.*, **7**, 1, 1928.
29. *J. Acoust. Soc.*, **4**, 322, 1933.
30. *Akad. Wiss. Wien, Ber.*, **111**, 402, 1902.
31. *Phys. Rev.*, **2**, 270, 1913. *See also* PARIS, *Phil. Mag.*, **4**, 917, 1927; *Proc. Roy. Soc.*, *A*, **115**, 407, 1927; *Proc. Phys. Soc.*, **39**, 269, 1927.
32. *Ann. Physik*, **33**, 763, 1910.
33. *Proc. Roy. Soc.*, *A*, **112**, 538, 1926.
34. *Ann. Physik*, **26**, 585, 1936.
35. *Proc. Phys. Soc.*, **40**, 206, 1928.
36. *Phys. Rev.*, **28**, 1038, 1926.
37. *Phys. Zeits.*, **35**, 408, 1934.
38. *Proc. Phys. Soc.*, **46**, 772, 1934.
39. *Bell. Syst. Tech. J.*, **11**, 402, 1932.
40. *Proc. Phys. Soc.*, **46**, 772, 1934; *Phil. Mag.*, **23**, 665, 1937.
41. *Proc. Phys. Soc.*, **48**, 100, 1936.
42. *Akust. Zeits.*, **5**, 77, 1940.
43. *J. Acoust. Soc.*, **12**, 31, 1940.

CHAPTER TEN

ULTRASONICS

The Singing Arc. When an electric arc is struck between two carbons fed by direct current, the incandescent spark produced is peculiarly sensitive to oscillations of the current supply. For example, any inductive effects superposed on the steady current will be repeated by the arc, and owing to the powerful disruptive effect of the spark, become audible, generally as a " spluttering " of the arc. To make the arc " sing " a constant tone it is necessary so to adjust the conditions that the arc is unstable, ready to oscillate, and then to include it as part of a resonant circuit. In this way Duddell [1] was able to make the arc produce powerful sounds of frequency $n = \frac{1}{2\pi\sqrt{LC}}$, where L is the self inductance, and C the capacity of a circuit placed in parallel with the arc (Fig. 91). Of course, an arc fed by alternating current will reproduce the frequency of the supply in the same way. The phenomenon is reversible, sound waves impinging on the arc will cause corresponding variations in the current through the arc. Thus if the single coil L and condenser C be replaced by two insulated but interwound coils, of which the secondary coil remains connected in the arc circuit, while the primary is connected to an ordinary telephone transmitter, any sounds—the notes of tuning forks, or even words—falling upon the latter will be forced upon the arc circuit and emitted as sounds of similar pitch and quality by the arc with increased vigour. The simple formula connecting the frequency of the " singing arc " with the constants of the simple shunt circuit suffices for general purposes but becomes untrue if large intensities and arc-lengths are employed.

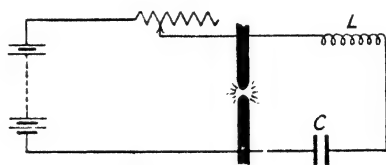



FIG. 91.—Singing Arc.

The singing arc, together with the electric spark (p. 15) and the Hartmann oscillator (p. 163) formed early sources of high frequency sound which could be made to function above the audible limit with suitable adjustment of the relevant factors. They

are now largely superseded by piezo-electric and magneto-strictive sources.

Piezo-electric Source of High Frequency. It was discovered by Haüy that a mechanical pressure exerted on certain crystals could give rise to a difference of electric potential in a perpendicular direction capable of giving rise to a current in a circuit connected to the faces of the crystal. A tension on the crystal reverses the direction of the current. The converse of this " piezo-electric effect " has been demonstrated, in particular if an alternating current be applied, rapid alternations of compression and distension occur in the two perpendicular directions. These forced vibrations will usually be insignificant unless their frequency coincides with one or other of the natural frequencies for longitudinal waves in the crystal, applying the word " longitudinal " to compressional waves in either of the two directions in the crystal, which are at right angles to the current.

The fundamental frequency for longitudinal waves of such a vibration is given by formula (36). When the crystal is of quartz, the velocity of such waves is found to be $5{\cdot}5 \times 10^5$ cm./sec., so that (36) becomes

$$n = \frac{1}{2l} \times 5{\cdot}5 \times 10^5,$$

that is, when each face is free to vibrate, and the centre of the crystal is a node. Thus, by cutting from a quartz crystal a section parallel to the optic axis, about 3 cm. deep, and mounting it between metal contacts or electrodes so that the faces are free to move, Cady [2] constructed a source of sound whose fundamental frequency was of the order of 100,000, having, of course, a series of overtones of even higher frequencies.

In use, this source is placed in the tuning circuit of a triode valve, in fact, it takes the place of the inductive circuit of Fig. 50. Pierce (*loc. cit.*, p. 252) used the circuit shown in Fig. 93, in which the quartz is self-excited and placed directly in the anode circuit of the valve. To get more power in the vibrations of the quartz, the resonant circuit shown in Fig. 92 may be used, employing two valves in " push-pull." The A.C. mains supply the two filaments through a transformer, the quartz is connected across the two grids, and the resonant circuit consists of the inductance L and the variable condenser C which lies across the anodes. To the midpoint of L is connected the high-tension battery through the milliammeters A_1 and A_2. It is convenient for the methods of measurement in fluids, described in the next section, to

have the normal reading of A_2 reduced to zero so that only changes in the amplitude of the oscillator are recorded. This is secured by a " backing battery " working through the 90 ohms resistance. By operating the switches the galvanometer G and its shunt may take over the duty of A_2, and give greater sensitivity to the measurements.

For laboratory or lecture demonstration purposes a slab of 100,000 cycles per second or a disc of 200,000 cycles per second is recommended. These can be obtained, in England, from the Quartz Crystal Co., New Malden. Osram PX 25 valves will be found effective, or American valves of similar characteristics. A coil of 200 turns wound on a former of 6 in. diameter and a variable condenser (up to 0·01 microfarad) will

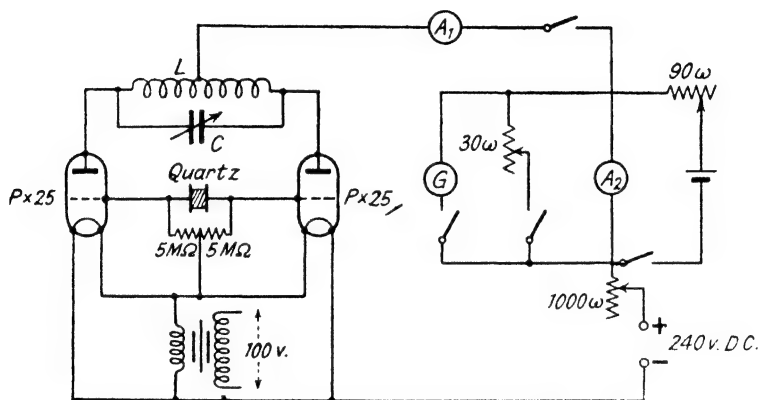



Fig. 92.—Ultrasonic Oscillator.

cover the range of frequency required. To exhibit the effects described in the final paragraph of this chapter, 250-watt valves for which the usual mains supply must be transformed up to 2,000 or 4,000 volts, and an oil-filled condenser will be a necessity.

Since piezo-electric oscillators are used as standards of high frequency, it is important to be able to locate their natural frequencies. If an acoustical method is used, the upper electrode is made as small as possible and central, the plate being sprinkled with lycopodium powder and the circuit varied until resonance and the consequent Chladni figures appear. Sometimes luminous appearances may be seen along the nodal lines. The frequency of the mode is then calculated from the known elasticity and density of quartz at the temperature in question. It is preferable, however, to find the natural frequency of the circuit in which the crystal is oscillating. For this purpose, a circuit containing a known inductance L and a variable

condenser is brought near the maintaining circuit and the capacity C of the latter varied until the two circuits are in tune. This is indicated by an alternating current galvanometer in the subsidiary circuit; then $n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ (cf. p. 222).

The quartz resonator under proper temperature control at once became a reliable standard of frequency and was adopted as such in stabilizing broadcasting frequencies and chronometers, competing in these spheres with the valve-maintained tuning fork. In stabilizing frequencies of audible pitch it is usual to exercise the control by synchronizing the ultrasonic oscillator with one of the high harmonics of the low-frequency vibrator. Dye [3] has given a very thorough examination, both theoretical and experimental, of the quartz resonator and its circuit, and has examined the modes of vibration of the crystal itself in a Michelson (light) interferometer. By adapting a thermionic valve oscillator, known as the "multivibrator," devised by Abraham and Bloch,[4] which is very rich in harmonics, Dye was able to produce a range of electric oscillations of sonic and ultrasonic frequencies, all governed by a standard tuning fork of 1000 cycles/sec. It is thus possible to have standard frequencies, all multiples of the fork frequency and only limited by its accuracy, up to one and a half megacycles per second. By obtaining beats between these and the quartz crystal frequencies, the latter can be measured with great accuracy.

Propagation of Ultrasonics in Gases. If a reflector (R, Fig. 93), e.g., a board, is placed at a distance opposite one face of the crystal, stationary waves will be set up in the space between if this is a multiple of half the wave-length. This condition will in fact be indicated by the response of the milliammeter (A) in the circuit, whose function is to indicate that portion of the valve current which is *direct*, for it is not able to follow the high-frequency oscillations. At positions such that the waves return in phase with those being sent out, the oscillations of the crystal will be encouraged and a minimum reading of A will be shown; if the contrary is the case, A will indicate a maximum since the alternating current is least. This apparatus, designed by Pierce,[5] accordingly serves as an interferometer for high-frequency waves, a series of maxima and minima being shown on A as the board is moved out, occurring at distances $\lambda/4$ apart. The radiation may be detected by its pressure on the reflector, particularly if the latter consists of a light disc at one end of a suspended lever (cf. p. 216).

Under certain conditions, the amplitude of the peaks may be used to measure the absorption in the medium, but a better way to do this is to follow Pielemeier [6] and use a quartz of identical frequency to the sender in place of the reflector and measure the reaction upon an oscillating circuit connected to the receiving quartz, as the separation between the two is varied.

The system quartz plus fluid column plus reflector must be considered as a complex impedance whose value passes through maxima and minima as the length l of the column is varied. In the Pierce interferometer the ratio of the anode current at any value to the maximum is measured. In gases this current keeps nearly constant except at multiples of $\lambda/2$ equal to l when there occurs a sharp peak, but in a liquid, where the specific impedance of the fluid is comparable

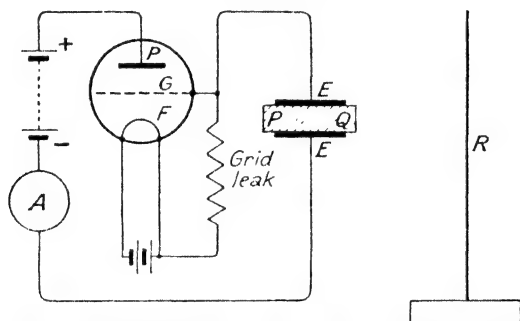



Fig. 93.—Piezo-electric Quartz Resonator (*Pierce*).

with those of its terminations, the current ratio remains close to unity except at locations where l is an odd multiple of $\lambda/4$.

Most of the detectors described in Chapters VII and IX may be used with equal facility in the ultrasonic region provided their bulk is small; torsion discs, resistance thermometers [7] and hot wires,[8] are particularly suitable. Brandt and Freund [9] showed that dust figures could be set up in the stationary wave-system between quartz and reflector, exhibiting the Kundt's tube phenomena on a small scale. In view of the small wave-length of the radiation relative to the aperture (formed by the electrodes) through which the energy is usually radiated, the confining tube of the low-frequency experiments is superfluous as the waves travel ahead in a narrow beam, as in the analogous system of a beam of light shining through a hole. In fact, many of the features of light propagation, such as diffraction, interference, passage through gratings, etc., may be illustrated by the

behaviour of radiation from a quartz oscillator of sufficiently high frequency.

To obtain simultaneous values of both the absorption coefficient and velocity in a fluid medium it is possible to keep the source and reflector in fixed relative positions, in order not to disturb the oscillations of the former, and to traverse a hot-wire detector through the intervening space. In this way the stationary wave-system will be traced out.

The absorption is measured in terms of an absorption coefficient α defined by $I = I_0 e^{-\alpha x}$, where I_0 is the initial amplitude (at the quartz face) and I that remaining after a distance x has been traversed. This will affect the stationary waves, and the equations will need modification for this decay of amplitude. Let us suppose the progressive and retrogressive waves of a dispersive gas are given by the expression

$$y = Be^{-\alpha x}e^{i(\omega t - \beta x)} + Ce^{\alpha x}e^{i(\omega t + \beta x)}$$

Since $y = O$ at $x = l$, viz., at the reflector,

$$O = Be^{-(\alpha + i\beta)l} + Ce^{(\alpha + i\beta)l}$$

Therefore $y = Ce^{(\alpha + i\beta)l}\left[e^{(\alpha + i\beta)(x-l)} - e^{-(\alpha + i\beta)(x-l)}\right]e^{i\omega}$

$$= 2Ce^{(\alpha + i\beta)l}\sinh\{(\alpha + i\beta)(x - l)\}\,e^{i\omega t}.$$

Whence (putting $A = Ce^{\alpha l}$ and omitting the time factor $e^{2i\omega t}$)

$$y^2 = 2A^2[\cosh\{2\alpha(x - l)\} - \cos\{2\beta(x - l)\}] \quad . \quad . \quad (93)$$

As α is usually small compared to β, the maximum and minimum values of y as x varies are given by $2A\cosh\{\alpha(x - l)\}$ and $2A\sinh\{\alpha(x - l)\}$, so that by tracing out the peaks and troughs in the pseudo-stationary waves, both α and β can be determined.

Propagation in Gases. Measurements in dry air have shown but slight differences between the velocity of propagation at ultrasonic frequencies and that normal to the audible range, except close to the source, when the enhanced velocity is apparently due to an amplitude effect. Hitchcock [10] found that he could vary the velocity in this region by altering the manner of excitation of the crystal. At high frequencies of the order of 200,000 cycles/sec., anomalies are found in certain gases of which carbon dioxide is the most noteworthy. Pierce himself had noticed that his interferometer gave diminishing response as the reflector was moved back in this gas. The experiments have been repeated by a number of investigators. Change of velocity with frequency, i.e., dispersion, is shown by these results (see Fig. 94) as well as absorption.

The principal gases in which this excessive absorption is observed are carbon dioxide—at 10^5 cycles/sec. it amounts to 40 times that which the classical theory, based on dissipation due to viscosity and heat conduction, would indicate—nitrous oxide and sulphur dioxide; but dispersion of the velocity is also ascribed to carbon monoxide, on which very careful measurements at two frequencies have been made by Sherratt and Griffiths [11] and to ammonia by Steil.[12]

The dispersion curve—variation of velocity with frequency—

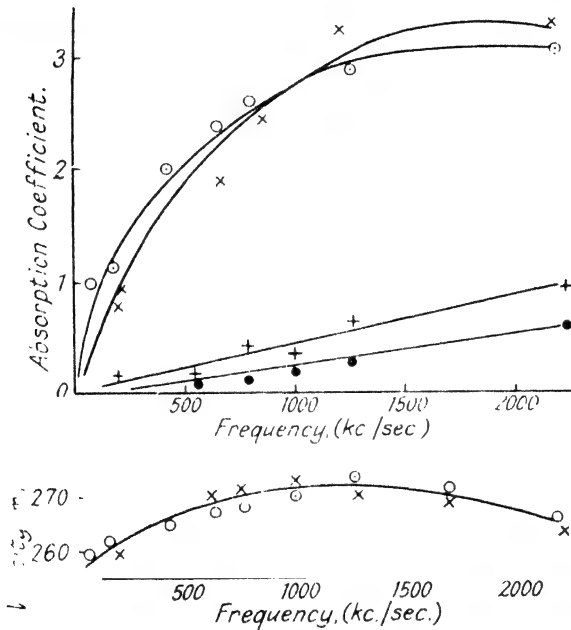



Fig. 94.—Ultrasonic Dispersion in Gases (● O_2, + SO_2, × N_2O, ⊙ CO_2).

shows often a slight dip below the normal value followed by a more considerable rise and, occasionally, another slight dip as the frequency goes to still higher values. It does not, however, return to normal, at least, within the range of frequency attainable. It must be borne in mind that the accuracy with which the wave-lengths can be measured decreases as they get smaller. It is important to know how pressure and temperature affect the dispersion curve. Richards and Reid [13] first obtained some data for the three first-mentioned gases, and more comprehensive data, enabling the complete curves to be plotted, were obtained by Railston and Richardson [14] for the pressure effect

and by Penman [15] and by Warner [16] for the temperature effect. If the velocity (reduced to 0° C.) is plotted against frequency divided by pressure (Fig. 95*a*) the results for carbon dioxide fall nicely on a single curve. Plotting the other results against the quotient of frequency and temperature is not so successful (Fig. 95*b*), though the general trend of the data can be outlined by a single curve; moreover, the crux of the velocity rise occurs at a value which increases in linear

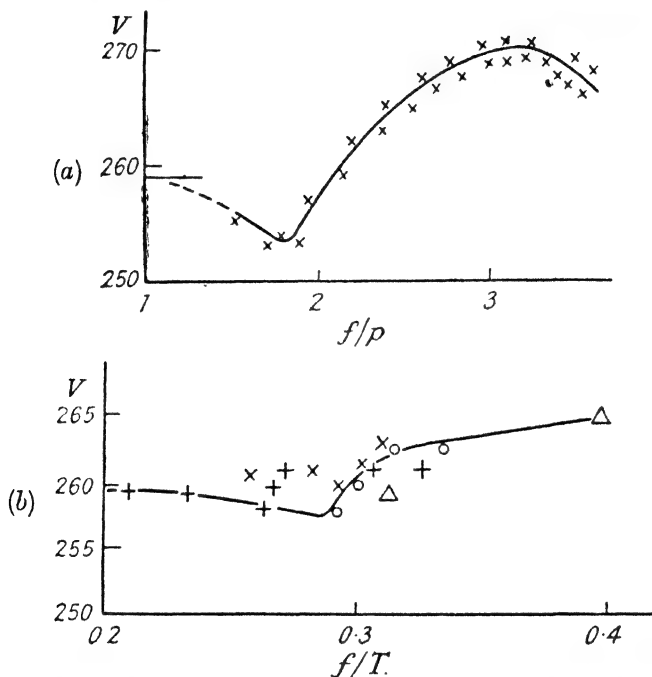



FIG. 95.—Effect of (*a*) Pressure and (*b*) Temperature on Ultrasonic Dispersion.

fashion. (It should be reiterated that this change of velocity has nothing to do with the change of density of the gas, which has been allowed for by the reduction of the values to 0° C. in accordance with the usual formula (6)).

Other instances of excessive absorption occur when a gas, which in the pure state behaves normally, has a small admixture of another gas or vapour. The most notable case is that of air containing water vapour. Knudsen [17] noticed that, even at audible frequencies, moist air absorbed sound to a much greater extent than dry air (cf. p. 319).

It has since been observed in the ultrasonic region, while Pielemeier [18] and Mokhtar and Richardson [19] have found dispersion of the velocity in moist air. There is usually for each gas or mixture of gases one value of the humidity for which the radiation suffers its maximum of absorption and of velocity.

Peculiar effects appear in the neighbourhood of the "critical point" at which a gas passes when compressed into a liquid in a continuous fashion without noticeable division into two phases, gas above and liquid below. Schneider [20] and also Parbrook and Richardson [21] have observed a large absorption at the critical point itself presumably due to abnormal scattering by the striations which Andrews originally observed in a gas under these special conditions.

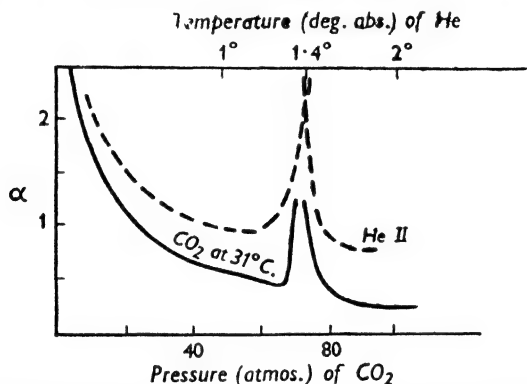



Fig. 96.—Comparative Absorption in Carbon Dioxide near critical point and in helium near transition point.

Of common fluids, carbon dioxide and ethylene lend themselves most readily to such investigations, their critical points being at 31° C. under 76 atmospheres and 10° C. under 50 atmospheres respectively. The velocity passes through a minimum at the pressure at which condensation occurs as Herget [22] first showed.

The absorption coefficient in the gas steadily decreases with increase of pressure as it should do according to equation (97), p. 261, but its actual value near the condensation point is some thousand times that which one would derive from this formula, if one inserted the ordinary kinematic viscosity derived from steady flow experiments (cf. Fig. 96).

Theories of Absorption and Dispersion. This phenomenon has been explained by Herzfeld and Rice [23] in terms of a delay in the change of translational energy into vibrational energy of the molecules

as the wave is propagated. When the time period of vibration becomes comparable with this "relaxation time" or "mean life of a sound quantum" the molecules become stiffer to the vibration (to use the expressive phrase of Hubbard) with a consequent absorption of energy and rise in γ, the ratio of specific heats. Probably the simplest mathematical treatment is that of Henry [24] as follows :—

Let E_x be the actual energy and E_T the energy which the molecules would have if at equilibrium at temp. T, and suppose that

$$\frac{dE_x}{dt} = \frac{1}{\beta}(E_T - E_x) \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

where β is the period of relaxation of the vibrational energy. If the gas is subject to adiabatic variations of temperature of frequency $\omega/2\pi$

$$T = T_0 + T_1 e^{i\omega t}$$

$$E_T = E_0 + C_\omega T_1 e^{i\omega t}$$

C_ω being the specific heat of vibrations. Substituting in (94) and solving, we find :—

$$E_x = E_0 + C_\omega T_1 (1 + i\omega\beta)^{-1} e^{i\omega t}$$

If the total specific heat be regarded as made up of two parts, C_ω due to vibration and C_1 due to translation (+ rotation)

$$C' = C_1 + C_\omega (1 + i\omega\beta)^{-1}$$

where C' is complex; also, as usual, $\gamma' = 1 + R/C'$, (R = gas constant), and the velocity of plane waves V is $\sqrt{\gamma' p/\rho}$.

Hence $$\gamma' = 1 + R[C_1 + C_\omega/(1 + i\omega\beta)]^{-1}$$

and is also complex.

To find the real part we write :—

$$\frac{1}{C_1 + C_\omega/(1 + i\omega\beta)} = \frac{1 + i\omega\beta}{C_\omega + C_1 + i\omega\beta C_1}$$

of which the real part is

$$\frac{C_0 + \omega^2\beta^2 C_1}{C_0^2 + \omega^2\beta^2 C_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

where $C_0 = C_\omega + C_1$, i.e., the total specific heat when $\omega \rightarrow 0$. Thus the usual specific heat at constant volume in the expression for the velocity of sound is replaced by the reciprocal of (95). When ω approaches the value $C_0/(C_1\beta)$ a rise in velocity occurs from the normal;

$$V_0^2 = (1 + R/C_0) . p/\rho$$

to an ultimate value $V_\infty^2 = (1 + R/C_1) . p/\rho$.

There is also absorption in the critical region due to a change of relative phase between pressure and condensation in the wave. From observations such as those of Fig. 94 it is then possible to predict the relaxation time β. Thus for carbon dioxide at N.T.P. $\beta = 10^{-5}$ sec., for nitrous oxide $\beta = 10^{-6}$ sec.

It will be noted that the theory as set out demands that the velocity should remain at its low-frequency level until the critical frequency is approached, when it should rise fairly steeply to its ultimate level and stay there.

Railston [25] has tested organic vapours. The work is difficult, for condensation tends to take place on the quartz and impede its oscillations. Though a number show abnormal absorption, dispersion of velocity has only been confirmed in carbon disulphide and benzene vapours.

Anomalous Dispersion.—Another type of dispersion is found when any type of radiation encounters a system which contains a set of resonators tuned to a common frequency, or—what amounts to nearly the same thing in acoustics—a set of obstacles spaced at wave-length distances. Such a phenomenon is accompanied by scattering of the radiation, even when the obstacles are much smaller than the wave-length of the radiation as when ultrasonics pass through a cloud of dust particles, and this scattering of the radiation causes a diminution of the intensity which may be likened to absorption. When, in addition, the size of the obstacles approaches the wave-length of the radiation, there is a characteristic see-saw in the velocity which is more familiar in light than in sound. This anomalous dispersion was first shown for sound by Belikov.[26] With ultresonics it can be demonstrated by passing a beam of the radiation from sources of various but near-by frequencies through a lattice made up of equidistant and parallel wire nails.

Interferometer Effects on Absorption. Now we turn to the tube effects on absorption, distinguishing between interferometers which are " wide " and " narrow." When the tube is effectively wide as in the above-mentioned researches the danger is of the formation of radial oscillations (cross-modes) like those well-known in the practice of wave guides. When the source and reflector have been set very carefully parallel, satellite peaks may yet be found alongside the main ones as the reflector path is increased or decreased. The existence of these is a sure proof of radial oscillation and, if they cannot be removed by changing the tube width or by lagging, will give rise to a

"spurious" absorption. Bell [27] has demonstrated this and he has also shown how one may deduce the true gas absorption by reducing the pressure of the gas.

Turning to the other extreme of interferometers formed in narrow tubes, there is no question of cross-mode interference save at impossibly high frequencies, but the two terms of the Kirchhoff equation now become equally important. Following on some earlier work started by May,[28] Lawley [29] has constructed an interferometer in which the source is a nickel needle in magneto-strictive oscillation feeding into glass tubes of diameters down to 0·3 mm. and the movable reflector is a similar rod of brass.

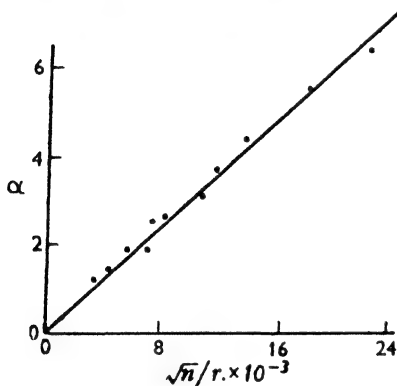



Fig. 97.—Absorption in Air in Narrow Tubes (*Lawley*).

Lawley has enclosed such an apparatus in a chamber in which the pressure can be reduced. In accordance with the fundamental equation (p. 171) he plots his absorption coefficients against $\sqrt{n}/r$. As the equation shows, α increases in linear fashion with this parameter and the straight line, Fig. 97 (which covers a number of frequencies and radii in air) extrapolates to the origin. The constant C_1 in the equation, however, derived from these data in oxygen, nitrogen and hydrogen varies between 4 to 8 per cent. above the theoretical value.

In the last few years, progress has been made in the use of pulse methods for absorption in which the source is actuated to send out a short train of waves, is directed at an obstacle and the reflected train caught on the sender on the same principle as the sonic apparatus already described (p. 204).

This, first used for ultrasonics in liquids by Biquard and Ahier,[30] has been much used by Pinkerton [31] and by Pellam and Galt [32] and their colleagues.

Attenuation of Plane Waves by Viscosity. From the Navier-Stokes equations of the dynamics of a viscous fluid, together with an equation of continuity,* we can derive the equation for the propaga-

* See, for example, the author's *Dynamics of Real Fluids*, p. 4.

tion of a plane wave in terms of the particle velocity $\dot{\xi}$, the pressure gradient $\partial p/\partial x$ and the kinematic viscosity ν:—

$$\ddot{\xi} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{4}{3}\nu\frac{\partial^2 u}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

This together with the usual expression relating the excess pressure and the condensation to the velocity of sound (c) transforms to

$$\frac{\partial^2\dot{\xi}}{\partial t^2} = c^2\frac{\partial^2\dot{\xi}}{\partial x^2} + \frac{4}{3}\nu\frac{\partial^3\dot{\xi}}{\partial x^2\partial t}.$$

Assuming a solution of the type

$$\dot{\xi} = \dot{\xi}_0 e^{i(\omega t - \beta x)} e^{\alpha x}$$

we derive

$$\alpha^2(c^2 + \tfrac{4}{3}i\nu\omega) + \omega^2 = 0,$$

or, neglecting the square of $\nu\omega/c^2$,

$$\alpha = -\frac{i\omega}{c} - \frac{3}{2}\frac{\nu\omega^2}{c^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

Taking the real part, the absorption coefficient is then $\frac{3}{2}\frac{\nu\omega^2}{c^3}$ or $\frac{6\pi^2\nu}{\lambda^2 c}$; but a correction may be added for the diffusion of heat by the fluid during the passage of sound waves. This amounts to an increase of ν (Kirchhoff).[33] The important fact is that, in either case, $\alpha\lambda^2$ should be constant in a gas, in so far as c is independent of frequency.

Propagation of Ultrasonics in Liquids. These studies were initiated by Boyle [34] and his collaborators in Canada, using the Pierce method and torsion vanes as well as dust figures. Hopwood [35] has exhibited a number of reflection and diffraction effects on a small scale by this method. Naturally, considerably more energy has to be put into the source than in the corresponding case of a gas, and considerable dissipation of energy as heat may occur, but no dispersion such as that described in the section on gases has yet been attested. In using the Pierce method for liquids the physical properties of the reflector have to be seriously considered. It was hoped by Boyle that it would be possible to detect an iceberg in sea water by the reflection of ultrasonic energy sent out beneath the sea from a ship. The reason why this method must fail is an interesting result of the theory we have given on p. 233. Sea water and ice have, in fact, nearly the same

characteristic impedance although water and steel are sufficiently different in properties to make it possible to detect the steel hull of another ship, in time of fog, by the use of a submarine ultrasonic beam. In the table the approximate velocities of sound, densities, characteristic impedances and in the final column the reflection coefficients: water-ice, water-steel, are set out.

	c	ρ	ρc	r
Water	$1 \cdot 4 \times 10^5$	1	$1 \cdot 4 \times 10^5$	—
Ice	1 ,,	0·9	0·9 ,,	0·03
Steel	5 ,,	8	40 ,,	0·93

Another factor which has a bearing on the reflection is the thickness of the reflector. Boyle and Froman [36] have found that the ratio of incident to reflected energy is least when the thickness equals an even multiple, and is greatest when it is an odd multiple of a quarter wave-length for the ultrasonic radiation in the material, an experimental fact which is readily confirmed by theory.

About a decade ago Brillouin predicted that if a liquid were penetrated by *progressive* waves of compression of short wave-length at the same time as it was irradiated by light, there would be diffraction of the light by the regular pattern of density variations in the liquid in analogous fashion to the diffraction of X-rays by a crystal in the familiar Bragg experiment. Thus in Fig. 98 suppose that a sinusodial ultrasonic wave is moving from left to right in the direction XX' giving at the instant pictured maxima of compression distant d apart at AA', with rarefactions at BB', and let light of wave-length λ be incident at an angle i to the direction of propagation and be reflected along paths such as SAR, and let $\theta = \pi/2 - i$ be the angle between the perpendicular $A'N$ from A' on to AR and the direction of propagation. Then the path difference $= 2AN = 2d \sin \theta$ and the expression for equality of phase in the light reflected along AR, $A'R'$ is, as in the Bragg experiment, $j\lambda = 2d \sin \theta$ with the restriction that for a sinusoidal wave $j = \pm l$, instead of any integral value. The condition for a maximum in the diffracted light is therefore $\sin \theta = \pm \lambda/2l$, l being the wave-length of the sound. If then a beam of light cross the path of the ultrasonic beam, we should see on the far side the central image, with two diffraction maxima, one on each side. Notice that l must be small to produce a reasonable separation,

θ. If $\lambda = 0{\cdot}6\mu$* and $l = 10\mu$, $\theta = 2°$. It should also be noted that the theory is independent of the motion of the layers A, A', as long as they conserve their separation. We are not here dealing with stationary waves. The necessary experiments have recently been undertaken by Debye and Sears [37] on the one hand and Lucas and Biquard [38] on the other, using light passing from a linear slit, through a trough of liquid and out to a photographic plate. The simplest pattern observed is made up of a central undeviated image of the slit from which the light comes and a diffracted line on either side; but others comprise five or more images. In fact, the above theory is too

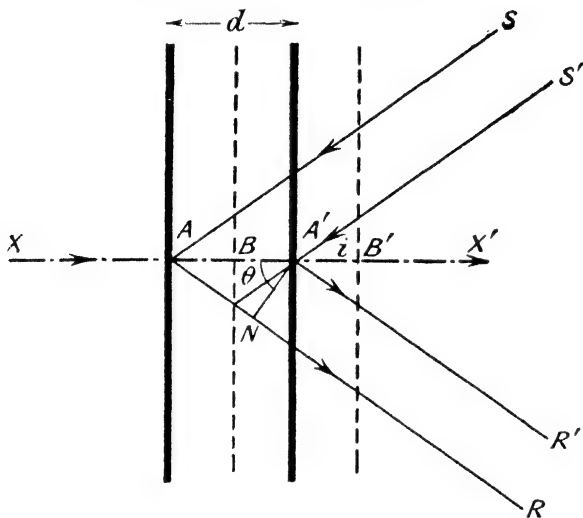



FIG. 98.—Diffraction of Light by Ultrasonic Waves.

simple to explain all the complexities of the phenomenon, though it suffices for the fundamentals. Raman and Nath [39] have given a more precise theory, but this is of interest to the student of optics and does not concern the applications of the discovery with which we are now dealing.

When stationary waves are formed between a quartz oscillator and a reflector in a liquid medium the compressions which act as diffracting centres to the light are, of course, immobile, and rather simpler apparatus can be set up to show their locations. A parallel beam of light of aperture sufficient to comprise a number of ultrasonic half-wave-lengths is cast athwart the trough containing the

* 1μ (micron) $= 10^{-3}$ mm.

liquid and made to fall on a viewing screen on the other side, whereon the nodal planes appear as shadows whose separation can be read off with a travelling microscope. The experiment is essentially of the same nature as the Schlieren method of Toepler (p. 16), and has been greatly developed by Hiedemann [40] and his collaborators, one of whom, Seifen,[41] has recently brought the technique to a high state of precision for ultrasonic velocity measurements. They have also used it on progressive waves, interrupting the (polarized) light from the lamp at the same frequency as the ultrasonic source by controlling a Kerr cell from some of the electric potential developed in the driving circuit. The Kerr cell interrupts the light at the same frequency, thus acting as an optical stroboscope.

An even simpler apparatus, suitable for a lecture demonstration, has been set up by Fox and Rock.[42] No lenses or slits are used, but the light source is a straight discharge tube containing mercury vapour from which the light passes athwart an ultrasonic oscillator which modulates the source with its own frequency, then through the liquid contained in a trough which is irradiated at the same ultrasonic frequency. A screen placed on the other side then receives shadows of the compressions in the progressive waves, which are "held" in position by the interrupted light of the same frequency. Thus the optical system is just like that which is used to measure the wave-length of low frequency ripples on a liquid, in which experiment the tuning-fork which originates the ripples by dipping into the surface also interrupts synchronously the light by which they are viewed, giving the ripples a stationary appearance. A feature of this optical system is its simplicity, which makes adjustment for plane-parallel light unnecessary and allows of the calculation of the wave-length in terms of geometrical constants which can be measured accurately. It is necessary to eliminate standing waves in the trough (just as in the ripple experiment). This is done by making the radiation pass into a chamber at the end of the trough in which the energy is frittered away by repeated reflection between walls covered by gravel.

In place of a slit, Bär and Meyer [43] use an opaque screen studded with holes giving a large number of light sources. Their plates have somewhat the appearance of Laue X-ray photographs, the dots being drawn out where the light has passed through the irradiated liquid. Since the intensity of the diffracted light is proportional to the intensity of the ultrasonics at any point it is possible to estimate the absorption suffered by the latter in the liquid. Bär and Meyer have also verified the wave-length of the ultrasonics by letting them fall on a wire grating, set in a plane parallel to the wave front. This causes diffraction of the ultrasonics in the liquid, changing their direction through the

angle ϕ. There remains, of course, still the direct radiation. The light from the section of the spotted source which passes through the diffracted beam *alone* then will have each spot drawn out in the same direction, viz., at an angle ϕ to the original beam. Measuring this angle and knowing the spacing of the wire grating they were able to calculate the ultrasonic wave-length, independently of a knowledge of the light wave-length.

All liquids show a greater absorption than that which theory would indicate (effects of viscosity and heat conduction alone), but some show an enormous absorption (e.g., benzene and chloroform), which recalls the behaviour of carbon dioxide and nitrous oxide gases. It is indeed striking that those substances which behave abnormally to ultrasonic radiation are just those which scatter light most readily, which leads one to speculate that there must be some connection between the mechanisms concerned. Possibly the common factor is the tendency to aggregation. Kneser [44] has shown that the attenuation is much greater than possible relaxations could explain.

On the basis of the absorption measurements of Biquard [45]—who, incidentally, considers that the radiation is "scattered" to a large extent—Lucas [46] envisages a liquid as being a medium always in the throes of casual fluctuations in density, caused by a series of thermal waves which ricochet from one boundary to another. The interactions of the ultrasonics with such a "speckled" medium—to use a homely phrase—cause a diversion from the straight path and diminution of amplitude at increasing distances from the source. The mechanism propounded is similar to that by which light is scattered from a fog or colloidal suspension.

The question of whether dispersion of the velocity takes place in liquids is doubtful. Certainly, nothing like the rise of velocity which occurs in carbon dioxide has been observed in any liquid. Parthasarathy [47] has examined a large number of organic liquids. Bär [48] has pushed the frequency up to the enormous value of 8×10^7 cycles/sec., but cannot detect dispersion in water, within the limits of experimental error, which mount up at such short wave-lengths (about 0·02 mm.). A change of velocity of the order of 1 per cent. is suspected by other workers in acetic acid, acetone, benzene and toluene.[49] Biquard,[50] too, has shown that the velocity rises in linear fashion as the pressure on the liquid is increased. We must await refinements in the technique of measurement of ultrasonic wave-lengths before these and similar results can be confirmed.

Propagation in Liquid Helium II. This is the place to introduce a subject which, though it does not necessarily involve ultrasonics, is as to its development closely connected with the matters we have been discussing. When helium is liquified it forms two phases I and II, the transition from one to the other occurring at what is called the lambda-point, 1·4° absolute. One feature of the propagation of sound waves in liquid helium recalls closely that in carbon dioxide at the critical point. Fig. 96 shows the similarity in the trend of the absorption coefficient at the two transitions, using the data of Schneider,[51] Parbrook and Richardson [52] for CO_2 and Atkins and Chase [53] for helium.

Evidently from the acoustic point of view the changes are similar.

Helium II, however, shows another unusual feature. If thermal waves are sent into it from a metallic strip heated by alternating current (a thermophone, p. 118) these are not absorbed in a few millimetres as in a normal fluid but can penetrate far enough for stationary waves to be set up between the source and a suitably placed reflector. These waves must be detected by a resistance thermometer (p. 186) and measurements show that the velocity of such thermal waves is of the order of 20 m./sec. To a first approximation, the speed of waves of frequency n in a medium of thermal diffusivity (ratio of thermal conductivity to specific heat of unit volume) k is $\sqrt{(4\pi kn)}$ and the damping coefficient $\sqrt{(\pi n/k)}$. In a gas such as air, such waves are damped out of existence in a few millimetres but in a fluid of supraconductivity such as liquid He II the thermal waves should be appreciable to a much greater distance, and the true sound waves might be of less significance as local expansions and contractions of the fluid would be more difficult to set up ; moreover, the latter would be of isothermal type. Peshkov [54] has made experiments with such a source in liquid helium near the λ-point using a resistance thermometer to plot out the thermal waves. His results indicate for the liquid a thermal diffusivity of order 10^3. He himself, following Landau [55] ascribes this radiation to a special type of sound propagation called " the second velocity of sound in He II," associated with its abnormal fluidity.

Miscellaneous Effects of Ultrasonic Waves. Besides purely acoustical measurements with ultrasonic waves, other experiments have been performed of a rather spectacular nature. Wood and Loomis [56] fed 2 kilowatts of electric power to an oscillator under oil, which allows of much greater power being employed without breakdown of the insulation. The pressure at the surface of the oil was

so great as to cause the oil to rise in water-spout form, emulsifying as it did so. The hydrostatic pressure is sufficient to drive all air out of the solution in the form of bubbles rising from nodal planes. Even if there is no dissolved air, cavitation [57] in the form of bubbles may occur in which the liquid is apparently evaporated by the intensity of the rapidly alternating forces. A catalytic effect on chemical reactions has been noted, reactions taking place at lower temperatures, when irradiated by intense ultrasonics, than under ordinary conditions; and Szalay [58] has been able to break down certain polymeric organic molecules into less complex ones by irradiation of aqueous solutions by ultrasonics.

The waves also possess marked biological action in which the immediate cause seems to be the intense energy dissipation in the track of the radiation. Their disruptive actions on a number of living specimens, plants, vaccines, suspensions of bacteria and of blood corpuscles have been examined by Wood and Loomis, Hopwood [59] and others, and in every case their action is to destroy living matter. An industrial application of this process is the sterilization of milk by ultrasonic energy.

References

1. *Electrician*, **46**, 269, 310 and 356, 1900.
2. *J. Opt. Soc.*, **10**, 475, 1925.
3. *Proc. Roy. Soc.*, *A*, **138**, 1, 1932.
4. *Comptes Rendus*, **168**, 1105, 1919.
5. *Proc. Amer. Acad. Sci.*, **60**, 271, 1925; **39**, 300, 1927.
6. *Phys. Rev.*, **34**, 1184, 1929; **36**, 1005, 1930; **38**, 1236, 1931; **41**, 833, 1932. *See also* Groszmann, *Ann. Physik*, **13**, 681, 1932; *Phys. Zeits.*, **35**, 83, 1934.
7. Malov, *Hochfreq.*, **42**, 115, 1933.
8. Richardson, *Proc. Roy. Soc.*, *A*, **146**, 56, 1934.
9. *Zeits. Phys.*, **92**, 385, 1934; **94**, 348, 1934. *See also* Pearson, *Proc. Phys. Soc.*, **47**, 136, 1935; Parker, *ibid.*, **49**, 95, 1937.
10. *Proc. Inst. Rad. Eng.*, **15**, 906, 1927.
11. *Proc. Roy. Soc.*, *A*, **147**, 292, 1934.
12. *Zeits. phys. Chem.*, **31**, 343, 1936.
13. *J. Chem. Phys.*, **2**, 193, 1934.
14. *Proc. Phys. Soc.*, **47**, 533, 1935.
15. *Ibid.*, **47**, 543, 1935.
16. *J. Acoust. Soc.*, **9**, 30, 1937.
17. *Ibid.*, **3**, 126, 1931, and **5**, 112, 1933; Kneser, *ibid.*, 122; Rogers, *Phys. Rev.*, **45**, 208, 1934.
18. *J. Acoust. Soc.*, **10**, 87 and 313, 1939.
19. *Proc. Roy. Soc.*, *A*, **184**, 117, 1945.
20. *Canad. J. Chem.*, **29**, 243, 1951.
21. *Proc. Phys. Soc.*, **65B**, 437, 1952.

22. *J. Chem. Phys.*, **11**, 37, 1940.
23. *Phys. Rev.*, **31**, 691, 1928. *See also* KNESER, *Ann. Physik*, **11**, 761, 1931; and HEIL, *Zeits. Phys.*, **74**, 31, 1932.
24. *Proc. Camb. Phil. Soc.*, **28**, 249, 1932.
25. *J. Acoust. Soc.*, **11**, 107, 1939.
26. *Zeits. Phys.*, **39**, 233, 1926.
27. *Proc. Phys. Soc.*, **63B**, 958, 1950.
28. *Ibid.*, **50**, 553, 1938.
29. *Ibid.*, **65B**, 181, 1952.
30. *Cahiers de Phys.*, **15**, 21, 1945.
31. *Proc. Phys. Soc.*, **62**, 286, 1949.
32. *J. Chem. Phys.*, **14**, 608, 1946.
33. *Ann. Physik*, **134**, 177, 1886.
34. *Several papers in Trans. Roy. Soc. Canada*, 1925–32.
35. *J. Sci. Instrum.*, **6**, 34, 1929.
36. *Canad. J. Res.*, **1**, 405, 1929, and **2**, 3, 1930.
37. *Proc. Nat. Acad. Sci.*, **18**, 409, 1932; *Comptes Rendus*, **198**, 237, 1934.
38. *Comptes Rendus*, **194**, 2132, 1932; **195**, 121, 1932; **196**, 257, 1933; **197**, 309, 1933; *J. Physique*, **3**, 464, 1932.
39. *Proc. Ind. Assoc. Sci.*, **2**, 406, 1935; **3**, 75 and 119, 1936.
40. *Zeits. Phys.*, **87**, 442, 1934, *et passim*, 1935–8.
41. *Ibid.*, **108**, 681, 1938.
42. *Rev. Sci. Instrum.*, **10**, 349, 1939.
43. *Phys. Zeits.*, **34**, 393, 1933.
44. *Ann. Physik*, **32**, 277, 1938.
45. *Ann. Physique*, **6**, 195, 1936. *See also* RICHARDSON, *Proc. Phys. Soc.*, **52**, 480, 1940.
46. *J. Physique*, **8**, 41, 1937.
47. *Proc. Ind. Acad. Sci.*, **2**, 497, 1935; **3**, 285, 482 and 519, 1936.
48. *Helv. Phys. Acta*, **10**, 332, 1937; **11**, 472, 1938.
49. SPAKOVSKY, *Acad. Sci. U.S.S.R.*, **18**, 169, 1938; DUTTA, *Phys. Zeits.*, **39**, 186, 1938; RAO, *Proc. Ind. Acad. Sci.*, **7**, 163, 1938.
50. *Comptes Rendus*, **205**, 897, 1938.
51. *Canad. J. Chem.*, **29**, 243, 1951.
52. *Proc. Phys. Soc.*, *B*, **65**, 437, 1952.
53. *Proc. Phys. Soc.*, *A*, **64**, 826, 1951.
54. *J. Phys. U.S.S.R.*, **8**, 381, 1944.
55. *Ibid.*, **5**, 71, 1941; **8**, 1, 1944.
56. *Phil. Mag.*, **4**, 417, 1927.
57. BOYLE, TAYLOR *et al.*, *Trans. Roy. Soc. Canada*, **23**, 91 and 187, 1929.
58. *Zeits. Phys. Chem.*, **164**, 234, 1933; *Phys. Zeits.*, **35**, 293, 1934.
59. *J. Sci. Instrum.*, **6**, 34, 1929.

CHAPTER ELEVEN

SUBJECTIVE SOUND

The Voice. The human organs engaged in the production of sound consist of (1) the lungs which by their expansion and contraction provide a blast of air, delivered through the miscalled (2) vocal cords in the larynx at which the vibration originates, (3) the cavities formed by the larynx itself, mouth, nose, and sinuses behind the forehead, which function as resonators to the sound produced in the larynx, and of which two at any rate, the mouth and larynx, are capable of adjustment or tuning at will. This system has often been compared to a reed instrument, in which the lungs form the bellows, the vocal cords the reed in its tube, the head cavities make up the resonant column of air; but the analogy fails in likening the vocal cords to a reed stretched across the larynx.

This organ consists of two flat membranous bands stretched across the larynx, spanned by two muscles which are able to close the slit which forms the air passage, and at the same time to put tension on the bands. The idea that these bands vibrate transversely in the fashion of the cords of a stringed instrument to produce the voice, is as old as Galen, but the range of tension and thickness actually available, even allowing for possible partial vibrations, seems too small to form, in accordance with the expression for the natural tones of strings, proper vibrations covering the two octaves over which the average voice ranges. It seems preferable to regard the sound as being engendered by a sort of jet tone (p. 153), in which however the membranous sides of the slit themselves take part, like the lips of a player on a brass instrument. By means of the laryngoscope invented by the famous singer Manuel Garcia [1]—a small mirror by which the cords can be viewed in the glottis while the patient is singing—it can be demonstrated that not only is the tension varied, but that the width of the slit between them is altered as the frequency of the note or the velocity of expulsion of the breath is changed, agreeing with our formula (55), p. 148. Records of the vibrating cords by stroboscopic methods,[2] or by the painful process (for the singer) of a recording instrument in the glottis,[3] show that these membranes do indeed execute a S.H.M., while the note emitted from the mouth is complex.

The reed-pipe analogy led scientists to ascribe the production of sound, or of the pitch characterizing a sung vowel, to the vocal cords, while to the neighbouring air-cavities, mouth, nose, throat, was ascribed the function of modifying the quality of the note to form the different vowel sounds. Apart from these modifications, there are two varieties of larynx tone production. In the lower part of a singer's range of pitch the "chest voice" is employed. The laryngoscope shows that the cords form a long fine slit by their juxtaposition in this range, while the walls of the upper part of the chest seem to be set in forced vibration—hence the name. The upper notes are produced by the "head voice," in which a part of the slit is completely closed by the small tensioning bones, which strain the vocal cords to a greater extent than in the chest voice. Between these two "registers" there is a break in the voice, in disguising which the art of the virtuoso has to be exercised. A rapid alternation between the registers on a note in their limited overlapping region, results in that peculiar sound known as "yodelling." In whispering, the vocal cords do not take part in the vibration—anyone may test this, by holding his "Adam's apple"—the mouth resonator is feebly excited merely by blowing breath into it through the narrow orifice of the throat.

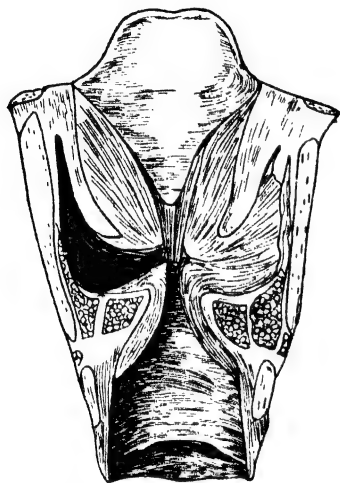



Fig. 99.—The Larynx.

Vowels. Granted that vowels are characterized by modification of the size and shape of the mouth and throat, we have yet to consider to what extent the vibration of the air in these cavities is coupled to the larynx-notes which we have just been discussing. One theory, the oldest and best borne out by experiment, ascribes certain fixed simple tones, characteristic of each vowel, to the complex note produced by singing the vowel to any pitch, and goes back to Willis,[4] 1832. This is what one would expect if the tonal modifications have their origin in the mouth which is set to a more or less definite position for each vowel. The alternative idea, that the quality of vowels

depends on the *relative* number and intensity of the overtones to each fundamental larynx-note has fallen into disrepute.

The Formant. It was Helmholtz [5] who suggested after extensive studies of vocal sounds that specific vowels were characterized by a specific set of resonances, which Hermann [6] called "formants." A vowel sound sung at a different pitch therefore had all or some of these resonances in common, corresponding to the injection into the vocal resonators of a fundamental tone of varying pitch. The theory in its original application is an over-simplification since a singer does not maintain his vocal resonators in the same form throughout the gamut, as he sings, but the theory is obviously apposite to many musical instruments whose function is to produce a large number of distinct tones from the same resonator or, at most, to inject tones from different sources into a common sound-board for radiation purposes.

Scripture [7] has emphasized that a Fourier analysis does not help much to characterize a vowel sound. The fact is, there are so many high overtones present that such an analysis even if it could be expeditiously carried out would not enable one to "see the wood for the trees." It is nevertheless possible from a mere inspection of the trace to recognize certain vowel types. The vowel *ah*, for instance, appears as a reiterated series of rapidly diminishing peaks of the relaxation type (p. 55) in which the vibrations of the air in the mouth cavities are started by the vocal cords, then relax in a series of rapid and damped oscillations, until caught up again by the succeeding impulse from the vocal cords.

Consonants. These may be divided into classes. Some are characterized by a complete temporary stoppage of sound (*B*, hard *C*, *D*). Others involve an incomplete cessation, the remaining sound anticipating the following vowel by a weak note of corresponding quality (*F*, *V*, *L*, *M*, *N*). *R* and *S* show a very small drop in intensity with anticipation of the following vowel.

Analysis of Speech. Phonetics. Almost all the analysers described in Chapter VIII have been used, in experiments too numerous for detailed mention, for tracing the sounds of speech and song in the neighbouring air. Miller,[8] who has done much work recently on the subject, finds his results to be in agreement with the fixed-region vowel theory, and also finds that the mouth, as a resonator, exhibits multiple resonance. Paget [9] has constructed models of the vocal system by ingenious combinations of resonators, formed of plasticine with an artificial larynx formed by an orifice over which

a rubber band is stretched edgewise. These, if blown, produce good imitations of vowels. The tongue divides the mouth into two cavities, the larger of which has resonances between 300 and 850, and the smaller between 600 and 2,500, depending on the position of the tongue. The resonance pitches of this double resonator lie farthest apart for *I* and nearest for *A* (as in calm). Most vowels involve a throat resonance, in addition.

The pitch and intensity at which a vowel is uttered naturally affect the wave-form of the resulting sound. This makes it more difficult to recognize a vowel from its wave-form, though with many vowels the change is not great at moderate ranges of pitch. Actually, it is not possible to utter certain vowels at extremes of pitch ; singers have to do the best they can by changing the vowel to some closely related one when such is the case. The effect of pitch on wave-form has been recently examined by Riddell [10] and Stout.[11]

The human being finds the correct position of the mouth for different sounds by instinct and imitation. It is the business of the science of phonetics to aid him in this process, especially when he is mastering a foreign language with foreign sounds, by discovering for him the positions and movements of the mouth in speech. These positions are found by probing instruments, when the system is set to produce the given sounds, and records or models are made. The pupil then endeavours to mould his mouth into the same shape.

Hearing. Ohm's Law. Before introducing the mechanism of the ear and possible explanations of the action of hearing, we shall describe the salient observations, physical and psychological, of the functioning of the human auditory system, considering the ears merely as two receivers of sound located on either side of the head, connected with the brain by a nervous system.

It is a matter of common observation that the ear is able to perform, qualitatively at least, the Fourier analysis of a complex note, meaning that we are able, within limits depending on our natural ability and training, to say what partial constituents are present in the note. Assistance in this direction may be obtained by sounding the expected partial alone, before listening to the note of which it forms a constituent, or by amplifying the partial by a suitable resonator. Ohm [12] enunciated this principle in 1843 in a statement generally known as Ohm's law of acoustics, to distinguish it from his more famous electrical law : " Every simple harmonic motion of the air is perceived by the ear as a simple tone ; all others are resolved by the

ear into a series of simple tones of different periods." That vague concept which we term the quality of the note depends then on the number and relative magnitude of the partial simple tones which the ear can find in it. When these are inharmonic and scattered indiscriminately through the audible pitch range, we describe the impression as a noise. As far as weak sounds are concerned, physically and physiologically this theory has stood the test of time fairly well; the instances where it seems to be untrue can be explained in the main as aural illusions, that is to say, that their cause is psychological. Early in this book it was shown that the response of an asymmetric recorder like the ear-drum to intense sounds is non-linear, and may therefore not be a faithful copy of the air vibrations. This is not a refutation of Ohm's law, at least in principle, but may simply imply the intrusion of other simple tones not in the external sound, into the quality of the note as perceived by the ear. An outstanding example of such intrusions are the subjective combination tones.

Phase and Quality. If Ohm's law be true, the question is naturally asked, what influence have the respective phases of the components of the note on the impression of its quality? Helmholtz [13] and also König [14] attacked this problem experimentally. Helmholtz employed a double siren by which he could produce two tones of different pitch to form a complex note of two components. The phase relation between them depended of course on the relative time of opening of the holes in the two discs of the siren: provided these rotate at the same rate the phase difference remains constant. But by a handle one disc could be made to advance slowly upon the other causing a progressive change of relative phase. No difference of quality could be detected. If phase has no influence on quality, synthetic wave forms, made up of the same components but differently spaced as to phase, will produce the same effect on the ear, though the resultant complex waves are of different shape. König constructed a siren having templates cut to such shapes and passing over the holes, and reached the same conclusion. Finally, with the development of polyphase alternating currents, Lloyd and Agnew [15] were able to reverse the phase of one component of a vibration imposed on a diaphragm transmitter; again no change in the quality heard.

It is curious that, in listening to an orchestra, no difficulty is experienced in assigning a particular partial tone in the mass of tone colour to the instrument which is evolving it. Localization plays perhaps a part in this, and the difference in duration of different notes

helps, but the ear seems to have an uncanny facility in this respect. Possibly because we are used to hearing the solo part at the top of the pitch scale, the ear tends to pick out the topmost part in the musical piece, and characterize this as the " tune " of the piece. On the other hand, the pitch of the note of a single instrument or voice is characterized by that of the fundamental, or lowest tone in the note, this being generally of much greater intensity than the upper partials. Fletcher [16] believes that he has shown by later experiments that the fundamental does not determine what pitch we mentally assign to a note. A diaphragm being excited by an alternating current of complex type, certain partials were removed by suddenly switching-in suitable filters to the electric circuit. The corresponding diaphragm and air tones being presumably removed by this action, a musician who was listening was asked to say whether he thought the " pitch of the note " had altered. Whenever one of the *lower* constituents had been removed the listener made answer that not the pitch, but only the quality of the note had been altered. If this experiment bears a physical or physiological explanation, it is of far-reaching consequence for our ideas of the function of the ear, but it is important to emphasize the control conditions, and to inquire to what extent the effect is psychological. The procedure was to remove the lower tones from the note and ask what *change* in pitch resulted ; not to produce an isolated note with the fundamental missing and ask what pitch the note had.

Physiological Intensity. Our instruments which measure intensity of sound actually measure the energy which falls upon them. The energy of a vibrating particle is proportional to the square of the velocity and therefore, in S.H.M., to n^2a^2 (cf. 10) ; or, if the frequency of the tone is fixed, to the square of the amplitude. When however we consider the human instrument, the case is altered, for our sensation of the loudness of a tone is not a linear function of the intensity as received by the ear-drum. Experiments to determine on what power or function of the amplitude the sensation of loudness or physiological intensity depends are fraught with difficulty.

The " Weber-Fechner law " relating to the stimulation of the senses may be stated, as follows : The increase of stimulus necessary to produce a just perceptible increase of the sensation δE bears a constant ratio to the total stimulus Σ, or :—

$$\delta E = k\frac{\delta\Sigma}{\Sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

or, in the integrated form : $E = k \log_e \Sigma$, the sensation is proportional to the logarithm of the stimulus. This law applies to all our senses. Steinberg [17] has found that k is not truly constant at all frequencies, as it is a function of the minimum stimulus required to produce any sensation of sound at all in the ear, and this varies with pitch, as we shall find shortly. This introduces a complication into the problem as we have not only to determine how the stimulus depends on the amplitude, but also the sensation is not itself a linear function of the stimulus. It is difficult to see how these two factors are to be separated.

Pitch Limits. It is a matter of common observation that when a body is vibrating sufficiently slowly the alternate condensations and rarefactions impressed by it on the air and received by the ear are perceived by it as distinct pulses, but that when these pressure changes take place sufficiently rapidly, the sense of their isolation is lost and they blend into a musical tone. This point represents the lower limit to the ear's power of analysis. This lower pitch limit depends on the individual ear and may be detected by a siren rotated at slow and then faster speeds. It is important to remember that a pulse in the mathematical sense of a compression followed by a single rarefaction is very rare in nature, and even the "pulse" produced by the air when a single hole of the siren is uncovered may involve tones continuing for a short while. In such an experiment these tones of higher pitch must be mentally excluded from perception. The lower pitch limit is about 16 vibrations per second.

Slow vibrations involving considerable movements of the air remain unperceived as tones if their rate of pulsation falls below this limit, though their existence can be demonstrated by manometric flames, etc. Such "infrasonic" waves have been extensively studied by Esclangon,[18] for this type of vibration is propagated from the muzzles of guns at the instant of firing. Though no sound in the strict sense of the word is heard, yet there is a sensation of detonation, when the initial compression made by such a wave is sufficiently precipitate. Such a sensation is merely one of pressure on the ear-drum, and its intensity is a function of the abruptness of the discontinuity in the air; the ear is in fact acting as a manometer. Thus large explosions may produce a small sensation if their rate of development is slow. On the other hand, proximity to the bursting of a shell may involve temporary loss of hearing, of memory, and more complicated nervous disorders due to the violent shock of pressure to the auditory system.

There is an upper frequency limit to the ear's powers of tone perception. When the frequency of vibration exceeds this limit, no sound is heard at all; it is as though the sounding body were absent. This limit lies round 20,000 vibrations per second, so that the audible pitch range covers 10 or 11 octaves. The upper limit falls with increasing age. Fundamental tones of this high order of pitch can of course be produced only by very small vibrating bodies; the noise made by the wings of a grasshopper is a well-known example. For laboratory work on this question, a number of sound sources have been employed.

(1) The longitudinal tones of short glass rods, estimated by Kundt's tube (cf. p. 27).

(2) The overtones of small and thin glass plates, clamped at their edges, determined by sprinkled sand.[19]

(3) The Galton [20] whistle.

The last instrument is most favoured by psychologists. It consists essentially of a very short cylindrical pipe, blown from an annular nozzle (cf. p. 161) of which the "height of the mouth," f, can be varied by turning the lower micrometer screw; so that as nf is constant and n is always one of the partial tones of the little pipe, a series of high-pitched notes is obtainable. The length of the pipe can also be varied in Edelmann's form of the instrument,[21] by twisting the upper screw.

Minimum and Maximum Audibility. Beside the frequency limits there are intensity limits to the sounds which the ear can perceive. The minimum or threshold audibility of the normal ear has been measured most carefully by Fletcher.[22] A common method for making the test on an individual ear is to compare the time during which a damped tuning-fork is heard by the ear with that of a normal ear. Less intensity for a sound to be audible is required in the middle of the pitch range; more is required at each end.

When a sound stimulus is very intense, the sensation becomes painful, and above a certain limit cannot be perceived as sound; there is merely an unpleasant feeling of pressure on the ear-drum. Exposure to such intense sounds causes temporary tinnitus, or "ringing in the head," which in some persons, due to a lesion of the auditory nerves, amounts to a permanent or recurring affection of the auditory system. The two lines forming the upper and lower limits of intensity for tone perception are shown on an audition diagram (Fig. 100, after Wegel [23]), plotted against the frequency. The region enclosed between

the two lines represents the auditory region of the ear, both as regards intensity and pitch.

Andrade and Parker [24] have devised a pipe maintained by a loud-speaker unit at one end, while the other end is open to the atmosphere, as a standard source suitable for making measurements of audibility. The intensity of the source is derived from observations of the amplitude of smoke particles held in suspension in the glass tube forming the pipe (cf. p. 187). The source was mounted on the top of a building facing another across a court, the floor of which was padded to avoid reflections. A listener stationed himself on the roof of the building

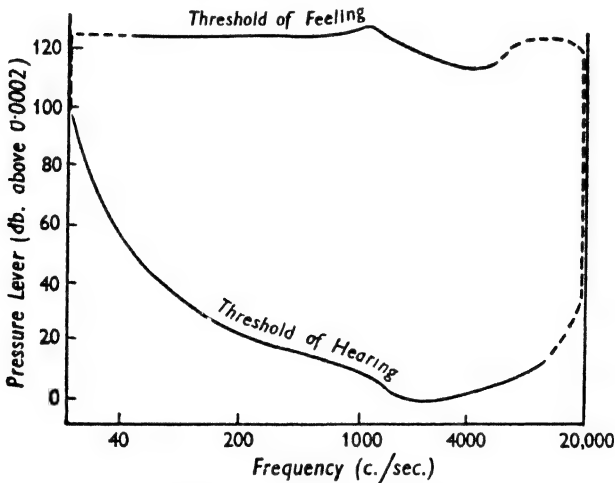



Fig. 100.—Audition Diagram.

opposite with his head in a specified position, and his threshold of audibility was determined in terms of the energy which would be radiated to the distant station from a piston source having an amplitude equal to that at the open end of the pipe. Since draughts would have prevented an actual measurement of the amplitude taken up by particles in the mouth, the pipe was run at its second harmonic and the measurement made at the internal antinode, one-third of the way along from the diaphragm. The averages of the minima for five auditors at two frequencies (410 and 646 cycles/sec.) agree with the American measurements (Fig. 100).

Under abnormal conditions, the threshold intensity may be higher than the average shown in Fig. 100. This occurs especially when

the tone is masked by the simultaneous loud sounding of another, or of an incoherent noise. Wegel and Lane [25] have conducted experiments in which one pure tone was masked by another pure tone, defining the masking as the logarithm of the ratio of the threshold intensity of the masked tone to that of the same tone unmasked. The effects are complicated by the additional masking introduced by the combination tones formed between the two tones. The question is of importance in connection with the intelligibility of speech in the presence of noise, as in listening to a telephone in a noisy office or works.

The Bel: Measurement of Loudness. Since the loudness of a noise is measured nowadays in terms of the masking which it can produce upon a note of variable intensity, it is important to be able to measure such an intensity in physiological units. A scale of loudness should be logarithmic, if the Weber–Fechner law (98) be true, and must bear a relation to the minimum audible loudness at the same pitch. The standard of intensity generally accepted is that of a tone whose objective intensity is ten times that of the just audible sound of the same pitch. This is called the *bel* after the telephone pioneer. Alternatively if I_1 and I_2 are two intensities, the difference in sensation level in bels is given by: $\delta E = \log_{10} [I_1/I_2]$. More often a unit one-tenth of the bel called the *decibel* (db.) is used.[26]

To measure "equivalent loudness" a frequency of 1,000 cycles/sec. and a sound pressure of 0·0002 dyne per sq. cm. is taken as threshold. The level of intensity of the 1,000 cycles/sec. reference tone is raised until it appears equally loud with the sound under test; then the rise of intensity of the standard tone (in decibels) is said to be equal to the equivalent loudness of the sound in *phons*. Broadly speaking, then, the units are decibels if reference is made to the minimum audibility at the same pitch, but phons if referred to that at 1,000 cycles/sec. Fig. 100 shows that over the frequency range 500 to 5,000 vibrations per sec. the objective intensity of a tone in db. will be numerically the same as the subjective intensity (loudness) in phons, but not at a lower pitch than 500 vibrations per sec.

The simple method of Davis [27] for measuring the sensation level of a noise consists in striking a tuning-fork at a determined amplitude in the vicinity and observing the time which elapses before its decaying amplitude is masked by the noise in question. There are noise meters now available in which the intensity of a "warble tone," viz., one whose frequency wanders over a certain range at about four periods per sec., can be varied until it is just masked, the masking

level being read off in decibels on the meter. Noises vary from about 10 db. for a whisper to 100 db. for a steam siren.

The method of measuring loudness employed at the Bell Telephone Laboratories is of considerable interest.[28] The experiments in essence involve a subjective judgment of "twice as loud," although it is not certain that this ratio possesses the same significance as the octave does in pitch perception. The first experiment deals with monaural versus binaural hearing. If the observer hears first with both ears and then with one only, it is assumed that the loudness has decreased to one-half. The r.m.s. sound pressure at the single open ear being p_1, it is necessary to raise it to p_2 before the loudness seems equal to what it was with both ears open. With a number of listeners, corre-

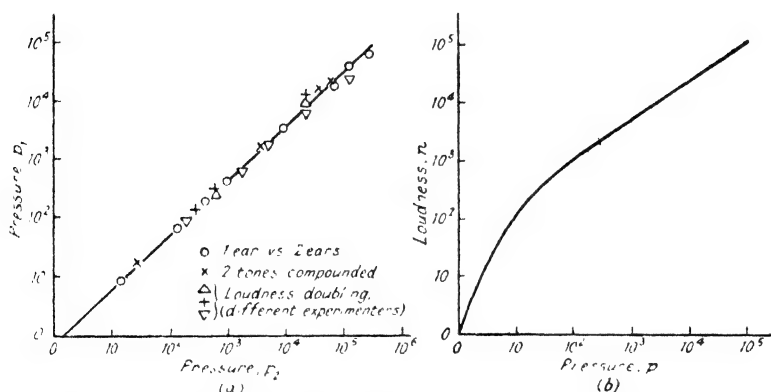



FIG. 101.—Experimental Basis of Loudness Scale (*after Fletcher*).

sponding pairs of values of p_1 and p_2 were measured, and are shown as **o** on Fig. 101 (a). In the second experiment a 1,000-cycle reference tone is balanced at r.m.s. pressure p_1 against a pure tone of remote frequency until they sound equally loud. They are then sounded together, and again it is assumed that the loudness is doubled. The 1,000 cycle-tone sounding alone is now raised to pressure p_2 until it sounds equally loud to the combination. Such points are marked **x** on the figure. In the third experiment, which has been used by four different sets of observers, although only three are shown in the graph, the reference tone alone is listened to at pressure p_1, and then raised to p_2, such that the loudness appears doubled. The consistency in the results obtained by these different methods is considered sufficient to warrant a loudness scale, based on successive doubling.

Thus we can calculate from this figure the relative pressure increases corresponding to increases of loudness $= 2^n$. This has been done, taking a pressure ratio of 100 to correspond arbitrarily to an increase of loudness of 1,000 times ($n = 3$), and the result shown by the curve on Fig. 101 (*b*). It will be noted that in accordance with the well-known relation between stimulus and sensation, the scales have been plotted logarithmically, and the straight lines obtained—at least over the range which matters in telephony, broadcasting, etc.—justify the choice of a logarithmic scale of loudness. It is important, however, as A. H. Davis has emphasized, to *specify the threshold* in every case, if we are to understand each other's scales.

Stevens [29] has shown that with a change of intensity, the mind associates a change of pitch. In the middle of the gamut, where the ear is most sensitive, no apparent change of pitch is produced by making a tone louder, but down at 150 cycles/sec. a rise of intensity of 50 db. involves a subjective *fall* of as much as 12 per cent., whereas at 12,000 c./sec. the pitch appears to rise by an equal amount. This subjective change of pitch is then paralleled by the decrease in sensitivity of the ear. The author suggests that excessive intensity acting at the ends of the basilar membrane changes the spatial stimulus pattern on it. Montgomery [30] stresses the effect of the method used on the results in such sensitivity measurements; particularly has the type of judgment demanded of the patient an important effect on the results. When the step in intensity or pitch is reduced until the subject reports " no change," the criterion is most difficult to determine. A large number of observations on different people is required to get a good statistical average ear.

Sensitivity of Ear. Psychologists measure sensitivity by the ratio $\frac{\delta\Sigma}{\Sigma}$ in the Weber-Fechner law (98). Both intensity sensitivity and pitch sensitivity have been measured. These have been defined as $\frac{\delta I}{I}$ and $\frac{\delta n}{n}$ respectively, where δI and δn are the least perceptible changes in I or n. From this form of definition arises the paradox that the smaller the sensitivity, the greater the numerical value of the quantity which defines it. For determining $\frac{\delta n}{n}$ the original method was to move a weight on a tuning-fork until the observer indicated that the pitch had been changed thereby—he was not asked to determine whether it had been raised or lowered; a more

difficult test. Max Wien,[31] who determined the relation between the amplitude of a telephone diaphragm in terms of the current through the exciting circuit (cf. p. 218), introduced the general method for $\frac{\delta I}{I}$, i.e., a sudden small change in intensity produced by change of current, to be judged by the observer. Knudsen [32] has recently made careful measurements of both sensitivities. The average results are shown in Fig. 102. The changes δI and δn were made by switching over the exciting circuits of valve-maintained telephone transmitters to alternate circuits having slightly different resistance (whereby I was altered), or having different inductance and capacity (causing an alteration of n in the circuit).

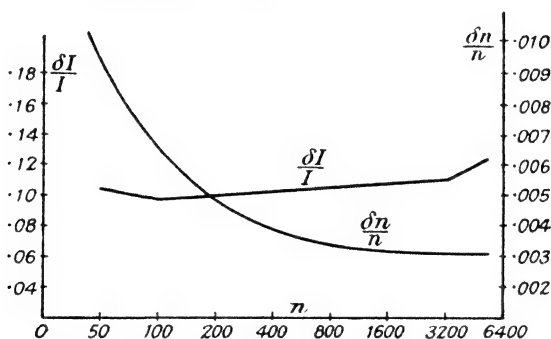



FIG. 102.—Sensitivity of Ear to Intensity and Pitch (*Knudsen*).

Minimum Number of Vibrations for Perception. It is easy to form an idea of the pitch of a sound, if it lasts several seconds; would the perception of pitch be possible if the sound lasted only long enough for the ear to receive a few vibrations? This question was first put by Savart, who satisfied himself that two vibrations only were enough to give an idea of the pitch, but more were necessary to perceive the quality of the note. More recent experimenters have used a telephone membrane excited for a very short period, i.e., by momentarily "making" the current circuit, or a siren in which all but a few consecutive holes were stopped. Stefanini [33] verifies Savart's statement by several methods, at least up to a limiting frequency. Leimbach [34] puts this limit at about 1,000 vibrations per second, so that the shortest time for pitch perception is about a thousandth of a second. On the other hand Gianfranceschi [35] concludes that the criterion for pitch perception is the absolute time during which the ear hears the tone, so that it depends on the absolute pitch; he sets

this time at 0·025 second down to 0·01 second for the most sensitive part of the audible region, corresponding closely to the figures obtained for the duration of the after-sound.

Interrupted Sounds. If a pure tone were interrupted for definite intervals and at definite instants, say a tone of frequency n interrupted m times per second, according to König a tone of frequency m would be perceived by the ear. This was thought to be an infringement of Ohm's law, but doubt has been cast on the observation by later investigations. König [36] employed a siren with, for example, 2 holes of every 5 in the ring plugged, so that m equalled $\frac{n}{5}$. This interruption-tone can be picked out by a resonator, so that it appears to be the siren which is producing it. Indeed Schaefer and Abraham [37] failed to detect it with a less dubitable apparatus, and remarked only a change in the quality when the sound was interrupted, not a new tone.

As the number of interruptions is increased, the intermittence ceases to be apparent; the sound seems to continue without break. This critical pulsation has been determined for different frequencies by Mollie Weinberg and Allen [38] by rotating a stroboscope disc in front of a hole in a box, from which a pure tone from a Stern "ton-variator" was issuing. At a critical speed of rotation of the disc the interruptions ceased to be audible. The incidence of this critical pulsation is dependent on the rate of damping of the aural mechanism, and the time during which the tone is interrupted at the critical pulsation represents the time during which the ear continues to respond after the tone itself has ceased. It is analogous to the "after-image" effect on the eye, and, as with the after-image, the time increases when the ear has been fatigued by the steady drone of the same or a nearby tone. The results show that for a tone of average intensity, this after-response falls from 0·02 second at a frequency of 140 to 0·015 second at 280 c./sec.

Binaural Location. The problems discussed above concern the ear as an individual receiver. There are others which involve the relation between the two ears. The most important of these is that faculty by which we determine the direction of a source of sound. Often we are aided in this by the senses of sight and touch, but in the absence of these we can form a good estimate of the direction of a sound. This has been explained in two ways:

(1) That there is a difference of *intensity* at the two ears, the

sound being judged on the side of that ear which receives the greater intensity.

(2) That there is a difference of *phase* at the two ears, the sound being deemed on that side where the phase is in advance.

Since in practice there is both phase and intensity difference at the ears, except when the source is directly in front or behind, the relative weight of the two factors has been a prolific source of investigation. Those of Stewart [39] demand detailed mention for his painstaking. Sounds from a tuning-fork were conducted to the two ears severally by two rubber tubes whose openings lay at equal distances from the fork. The observer then judged the sound to lie in the median plane through the head. The intensity of one component was changed without altering the phase difference, by pushing the mouth of one tube further from the fork, and the observer pointed out on a large protractor encircling his head, the angle ϕ through which he judged the sound to have turned. The relation :—

$$\phi = K \log \frac{I_R}{I_L} \qquad (99)$$

was found, I_R and I_L being the intensities received at the right and left ear respectively, and K a constant for the individual. To produce phase-differences alone Stewart used an instrument which he named a "phaser." A toothed iron wheel revolved in front of two independent electro-magnets, somewhat as in the phonic motor (p. 111). As the teeth passed they induced alternating currents in the coils of the electro-magnets, and excited telephones held to each ear. By altering the relative positions of the electro-magnets, any desired phase-difference δ between the tones heard could be produced. He found :—

$$\phi = K'\delta \qquad (100)$$

The two effects, intensity and phase, were discrete, for they did not interfere with each other's effects when both were employed together. But it was concluded—and the results of other workers generally support the conclusion—that up to a frequency between 1,000 and 1,500 vibrations per second, direction is judged almost entirely by phase-difference. At higher frequencies the phase effect fails and the intensity difference must be called upon to explain binaural location.

An illuminating experiment was performed by Lo Surdo.[40] He led the sound from a tuning-fork of frequency below 600 to both ears by tubes, and then lengthened one tube so much that the phase at the

left ear was in advance of that on the right. In spite of the diminished intensity at the left ear due to the longer tube which this component had to traverse, the sound was judged to come from the left.

In assuring himself of the direction and distance of a source the listener commonly moves his head about a vertical axis and listens to the change in sensation. Wallach [41] has shown the importance of this action for purposes of localization. Distance is apparently judged by the direction and magnitude of local echoes in relation to the direct sound.

If two notes of slightly different pitch are led to the two ears severally, " binaural beats " [42] are heard, involving rapid changes of the relative phase at the two ears. This gives an impression of a sound hovering or revolving round the head, unless the beats become too rapid.

The idea that the brain itself can appreciate a difference of phase between the sensations brought by the nervous mechanism of two independent organs has been a stumbling-block to physiologists. Some, e.g., Myers and Wilson,[43] have therefore supposed that the phase difference is apprehended as a difference of intensity at the two ears due to conduction across the head of the sound from the side on which the source lies. Banister,[44] who has thoroughly examined the pros and cons of this question, considers that the existence of such " bone conduction " from side to side through the head of aerial vibrations impinging on one side remains to be demonstrated.

Anatomy of the Ear. Externally the ear presents a trumpet-shaped channel to the sound. This narrows down to a nearly cylindrical tube at the end of which is found the membrane known as the ear-drum, which receives the aerial vibrations. The ear is shown in section in Fig. 103. To the back of the membrane *M* and at its centre is attached one end of the stapes, *S*, a couple of levers which transmit the vibrations of *M* to the " oval window " *O*, which communicates with the inner ear. The inner ear, or labyrinth, is filled with a liquid, the endolymph, having physical properties similar to water. The principal part of this inner organ is the cochlea *C*, which consists of two galleries, one over the other, divided by the basilar membrane, and communicating with each other at the far end, where there is a break in the basilar membrane. At the nearer ends of the galleries are the oval window *O*, in the upper gallery, and the " round window " *R* in the lower, which debouches on the upper end of the Eustachian tube *E*. Instead of stretching straight out from the two windows

the whole of the cochlea is coiled round on itself like the shell of a snail, hence the name. A system of nerves, *N*, connects the cochlea with the brain. The pressure on both sides should be normally atmo-

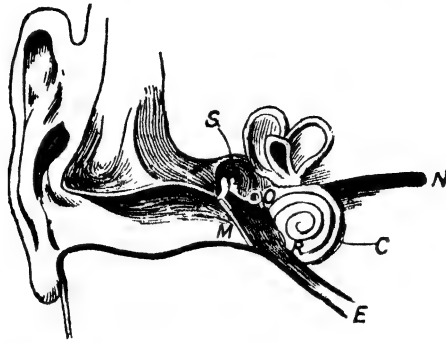



Fig. 103.—The Ear.

spheric. To secure this the lower end of the Eustachian tube can be occasionally opened to the atmosphere, e.g., in the act of swallowing.

The inside of the cochlea must be considered in detail. Fig. 104 shows a section across one of the folds, showing the two galleries with

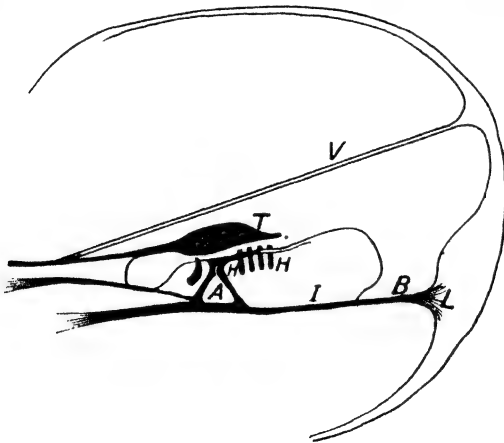



Fig. 104.—Section of the Cochlea.

the narrow basilar membrane *B* lying between. The size of the section decreases three-fold from the near to the far end of the galleries, with corresponding diminution in the width of the basilar membrane. This membrane has embedded in it a number of tendons or " strings " which therefore decrease in length from one end of the

membrane to the other; their direction lying in the plane of the figure. This membrane is attached to the wall of the cochlea by a series of fine fibres, known as the spiral ligament *L*. These serve to put tension on the basilar membrane, and on the fibres embedded in it. Gray [45] has shown that this ligament decreases in stoutness from the lower to the upper end of the cochlea, leading him to suggest that the fibres in the membrane are graduated both for length and tension from end to end of the cochlea. Overshadowing the membrane lie "Corti's arches" *A*, rods set at an angle to the membrane and which support a structure containing minute hair cells *H*, over which the "tectorial membrane" *T* is stretched. Higher up still and distinct from both of these lies the "vestibular membrane" *V*. All these structures progressively diminish in size from the base to the apex of the cochlea. Nerve bundles lead away from the neighbourhood of the roots of the hair cells.

The Resonance Theory of Hearing. This theory is generally associated with the name of Helmholtz, but the germ of it is to be found in Cotugno, *De acquaeductibus aurae humanae internae* of 1760, though a hint had been given by Du Verney 80 years earlier. Two sentences translated from Cotugno will serve to show his idea: "It appears that the sensation of hearing consists in the vibration of acoustic tendons (*nervi*) exactly corresponding to the vibrations of the sounding body. . . . The acoustic tendons are like cords oscillating with the vibrations of the sounding body, so that the impressions which the brain receives are as many as the vibrations of the sonorous body."

Helmholtz [46] elaborated this idea into what is known as "the resonance theory of hearing," likening the cochlea to an instrument containing a series of resonators, which could be excited by vibrations of appropriate frequency communicated to the cochlea via the stapes, the corresponding nerves then transmitting the sensation to the brain. At first he postulated Corti's arches as the resonators, but on finding that these were absent in certain creatures, he adopted a suggestion of Hensen that the fibres of the basilar membrane were the resonators. Objections to this theory were not wanting. The tenuity of the "strings," the strong damping due to the fact that they have to drag the membrane with them in their movements, the narrow differentiation of length and the small number of vibrators to cover such a wide range of pitch (10 or 11 octaves), their being surrounded by liquid, were all objected against Helmholtz, but he and his followers have

been able to overcome most of these difficulties. The most sedulous exponent of this theory in recent years is Wilkinson.[47] In order to explain how the resonant response of the membrane can cover the total audible pitch range he points out that these fibres vibrate under quite special conditions, the analogy with a pianoforte being quite out of the question.

To explain the extremely high pitch sensitivity of the ear, having in view the small number of fibres, Wilkinson employs the principle of maximum stimulation of the nerves. To illustrate this, he suggests the experiment of pressing a point hard into the finger; although there is considerable pressure all around the point, so that a large area surrounding the point is anæmic, yet the sensation is of pressure at a single point. "The point at which we feel the nerve ending to be stimulated is the point at which the maximum degree of stimulation is taking place." So in the cochlea, a pure tone will excite a *region* of the membrane, with maximum response at the level where a cross-section of the membrane has a natural frequency of the same period. The pitch sensitivity of the ear is limited by the degree with which the brain can distinguish *two* points of maximum stimulation together. This faculty may well vary in different parts of the scale, just as the application of two pointed objects close together can be felt as two points on the hand, say, whereas they are felt as one point if applied to a less sensitive region like the middle of the back. The resonance theory requires the assumption of very heavy damping on the basilar membrane, otherwise a rapid performance on a musical instrument would be audible as a jangle of notes.

Indirect Evidence for the Resonance Theory. On examination, we find that most of the facts about the sense of hearing fall into line with the resonance theory, in its developed form. The salient criteria are:

(1) Subjective combination tones: The ear-drum and trumpet form a system whose response is non-linear. In fact, owing to the one-sided load of the ear-drum, combination tones $p - q$, $p + q$, $p - 2q$, etc. are added to the vibrations due to two simple tones led to it, if Waetzmann's analogy of the asymmetrically loaded membrane be correct (cf. p. 62). The difficulty of the recognition of König's beat tones as simple tones then disappears, as the difference or summation tone is already present, to an extent dependent on the intensity, in the vibrations impressed on the cochlea, even if absent in the surrounding air.

(2) The variations in sensitivity and damping at different frequencies are what would be expected of an instrument consisting of a set of resonators.

(3) The possibility of fatiguing and even permanently injuring a definite level of the basilar membrane by continued sounding of tones of the corresponding frequency.

(4) The definite pitch and intensity limits.

(5) Deafness to certain definite ranges of pitch, and therefore to certain vowels, due to degradation or destruction of a certain length of the resonating strip.

Perception of Loudness. The analysis given above explains satisfactorily the perception of pitch and quality, but how then is loudness perceived? Physiologists believe that loudness is judged by the frequency with which signals pass along the aural nerves to the brain. An electronic model may be constructed to illustrate this. The amplified signal from a microphone is passed through appropriate filters—to represent the resonators in the ear—to a condenser which discharges through a thyatron valve every time that the potential on the condenser reaches a certain value, the flash of the discharge representing one signal to the brain. If the sound from a tuning-fork held at a distance falls upon the microphone (ear) infrequent discharges occur, but as the tuning-fork is brought nearer so does the frequency of the discharges (signals to the brain) increase.

Central Analysis Theories of Hearing. In 1865, Rinne [48] put forward the theory that the ear-drum and organs connected to it in the inner ear merely copied the vibrations which fell upon them, exciting *all* the auditory nerves to a greater or less extent, and that the analysis into component tones was performed by the brain itself. Such a theory is rather unsatisfactory to physicists as it begs the question of the purpose of all the intricate structure of the inner ear, for why is not then the nervous system applied directly to the ear-drum? Apart from this, the theory requires that a nerve shall be capable of transmitting impulses up to 20,000 per second, and that the brain shall recognize the numbers of these impulses, even if overlaid with impulses at other frequencies. Whereas it is known that, for at least one-thousandth of a second after each stimulation, a nerve fibre is impotent to transmit a further stimulus.

It has been impossible to give more than the main pros and cons of the several theories of hearing. On the whole, the resonance theory holds the field, although modifications may be

necessary as research, both physical and physiological, brings further evidence.

The Musical Scale. In spite of the extreme sensitivity of the ear in discriminating pitch, only a limited number of notes is used in music. The ear recognizes that the whole range of notes divides itself up into successive groups, and that each group repeats the musical effect of the others. A group is limited between two notes, the higher being the octave of the lower. The construction of a scale is therefore a question as to how many notes there shall be in the group. The "interval" between two notes is defined by the ratio of their respective frequencies. The smallest interval employed is the semitone ($\frac{16}{15}$), although quarter tones have occasionally been employed. This limit is set by the requirements of musical instruments having a separate key or hole for each note. On a violin or trombone, on the other hand, a player can himself subdivide the semitone. The octave has been divided in a number of ways. Each of these is known as a musical scale or mode, but of them only three survive in current musical practice: (1) the major scale, (2) the minor ascending scale, and (3) the minor descending scale. Each of these three scales starts from its "key note," and by means of a series of intervals which includes two semitones, arrives at the octave, which is the key-note of the group above. The scales differ from each other in the relative positions of the two semitone intervals in the group.

The major scale in the table on this page is shown as the interval from one note to the next, then as the interval from the first or key-note, and then as a relative number of vibrations.

While music remains within the compass of such a series of notes all based on one key-note—all in one key—there is no difficulty in constructing a musical instrument having octaves of notes, all in the series of intervals of the table below. But composers employ different

	—	Major-tone.	Minor-tone.	Semi-tone.	Major-tone.	Minor-tone.	Major-tone.	Semi-tone.
Interval from note below	—	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$
Interval from key-note	—	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Possible frequencies	240	270	300	320	360	400	450	480

keys, with frequent changes from one to another, so that, for instance, after a musical composition has been confined to a series of notes based on a key-note of frequency 240, the key may change to one based on the second note of the old key, i.e., on 270. By applying the same interval ratios to this note, we get the series:—270, 304, 338, 360, 405, 450, 506, 540 (major scale), for the notes comprising one octave of the new key, of which only four coincide exactly with any one of the old notes in the table. At this rate a large number of additional notes would be required for every key in which an instrument had to play. To obviate this complication, many schemes for adjusting the intervals in the octave have been proposed, but that which holds the field is the equi-tempered scale. In this scale the octave is divided into twelve equal semitones, so that the value of the equi-tempered semitone becomes equal to $\sqrt[12]{2} = 1{\cdot}0594$ and a whole tone is $\sqrt[6]{2} = 1{\cdot}1325$, the distinction between major and minor tones being sunk. Since every interval in this scale is a multiple of $\sqrt[12]{2}$, it is now feasible to employ 12 notes to the octave, and yet be able to use keys founded on any one of them as key-note.

Harmony. The basis of classical harmony was the common chord, formed of the first, third and fifth of the notes of the scale, the interval between the first and third being $\frac{5}{4}$ (4 semitones) or $\frac{6}{5}$ (3 semitones), and between the first and fifth $\frac{3}{2}$ (7 semitones). These intervals are known as the major third, minor third, and the perfect fifth respectively. Those intervals which the æsthetic taste judges to be concords are all expressible by the ratio of small numbers. Thus the fifth is the most perfect concord after the octave. The equi-tempered scale retains the perfect octave (interval 2), but spoils the perfect fifth, which now becomes $2^{\frac{7}{12}} = 1{\cdot}498$ instead of $1{\cdot}500$. On the other hand it makes all fifths (i.e., 7 semitones) equal, wherever they are chosen on the scale, while all major thirds become $2^{\frac{4}{12}} = 1{\cdot}260$, and minor thirds $2^{\frac{3}{12}} = 1{\cdot}189$. The common chord in the major scale is then made of a major third followed by a minor third, and in the minor scale is made of a minor third followed by a major third. All concords are made up of these three notes in various positions. Any note which is alien to this combination is made to lead in certain stipulated ways to a note which forms part of a concord. Chords having these intruding notes count as discords, and the process by which the return is made to a concord is known as "resolving the discord."

Modern composers have built up scales in which other notes, such as the seventh or the ninth in the scale are regarded as regular constituents of the chord characterizing a key. They do not regard such chords as discords, and consequently leave them unresolved in their compositions. Another departure has been made by Scriabin, who builds up chords based on the upper constituents of the harmonic series of tones—the 8th to the 14th partials, in fact. These he arranges in ascending fourths.

His "Promethean chord" based on 240 (fundamental 40), for example, would contain the following notes (using the frequencies of the natural scale), 240, 338, 427, 600, 800, 1,280. This "synthetic harmony" embraces both forms of common chord. Such innovations leave the key a rather undefined quantity, and the natural development in modern music is to ignore the question of key altogether. For this reason it appears to be beside the point to detail the elaborate theory of harmony developed by Helmholtz on the common chord.

Standards of Pitch. The infinite local variations in tuning pitch began to settle down to some measure of uniformity in the early part of the nineteenth century. A conference of physicists at Stuttgart in 1834 proposed that the A for tuning should be set at 440 cycles/sec. This was endorsed for England by the Society of Arts in 1850.

Unfortunately, the 1850 recommendation remained little more than a record on paper, and it appears, looking back, that the Society itself was partly responsible for this state of affairs through an unfortunate ambiguity in its proposals. At that time it was common to use a C tuning-fork for determining instrumental pitch. Now if you say that A shall be 440, it is natural to determine the C above from its relation in just temperament, which is a minor third ($=\frac{6}{5}$) higher. It is, however, well known that the exigencies of playing on keyboard instruments demand a modification of just temperament. On the pianoforte, then, the R.S.A. standard would require C = 524 instead of C = 528 of the orchestra. Standard tuning-forks were for a time adopted in orchestral tuning (in just temperament) corresponding to a pitch for A, if it had persisted, of 444 instead of the 440 intended by the Committee.

The situation became complicated when a French Commission containing such well-known names as those of Berlioz, Meyerbeer and Rossini decided in 1858 for A = 435, and entrusted to Lissajous the construction of a standard fork of this frequency.

The Army was the last stronghold of high pitch, but their pitch was lowered in 1927, so that—with the exception of a few local choral societies and orchestras—the pitch of musical performance hovered round A = 440, within a few vibrations, over the whole of Europe at any rate, as tests of broadcast performances within the last few years have shown. In 1938 a Committee of the British Standards Institution, having members from the scientific and musical professions and from the musical instrument makers, recommended 440 for A, and this was adopted by an international gathering of similar interests. This has now been adopted as the standard broadcast pitch in many countries.[49]

Van der Pol and Addink [50] have made an interesting series of measurements of the variations in pitch between different orchestras and in the same orchestra during a performance. To compare the pitch of the tuning-note it is sufficient to compare the pitch during tuning with the standard 440 by counting beats or by the use of a wave-meter. In order to assess the variation in pitch during performance, the music from a wireless receiver was passed through a filter circuit which limited the electrical vibrations passing to those in the vicinity of A—actually from A♭ to A♯—and the frequency of such vibrations as passed the filter compared with the standard.

The pitch generally tends to rise above the tuning-note during forte passages, probably because the notes of the wind instruments tend to rise in pitch with the blowing pressure. The tests also showed —what has been long known—that the human voice at its best is by no means so precise in maintaining constancy of pitch as the mechanical instruments.

References

1. *Phil. Mag.*, **10**, 218, 1855.
2. Musehold, *Arch. Laryng.*, **7**, 1898.
3. Katzenstein, *Beitr. Anat. Ohr.*, **3**, 291, 1910.
4. *Ann. Physik*, **24**, 397, 1832.
5. *Sensations of Tone*, p. 105.
6. *Pflüger's Archiv.*, 1897 to 1902 *passim.*
7. *Phys. Zeits.*, **29**, 911, 1928; *J. Acoust. Soc.*, **5**, 148, 1933.
8. *Science of Musical Sounds*, 215, 1922.
9. *Proc. Roy. Soc., A*, **102**, 752, 1923; **106**, 150, 1924; **119**, 157, 1928.
10. *Thesis, Durham*, 1938.
11. *J. Acoust. Soc.*, **10**, 137, 1938.
12. *Ann. Physik*, **59**, 497, 1843.
13. *Sensations of Tone*, p. 103, etc.
14. *Ann. Physik*, **12**, 344, 1881, and **57**, 339, 1896.
15. *Bureau of Standards Bull.*, **6**, 255, 1909.

16. *Phys. Rev.*, **23**, 427, 1924.
17. *Ibid.*, **26**, 507, 1925. *See also* FLETCHER and STEINBERG, *ibid.*, **24**, 306, 1924; FLETCHER, *J. Acoust. Soc.*, **6**, 59, 1934.
18. *Comptes Rendus*, **168**, 699, 1919.
19. SCHULZE, *Ann. Physik*, **68**, 99 and 869, 1899, and **24**, 785, 1907.
20. *Inquiries into Human Faculty*, p. 375, 1883.
21. *Ann. Physik*, **2**, 469, 1900.
22. *J. Frank. Inst.*, **193**, 720, 1922, and **196**, 289, 1923.
23. *Proc. Nat. Acad. Sci.*, **8**, 5, 1922.
24. *Proc. Roy. Soc.*, *A*, **159**, 507, 1937.
25. *Phys. Rev.*, **23**, 266, 1924.
26. *See* FLETCHER and MUNSON, *J. Acoust. Soc.*, **5**, 82, 1933.
27. *Nature*, **125**, 48, 1930.
28. FLETCHER, *J. Frank. Inst.*, **220**, 405, 1935; *J. Acoust. Soc.*, **9**, 274, 1938.
29. *J. Acoust. Soc.*, **6**, 150, 1935.
30. *Ibid.*, **7**, 39, 1935.
31. *Ann. Physik*, **36**, 834, 1888.
32. *Phys. Rev.*, **21**, 84, 1923.
33. *N. Cimento*, **13**, 65, 1917.
34. *Ann. Physik*, **39**, 251, 1912.
35. *Atti Accad. Lincci.*, **23**, 704, 1914.
36. *Ann. Physik*, **157**, 228, 1876.
37. *Arch. ges. Physiol.*, **183**, 207 and **85**, 536.
38. *Phil. Mag.*, **47**, 50, 126, 141 and 941, 1924.
39. *Phys. Rev.*, **15**, 425 and 432, 1920.
40. *Atti Accad. Lincei*, **30**, 125, 1921.
41. *J. Acoust. Soc.*, **10**, 270, 1939.
42. STEWART, *Phys. Rev.*, **9**, 502 and 514, 1917. *See also* LANE, *ibid.*, **26**, 401, 1925.
43. *Proc. Roy. Soc.*, *A*, **80**, 261, 1908.
44. *Phil. Mag.*, **2**, 144 and 402, 1926. *See also* VON BÉKÉSY, *Ann. Physik*, **13**, 111, 1932, and **14**, 51, 1932.
45. *J. Anat. Phys.*, **34**, 324, 1900.
46. *Sensations of Tone*, p. 145. *See also* WAETZMANN, *Phys. Zeits.*, **9**, 307, 1908; WEGEL, *Nature*, **116**, 393, 1925; VON BÉKÉSY, *Phys. Zeits.*, **29**, 793, 1928; **30**, 115 and 721, 1929, and **34**, 577, 1933; FLETCHER. *J. Acoust. Soc.*, **1**, 311, 1930.
47. *J. Laryng. Otol.*, 1921, and 365, 1927.
48. *Zeits. rath. Med.*, **24**, 12, 1865.
49. *See* KAYE, *Nature*, **142**, 820, 1938; **143**, 905, 1939; RICHARDSON, *J. Roy. Soc. Arts*, **88**, 851, 1940.
50. *Philips' Tech. Rev.*, **4**, 205, 1939. *See also* MADELLA, *Alta Freq.*, **8**, 300, 1939.

CHAPTER TWELVE

ACOUSTICS OF BUILDINGS

Reverberation in an Auditorium. Suppose a "pulse" to be produced at some point in a room. A wave of compression spreads out in all directions. In the absence of any reflecting bodies, an auditor would receive a single sharp impression, but the walls of the room reflect the greater portion of the sound, so that a series of waves, generally diminishing in amplitude, and formed by subsequent reflections, pass the observer's ears, until all the energy of the original wave has been dissipated by friction. In place of the single compression, the observer hears a roll of sound, and the time taken for this to

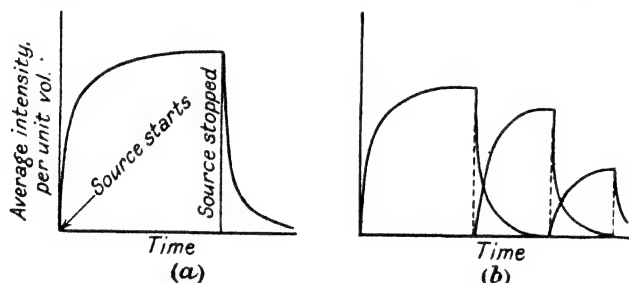



FIG. 105.—Rise and Fall of Average Intensity in a Room.

die away, i.e., fall below the threshold of audibility, is known as the "time of reverberation" of the sound in this particular room, reckoned from the time that the original pulse was produced, or, in the case of a continuous note, the time that the source stopped sounding. Now suppose such a continuous source, e.g., an organ pipe, to be started and to continue at constant output at some point in the room. The waves spread out and the auditor is conscious of the commencement of the sound when the first of these reaches his ears, and of an addition to the energy received for every reflected wave which subsequently adds itself to the incident waves still emitted from the source. During this time part of the energy is being absorbed by the walls and other materials in the room, and part is escaping through these walls and via open windows or ventilators. After a few seconds a balance is reached between the energy emitted per second and that lost outside

or dissipated ; then the intensity at the listener's ear, represented in energy incident on unit area per second, attains a steady average value. If now the source is cut off, this intensity is rapidly diminished, first by loss of the incident radiation and then by successive removals of the reflections of the last waves, until it falls below minimum audibility. The rise and fall of the sound is shown graphically in Fig. 105 (*a*). When a succession of different vowels or notes are sounded the effect is as in Fig. 105 (*b*). Now it is important for distinct rendering of music or speech that each separate sound should give rise to a sufficient intensity in every part of the auditorium and then rapidly decay to give place to the next sound. This is particularly necessary with speech ; for music more reverberation is permissible.

Reverberation Formula. A formula expressing the rise and fall of sound in a room was developed by Wallace Sabine [1] on the following assumptions :—

(1) That an average energy per unit volume σ could be conceived.

(2) That dissipative influences were negligible, energy being lost only by "absorption," using this word to include transmission to the outside.

Following Jäger,[2] we can calculate, on these assumptions, the energy falling on unit area of the walls per sec., for if σ is the energy in unit volume, that which comes within a solid angle $d\phi$ is $\sigma\, d\phi/4\pi$. That which falls per sec. on a unit surface (of the wall) at an angle θ will comprise $\sigma c \cos\theta\, d\phi/4\pi$ (c, velocity of sound). The total falling on the surface within a hemi-sphere will be $\frac{\sigma c}{4\pi}\int \cos\theta\, d\phi$, or since $\phi = 2\pi(1 - \cos\theta)$, it will be

$$\frac{\sigma c}{4\pi}\int_0^{\pi/2} 2\pi \cos\theta \sin\theta\, d\theta = \frac{\sigma c}{2}\left[\frac{1}{2}\cos^2\theta\right]_0^{\pi/2} = \frac{\sigma c}{4}.$$

If α is the average "absorption coefficient" of the walls, defined as the fraction of incident energy which is not reflected, the rate at which sound energy is removed from the room is $\frac{\alpha S c \sigma}{4} = \Omega\sigma$ ergs per second, where S is the total area of the walls and other absorbing materials. The rate at which the total energy $v\sigma$ in the room increases is the difference between the rate of supply from the source ($= Q$ ergs per second, a constant) and the rate of removal by absorption, so that

$$v\frac{d\sigma}{dt} = Q - \Omega\sigma.$$

Substituting $\sigma = B + be^{-\beta t}$ as a solution, we find :—

$$\beta = \frac{\Omega}{v},\ B = \frac{Q}{\Omega}.$$

Also, when $t = 0$, $\sigma = 0$, therefore $b = -B$, so that the rise of average sound energy per unit volume from the time the source commences to sound, is given by :—

$$\sigma = \frac{4Q}{\alpha Sc}\left(1 - e^{-\frac{\alpha Sc}{4v}t}\right)$$

and attains a steady value $\sigma_{max} = \frac{4Q}{\alpha Sc}$. Similarly, after the sound ceases, we find :—

$$\sigma = \frac{4Qe^{-\frac{\alpha Sc}{4v}t}}{\alpha Sc} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

to represent the decay of σ.

For a source of standard frequency and standard output measured by the energy produced per second (Q), the average steady intensity of sound (σ_{max}) produced in the room will be inversely as the absorption + transmission αS by the surfaces. On the other hand, if the material of the exposed surfaces be kept the same, the intensity is weaker in a room of larger surface area.

The factor $\frac{\alpha Sc}{4v}$ determines the time of reverberation in the room. Thus if σ_0 denotes the minimum audible intensity, equation (101) can be written :—

$$\sigma_0 = \sigma_{max}e^{-\frac{\alpha Sc}{4v}t_1}$$

where t_1 is the time from the cutting-off of the sound to the time at which the sound becomes inaudible. This gives :—

$$\log_e \frac{\sigma_{max}}{\sigma_0} = \frac{\alpha Sc}{4v}t_1.$$

Both α and σ_0 vary with pitch (cf. p. 307). The standard frequency chosen by Sabine was 256 (middle C).

For reckoning the time of reverberation, a standard steady intensity is required. For this Sabine took "a million times minimum audibility." His method of determining the maximum intensity in a room in terms of the standard will be explained presently. The

quantity on the left-hand side then becomes $\log_e 10^6 = 2{\cdot}3 \times 6$, and putting $c = 1{,}120$ ft. per second, we obtain approximately :—

$$t_1 = 0{\cdot}05\frac{v}{\alpha S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

the foot now being taken as the linear unit.

For a source of constant output, therefore, the time of reverberation should vary directly as the volume of the room, and inversely as the total absorption + transmission of the exposed surfaces. This formula has been verified by Sabine and others experimentally ; t_1 was found by a revolving drum chronograph, the turning-off of the source, an organ pipe, made a mark on the drum, and a second mark was made by the observer himself when the reverberating sound became inaudible. As standard material Sabine took an open window as having zero reflecting power. The coefficients of other materials were found in comparison with this ; e.g., Sabine found what area of surface of cushions with shut windows would produce the same effect as the maximum possible area of open window and no cushions, and obtained an absorption coefficient of 0·80 for the material of the cushions. To refer all values to his standard steady intensity, Sabine used two organ pipes of equal output and frequency as far as could be judged, first each singly, then two together. Calling the respective reverberations t_1 and t_2,

$$\sigma_0 = \sigma_1 e^{-\beta t_1}$$

$$\sigma_0 = 2\sigma_1 e^{-\beta t_2}.$$

From the first equation $\beta = \frac{1}{t_1}\log_e\frac{\sigma_1}{\sigma_0}$,

from the second $\beta = \frac{1}{t_2}\log\frac{2\sigma_1}{\sigma_0}$

whence $\frac{1}{t_1}\log_e\frac{\sigma_1}{\sigma_0} = \frac{1}{t_2}\left(\log_e 2 + \log_e\frac{\sigma_1}{\sigma_0}\right)$,

or $\log_e\frac{\sigma_1}{\sigma_0} = \frac{t_1}{t_2 - t_1}\log_e 2.$

By measuring the "times of reverberation" t_1 and t_2, in the same unchanged room, he was able to determine the value of the intensity, measured as energy σ_1 per unit volume, produced by a single source, in terms of the minimum audible intensity σ_0, and hence of the standard intensity $\sigma_0 \times 10^6$. The standard source chosen was perhaps unreliable, but was sufficiently accurate for this branch of applied

acoustics, in which approximate values of the quantities concerned suffice, in view of the large scale of the experiments.

Optimum Reverberation. The acceptable time of reverberation in a room is a matter of taste. Statistics have been compiled by Watson [3] and by Lifschitz [4] of the times of reverberation (always reckoned on Sabine's standard) and volumes of halls pronounced by public opinion to be acoustically good. It is found that this optimum time of reverberation increases with the volume. From (101) and (102) the actual time of reverberation varies as $\frac{v}{\alpha S}$; if the absorption + transmission per unit area α is kept constant for all halls, then this time will be proportional to the volume divided by the surface area, or, since the latter varies as $\sqrt[3]{v^2}$, if the hall is changed in scale, but not in shape, t_1 will vary as $\sqrt[3]{v}$. Plotting the optimum reverberation against the cube root of the volume of the halls in the collected data, Watson finds that the points lie roughly on a straight line (volumes between 10^5 and 10^6 cu. ft.), which does not, however, pass through the origin. It is therefore proposed to make α the same in all halls; referring to (101), it appears that when this is done, as the volume of the hall increases the output of the source must increase as $v^{\frac{2}{3}}$, if the same average intensity is to be kept. For speech the optimum reverberation, when the hall is full, is given by the statistics in the form :—

$$t' = 0{\cdot}75 + 0{\cdot}175\sqrt[3]{v} \quad . \quad . \quad . \quad . \quad . \quad (103)$$

Lifschitz finds that expert criticism does not think this law to be satisfied when the room is of small volume. A limiting reverberation of 1·03 seconds seems best for all rooms having less than 10,000 cu. ft. of volume.

Correction of Reverberation. The work of correcting an auditorium for too much reverberation on the one hand or for deadness on the other consists in practice of assimilating the times in formulæ (102) and (103); a mere matter of calculation. The volume of the hall is calculated, and substituted in (103) to get the optimum reverberation; going back to (102) it is necessary to adjust αS so that the value of t_1 so obtained shall equal the calculated t'. In all these calculations a latitude of 5 per cent. is permissible. As the total surface of the room cannot generally be altered, the walls, floor or ceiling must be covered or replaced by material of greater or less absorbing power.

Combining the results of Watson for optimum reverberation with Sabine's formula (102), we can draw a graph (Fig. 106) which gives the number of "absorption units," reckoned by the value of αS, which a room should contain, as a function of its volume. Under αS must be included the absorption produced by the average audience; it is usual to reckon this at 4·7 units per head, to tally with corresponding values of αS for *materials*, measured in square feet. Upholstered seats are useful as they provide absorption to take the place of the absent audience when the hall is sparsely filled. A table of absorption coefficients will be found on p. 308.

The Sabine formula has stood the test of thirty years of application with but slight modification but it has become apparent of late that

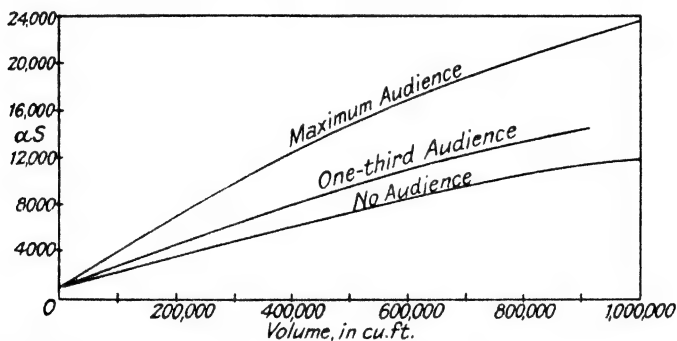



Fig. 106.—Variation of Absorption required in Halls with Volume (*Watson*).

getting the correct time of reverberation for a frequency about 500 c./sec. is not all that is necessary to give a building "good acoustics." Sabine himself recognized this when he adopted from pure physics the methods of *schlieren* and of the ripple tank to observe in model sections of buildings where the sound waves went to after being sent out; he also repeated his reverberation measurements over five octaves.

As regards the latter point, it is now customary to quote absorption coefficients for materials and to calculate reverberation times at three frequencies round about 100, 500 and 4,000 c./sec. to represent the extreme bass and treble as well as the middle frequencies, but research is still in progress to decide what is the desirable reverberation : frequency characteristic for auditoria is designed for different purposes. It is recognized that the curves for speech and music should not be the same. Whereas the musician likes strong reinforcement in the bass,

the intelligibility of speech often depends on the amount of "high frequency" that the listener picks up, since these are prominent in the transients which characterize most of the consonant sounds. In broadcast studios, however, it is generally thought that the ideal to be aimed at, at least when speech and chamber music are being radiated, is an acoustic illusion to give the listener the impression that the music or speech is being made in his own room. Unless the listener forms part of a discussion group or is at home in a baronial hall, this requirement demands in a large class-room rapid damping of sounds, except that the treble may with advantage for ease of being understood be more lingering when the voice is being broadcast. In small studios these conditions would obtain without the necessity for much

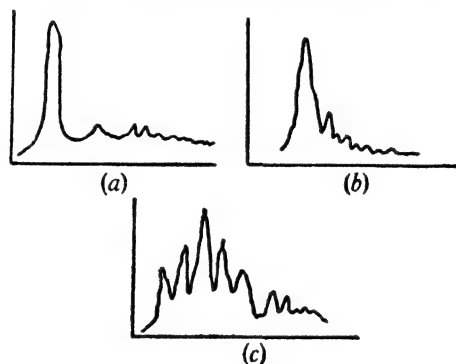



Fig. 107.—Decay of Sound in Auditorium.

correction but something has to be done whenever a few players or speakers are grouped in a large studio.

In Sabine's theory it is assumed that sound decays exponentially in a room. Research by Mason and Moir [5] has shown that, in those having large protuberances such as galleries comparable with, or larger in average dimension than, the wave-length of the sound, the decay is markedly discontinuous, often having a series of peaks of intensity. These workers were able to trace out the sound envelope as received by a microphone from the loud-speakers in a cinema to which a short wave-train of constant frequency has been supplied. Three traces from their records are shown in Fig. 107, (*a*) in a theatre well lined with absorbent so that all reflections are weak and only the primary impulse stands out; (*b*) on one of moderately good acoustics having some fairly large reflecting surfaces; (*c*) in a simple box-shaped room having hard parallel surfaces producing strong reflections, some of which

interfere favourably at the microphone to produce peaks higher than the direct radiation.

They also point out that the sound heard by the ear depends greatly on the form of the reverberation : time curve in the first few milliseconds of the decay, that is, up to the instant when it begins to be masked by the oncoming next syllable of speech or note of music. It is important that any reflected sounds falling on the ear at this period should either come so hot on the heels of the direct sound as to add to it without loss of intelligibility or be quite absent if open-air conditions are looked for.

This brings us to the question of prominent reflections. If these are established between hard and parallel surfaces, standing waves will be set up whenever a natural frequency of the air column is struck. Even " off the note " disturbance of the average reverberation may be expected, so that such a room will have a much indented (reverberation : frequency) curve. Moreover, what are known as " flutter echoes " may follow the emission of a pulse of sound between the two walls, as the ear is bombarded from left and right alternately with periodic waves having slowly decaying intensity. In a small room, these echoes can turn a pulse into a musical note, just as a fence of overlapping slats can turn the sound of a nearby footfall into a note ; in a larger room the impulse will be overlaid with a waxing and waning intensity as its sound dies away.

Two remedies for this state of affairs are available. One is to cover the offending walls with absorbent, but not to an extent unjustified by the requisite reverberation time. The other involves inclining the walls at a small angle of about 10 degrees or attaching panels similarly skewed to them. Rather unsightly in an auditorium, the latter expedient is commonly adopted in small studios for recording and broadcasting (Content and Green).[6]

In any event, the best treatment for most auditoria is to diffuse the sound outside of the stage, regarding this as a radiator and, therefore, providing it with hard surfaces to send out the energy into the body of the hall. The absorbent, if any, required to give a reasonable amount of reverberation is then distributed fairly evenly over the portion occupied by the audience. The convex reflecting surface is a device to aid general diffusion. Such surfaces are often set vertically at the sides of the dais or proscenium arch, cf. the former Queen's Hall, London—but there is now a suggestion to set half cylinders with axes horizontal on the ceilings and along the side walls of concert halls.

Such poly-cylindrical reflectors have already appeared in studios for recording and broadcasting. Volkmann and Boner [7] describe experimental studios in the United States where reflectors of quarter-inch plywood bent into arcs of about 3 ft. radius have been successfully used. The latter author prefers to leave spaces for flat surfaces between the cylinders, or to use different radii of curvature in adjacent cylinders, as, if the pattern is repeated all over the room, the structure acts selectively on wavelengths of sound approximating to the width of the reflector; it acts in fact as a diffraction grating. As a matter of experience, Boner finds that the decay in rooms so constructed approximates more closely to a steady exponential fall without peaks than in the conventional room with flat or steadily curved walls. In this construction, rooms up to 8,000 cu. ft. in volume may have a uniform reverberation time for frequencies from 40 to 17,000 c./sec. Furthermore, with a well-diffused sound pattern, the search for the best microphone position, which wasted a lot of the sound engineer's time, is obviated.

Other problems arise in large cinemas, where the sound from the loud-speaker has to be synchronized with movements on the screen. The loud-speakers are usually mounted one on each side of the screen or behind it, in order to give the illusion that the sound is coming from the pictured characters on the screen, a condition known to the acoustic engineer as intimacy. If an auditor is bombarded with diffuse sound coming from various directions, there is a lack of this sensation of intimacy even if the reflections follow the direct sound so quickly that no loss of intelligibility results. If the reflected sound is not entirely removed by absorbent materials on the walls, which may cause deadness to musical rendering, it is evident that those reflections which carry a fair amount of energy must come to the listeners' ears in a direction not widely different from the direct sound. The condition for intimacy is then that the prominent reflected ray must subtend with the direct one as small an angle as possible. This is to be secured by having a low ceiling and by damping any rays that might come off the side walls near the screen or off the rear wall by the application of absorbent material to these parts (Mason and Moir).

It has recently been found that when loud-speakers are installed in a building to amplify the source, if the amplified sound arrives at a listener between 5 and 35 milli-seconds after the direct sound, the level of the former can be considerably greater than the latter without the presence of the loud-speakers being noticed. In other words all the

sound appears to come from the primary source. This, so-called, Haas effect is clearly important in the setting up of amplifying systems.

As there are now suggestions to locate the source of sound at a definite point on the screen, and thence to move it about following the image for stereophonic effects, quite complex problems in applied acoustics will have to be considered when to a phase difference between the sounds from two loud-speakers are added the phase differences due to reflections (cf. what has been written on " binaural location," p. 283).

Effect of Frequency on Time of Reverberation. The calculations of Sabine were based on a pitch of 256 (" middle C ") taken as the average musical pitch. Such a compromise has to be adopted, in view of the fact that the sounds in the auditorium cover the complete musical range. When Sabine experimented with organ-pipes of other frequencies he found a change in the time of reverberation. This variation of reverberation with pitch results from two influences : (1) the variation of reflecting power of materials for sounds of different frequency ; (2) the variation of the minimum audibility with frequency. The latter influence is indicated on Fig. 100, p. 277 ; the other influence will be discussed in a later section.

Concentration of Sound. We have spoken at present as though the intensity reached after the sound had become established were the same all over the room. This is by no means the case if there are large concave surfaces which tend to bring the sound to a focus, concentrating the sound in some parts to the detriment of others. Where such is the case, an auditor moving about in the room during the production of a steady sound will hear marked oases and deserts in the sound intensity. " Dead spaces " caused by concentration of the sound into other parts are to be avoided ; and large smooth surfaces are detrimental, except behind the source of sound, where they may serve to send out plane waves to distant parts of the auditorium. When defects due to these are detected, it is advantageous to cover the surface with poorly reflecting material, or to break it up by heavily embossed decoration, the size of the protuberances being of the order of the long wave-lengths, a foot or more across.

The acoustical effect of a proposed design can be studied in advance of construction by making sectional models, both vertical and horizontal, through the proposed position of the source. The paths of sound waves through the room can be studied in the model sections by the spark-pulse method (p. 16), or by making the model a shallow

tank of water, and studying the progress of ripples over the surface (p. 18). The first method is more tedious, but gives more definite results and was extensively employed by Sabine in the elucidation of acoustical defects in existing buildings. If the walls of the model are made of some plastic material, the second method allows rapid adjustments of the section, until focussing of the waves is reduced or eliminated.

The paths by which the sound can reach an auditor, e.g., the direct wave and the once-reflected wave, should be made as equal as possible, otherwise, in place of the desirable reverberation the sound may rise again as an echo after it has once died away. The time of elapse between the direct and once-reflected waves to give the impression of a distinct echo is a personal quantity, but it is of the order of a fifth of a second (cf. p. 12). Echoes may be present in a building having a very high ceiling in relation to its horizontal dimensions, and may be remedied by the same means as those adopted for curved surfaces, i.e., by spoiling the reflection.[8]

Other Influences in Buildings. Resonance and interference play a minor part in auditorium acoustics. A certain amount of distortion is produced by the resonance of sections of partition or wall acting as soundboards to an emitted tone of the right frequency, or of volumes of air contained in small rooms, alcoves, etc. The effect is not very noticeable, as the resonant bodies in question are highly damped.

In the reverberation in small rooms, however, natural frequencies of the air space occupying the room are noticeable. When an oscillograph record of the reverberation is taken, Knudsen[9] finds these resonances present no matter what the frequency of the exciting source. These natural oscillations are particularly noticeable when a " reverberation meter "[10] is used in place of the more primitive but effective stop-watch. These mechanical recorders usually employ a warble tone (p. 278) which is turned on for a while, then switched off at the same time as a chronograph is started. The chronograph is stopped again when the intensity received by a microphone falls below a level corresponding to minimum audibility.

When a steady tone of constant frequency is sounding, an interference pattern is produced in the room. The variation in intensity throughout the room due to this cause may be shown by moving a tuned resonator, e.g., a hot-wire microphone, about the room, and noting the difference in response at different points even when there are no focussing surfaces present. During the rendering of speech

or music, this pattern is constantly shifting with the wave-length, and moreover is close-woven in the sense that the distances between maxima and minima due to diffraction are small, so that the effect generally passes unnoticed by the listener.

The acceptable time of reverberation varies by about 10 per cent. between music and speaking. Discrimination is now being made between different classes of musical sound, e.g., musical taste requires for the staccato notes of the pianoforte a reverberation differing from that of the sustained notes of the organ.

Measurement of Reflecting Power. In order to find, for building or other purposes, the fraction of incident sound energy reflected by a surface, we have at our disposal both small-scale (cf. p. 240) and full-scale experiments. Both types are open to criticism, but the results obtained by them show concordance, and are probably accurate enough for technical use.

Of methods using large sheets of the material, the substitution method used by Sabine in his reverberation experiments has been already mentioned. Other investigators rely, in principle, on the measurement of the response of a suitable detector to a sound produced nearby with and without a sheet of the material in the vicinity. The method is not so simple as it sounds. If the experiment is performed indoors, as is usual, or even if it is performed out of doors near the ground, reflections are produced by all neighbouring objects, forming, when the source is steady, interference patterns on both sides of the sheet. This makes possible only a comparative method, in which nothing is moved between two measurements except the sheet of material. Watson [11] placed this over the doorway between two rooms, focused upon it the sound of an organ pipe blown at constant pressure, and measured by Rayleigh discs the intensity reflected to A in the first room, and transmitted to B in the second (Fig. 108). The observer was enclosed in a box in order that his movements should not alter the diffraction pattern. As a working assumption, the deflection of the disc had to be taken as proportional to reflection or transmission produced by each sheet.

Results for specimens of the same material do not very precisely agree. Not only are there discrepancies when one compares different methods or small-scale with full-scale, but even between results reported by different laboratories using the same method. Assuming that the specimens have been mounted in the same way the remaining sources of error reduce to (1) differences in size of speci-

mens, (2) differences in location with respect to the incident sound. The former arise in comparing results, for example, by the pipe and the reverberation methods, the latter in the full-scale (reverberation) methods in which laboratories are of different size and shape and the source not at the same relative point. To get the reverberation to take place under as nearly universal conditions as possible, it is the practice in acoustic laboratories to use sound distributors in the form of large boards which are slowly rotated about a horizontal or vertical axis while the test is being made. In this way the sound

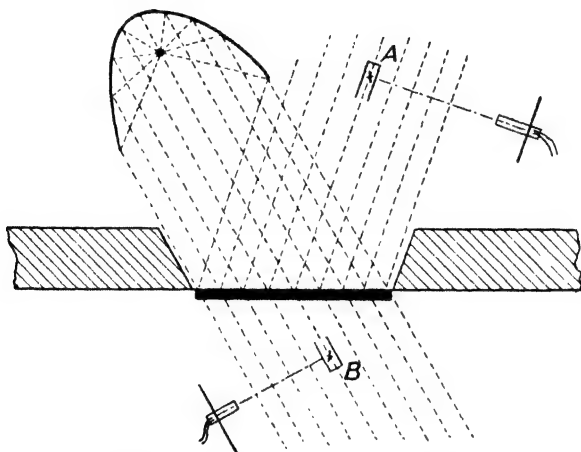



FIG. 108.—Measurement of Absorption Coefficients (*Watson*).

falls at various angles upon the specimen during the period of reverberation.

In applying the measured coefficients (*vide infra*) to a design, an architect needs to know the values for several frequencies in order to notice how consistent they are, as a specimen showing selective absorption will distort the sound.

Other workers have followed similar lines. An organ pipe is a rather unreliable source to use with a resonator, seeing that the latter is so sensitive to change of frequency ; others have employed tuning-forks as sources, and telephone receivers. It is necessary to ensure constant amplitude of the source as the varying conditions tend to react on it.

The measuring of absorption coefficients of porous materials in such a laboratory leads one to ask : what, fundamentally, is the action

of a porous material to sound? This problem has been treated theoretically by Rayleigh and others, using the concept of a nest of capillary tubes to replace the actual texture of a real material. In an attempt to test such theories Penman and Richardson [12] measured the absorption coefficient—or, more precisely, the acoustic impedance—of such an artificial material. A nest of glass capillary tubes, all of the same diameter and having the spaces between the tubes blocked out with wax, was placed at one end of a wide tube and a loud-speaker as source at the other. The pressure amplitude in the stationary waves set up in the wide tube was explored and hence the impedance of the nest of capillary tubes calculated.

While the general behaviour of a porous material to sound—variation with frequency and thickness—is aptly described by Rayleigh's theory in practice, yet the maximum absorption for any given frequency is reached sooner as the pores are lengthened than the theory would indicate. Indeed at high frequencies this maximum is reached in quite a small thickness by a real absorbent. One is forced to the conclusion that the actual damping in narrow tubes and in perforated structures is greater than that in theory, especially at high frequencies (cf. p. 260).

Scott [13] of "Metro-Vick." has recently used a modification of the original tube method (of H. O. Taylor) for studying the absorption of materials to make measurements on conduits lined with absorbent such as used in certain pipeline constructions. He points out that in this case, it is not sufficient to consider the lining as absorbing a certain proportion of the sound energy which falls upon its surface but that, for accuracy, a theory must envisage an attenuation of sound within the walls of the tube itself. Thus, when a lined conduit is in question, the theory must include the sound waves which pass along the material of the lining as well as through the air in the pipe.

The total attenuation is small at low frequencies because of the high impedance of the surface of the lining and also at high frequencies because then the sound from the loud-speaker is directed mainly along the axis. The intermediate peak of absorption can be adjusted at will by suitable choice of the ratio of air space to lining space.

Absorption Coefficients and their Use. Absorption coefficients (given by the fraction of incident energy not returned) for some common materials are shown in a table for $n = 512$ c./sec. The numbers represent average values obtained by the above methods, and do not distinguish between absorption and transmission,

that is, they are actually reflection coefficients subtracted from unity.

Asbestos cloth (¾ in.)	·26
Carpet (½ in.)	·30
Concrete	·17
Cork (2 in.)	·23
Glass	·027
Hair felt (1½ in.)	·58
Marble	·01
Common plaster	·03
"Acoustic" plaster	·30
Wood sheathing	·06

To find the number of absorption units in a room it is then necessary to take each area and multiply by the appropriate coefficient. The total gives the quantity αS. An example will make this clear :—

Acoustic Data of Great Hall, University College, London.
Volume : 170,000 cu. ft.

		Units
Wood, ceiling, floor, wall panels, etc.	16,000 sq. ft. @ ·06 =	1,000
Plaster ; upper part of walls	9,250 sq. ft. @ ·03 =	280
Glass ; windows	2,820 sq. ft. @ ·027 =	70
	Total =	1,350

With a full audience of a thousand persons, and an extra 4·7 units for each, the total absorption equals 6,000 units. With one-third audience, 3,000. Reference to Fig. 106 shows that more absorption must be introduced (1,000 more units) to reduce the reverberation at a sparsely attended meeting. It was therefore recommended to cover the upper part of the walls with "acoustic plaster," which is marketed for the purpose.

Acoustical Properties of Materials. As regards their action on incident sound waves, materials may be divided into two classes, porous and hard. We saw in Chapter VII that the propagation of sound through narrow channels is attended by rapid reduction of amplitude, by reason of the energy lost in friction. This is the accepted explanation of the true absorbent qualities of porous materials. The absorption is a function of the size of the pores, and a geometric function of the length of the pores, which is the thickness of the material, i.e., if one inch of material absorbs a quarter of the incident energy, two inches will absorb also a quarter of the remainder, making a loss of seven-sixteenths in all. The intensity thus falls in passing through a material according to an exponential law :—

$$I_x = I_0 e^{-\alpha x},$$

representing the intensity at a distance x from the datum where it is I_0. Some of the sound reflected from porous materials is not turned back at the surface but penetrates the material first. This is shown in Fig. 109 (after Watson [14]), wherein we see that the fraction reflected increases first and tends to a limit.

A partition impervious to air cannot reduce the sound intensity by damping air waves set up in it. A rigid partition, prevented from vibrating to and fro, can only transmit the sound by longitudinal waves such that the displacements are at right angles to the surface of the material, and can absorb the energy only by the friction induced by such vibrations. Both these effects are very small since the forces on the surface due to the air waves are generally quite small, so that

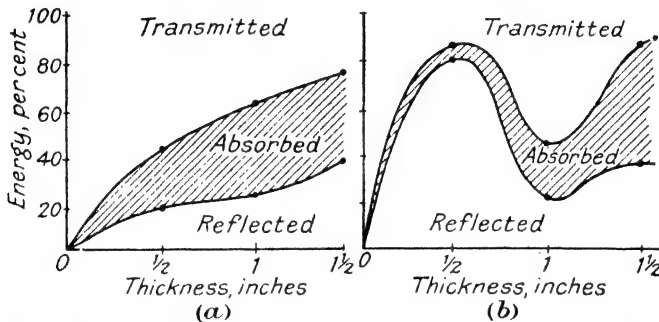



FIG. 109.—Action of Materials on Incident Sound, (*a*) Porous, (*b*) Non-Porous (*Watson*).

such a body forms a nearly perfect reflector. In practice, such walls, especially wooden partitions and floors, are only held tightly by their edges and are free to execute lateral vibrations like a drum, under the forcing of the incident air waves, and so to send out corresponding air waves both in front and behind, but mainly in the latter direction. The extent of this response for a given partition will naturally depend on the frequency, and at one pitch there will be resonance, or if the frequency of the source be kept constant, and successive layers be added to the partition, the response will depend on the thickness—at one particular thickness there will be resonance. These considerations explain the anomalous reverberation in buildings at certain frequencies, and why a partition of double thickness may transmit more sound than a single partition. A case of the latter is shown in Fig. 109. Some experiments by Watson on plaster partitions also gave support to this theory of transmission by forced vibration. He

arranged a series of wooden beams to press on the partition with a force which could be gradually increased. As the partition was "tuned up" in this way, its power of transmitting notes was changed, and moreover the tone which it transmitted best, its own natural frequency, changed. When the pressure caused cracks to develop in the partition, there were sudden drops in the transmission, until the cleavage was sufficient to open vents in the partition, when of course a considerable proportion of sound got through.

Erwin Meyer and his colleagues at Göttingen advocate a wall absorbent in which the "pores" are tapered channels. This is secured by covering the walls with large pyramids of rock wool, having their vertices facing outwards. In this way an effective "dead" room was constructed and tested for its "free-field" characteristics by showing that the sound level decreased inversely as the distance from the source, as for spherical radiation, except at the lowest frequencies.

Silencing.* Before passing to the insulation of buildings from noise a few words may be said on the prevention of noise in machinery, etc. This problem is becoming an acute one in modern urban life, but the actual remedies lie mainly outside the domain of sound. The main considerations which affect the problems are as follows:

(1) Prevention of moving contact between hard unyielding bodies, e.g., of wheels on road or rail in traction.

(2) Prevention of sudden discontinuities or accelerations in the motion, the "chattering" of valves, the sudden exhaust of high-pressure gases into the air, to be obviated by gradual lowering of the pressure through a silencer.

(3) Oiling of moving parts to prevent squeaks induced by friction.

(4) Balancing of the machinery, so that as far as possible moving parts have equal inertia.

Another noise which it is difficult to eliminate in the world of to-day is that of a rotating propeller. In the case of the airscrew, the sound is mainly that of the Æolian tones of the blades. Stowell and Deming [15] have simulated the effect by rotating a cylindrical rod about an axle passing through the mid-point at right angles to the axis of the cylinder. Although the diameter of the rod to be inserted in the Æolian tone formula (55) is fixed, the velocity of the air past the rod increases from the centre to the tips, since this velocity is compounded of the speed of advance of the whole propeller and the velocity of each point on the rod round the axle. Consequently, as

* *See also* p. 226.

these authors found, the resulting sound contains all tones corresponding to this range of speed with the highest pitch corresponding to the tip speed. The sound energy increases up to a maximum for a radius of the rod slightly less than that of the tips. (This is probably due to " spilling " of the eddies over the tips reducing the energy at the tips themselves.) Hilton [16] has continued this research with models in a high-speed wind tunnel. When the tip speed approaches the speed of sound in air, shock waves are produced (cf. p. 24), and the timbre of the sound suddenly changes.

Another troublesome noise arises at certain speeds from a ship's propeller, particularly when it is made of a well-resounding material like bronze. This phenomenon is known as the " singing propeller." From the periodic decaying and resurgence of the sound it appears to be a relaxation phenomenon, probably also connected with strong vortex formation behind the blades at one or more points in their path. Shannon, Conn and Arnold [17] have made an exhaustive study of this problem and find that variation in the wake over the propeller disc is the fundamental cause of " singing " and that this can be reduced by avoiding all obstructions to the flow through the disc and setting the propeller well clear of the stern post and rudder. The form of the leading edge of the outer third of each blade is important as it controls the breakdown of flow. The blade is bent so as to make this edge sharp and free from sudden reversals of curvature.

Insulation of Sound. Having reduced the noise of machinery to the greatest possible extent, the next problem, if the machinery is located near or in a building, is to insulate it so that as little of the vibrations as possible is transmitted through the floor and walls. The remedy is to introduce discontinuities in the solid path of the waves between the two systems. A motor or stationary engine has its bedplate bolted to a concrete floor with layers of wood and felt between. Occasionally it is necessary to construct sound-proof rooms for experiments. In building the walls of such a room the insulation principles already laid down are followed, viz., rigidity of casing, discontinuities in the material, inner materials containing air spaces or pores.

Bedell [18] describes the construction of a room at the Bell Telephone Laboratories designed to simulate an unlimited acoustic field. The walls of the room are covered all over with a number of layers of flannel and muslin separated by air spaces, varying from ½ to 3 in. in thickness, the floor having a grating supported free of the absorbent

for the experimenters to walk on. Measurements of the absorption coefficient of the material both by tube methods and in the complete room showed this to average 97 per cent. above 150 cycles/sec. Such rooms are, of course, of great interest in measurements of audition.

Meyer [19] has tested the frequency characteristics of this type of absorbent, more particularly in the case where the membrane enclosing the air space is not porous. In such a case the natural damped vibrations of the air cell play a part in its sound-absorbing qualities. The electrical analogue of this system is a resistance in series with an inductance and a condenser. The mass m of the membrane fulfils the function of the inductance; the capacitance is that of the cavity and $= l/\rho c^2$, where l is the depth, supposed small compared to the wave-length of the sound. The stiffness of the membrane contributes to the resistance. The natural (fundamental) frequency is then $\frac{c}{2\pi}\cdot\sqrt{\frac{\rho}{ml}}$ when the resistance is neglected. The absorption is naturally very selective. Damping of the air cavity by introduction of cotton-wool increases the absorption but makes the resonance peak sharper; this effect the author ascribes to coupling between membrane and air-cell. He suggests that such a system—e.g., a paper membrane 5 cm. from a wall, with the interspace stuffed with cotton-wool—would be useful in technical acoustics when it is desired to absorb the lower frequencies at the expense of the higher.

Constable [20] has also considered the effect of an absorbent lining between double partitions, both of which may drum. His theoretical treatment is more general than Meyer's, embracing as it does the ricochetting of the sound energy within the cavity between the two panels, some being absorbed by the lining and another fraction being transmitted through the panel at each rebound. If the absorption coefficient of the lining is not too great, a simple formula for the insulation is obtained, viz.:

$$R = \frac{r_1 r_2}{A} \times \text{(the total absorbing power of cavity)},$$

where R is the net sound reduction, r_1 and r_2 are those due to the panels and A is the area of the panels. He confirmed this formula by tests of double aluminium partitions lined with felt. In conjunction with Aston,[21] this author has also obtained vibration patterns of glass windows and brick walls. In the former case, the vibrations were excited by a near-by loud-speaker, and the amplitude measured by search coils stuck on the glass within the field of an electro-magnet.

For the brick wall a brass rod attached to a moving-coil loud-speaker was pressed against the wall. The amplitude contours fall into patterns which have a rather remote resemblance to Chladni figures,

For ordinary building an elaborate construction is unnecessary, sufficient insulation being obtainable by using double partitions. floors and doors, with hairfelt or some other insulator between, and making the outer walls massive and rigid. Pipes and metal supports running from one room to another are efficient conductors of sound in the direction of their length, especially if merely suspended instead of being encased in plaster, and should not run directly from one room to the next.

Particular attention has to be paid to the ventilating system, as the pipes composing this debouch directly into the rooms. Transmission of sound by the material of the pipe may be prevented by discontinuities and plaster casing, but the enclosed air acts as a speaking tube between different rooms unless the vibrations of the air column are broken up. This may be accomplished by introducing baffles either in the form of hairfelt, covered with cloth that hair may not be blown along the duct, or of balls of metal gauze. A spreader in the form of an absorbent sheet placed in front of ventilator openings has been found effective. Such devices divert or damp the rapid vibrations which transmit sound, while impeding very little the direct air-flow induced by a powerful fan.

References

1. *Amer. Architect*, **68**, 1900, 1911.
2. *Akad. Wiss. Wien, Ber.*, **120**, 613, 1911.
3. *J. Frank. Inst.*, **198**, 73, 1924.
4. *Phys. Rev.*, **25**, 391, 1925, and **27**, 620, 1926.
5. *J. Inst. Elect. Eng.*, **88**, 175, 1941. *See also* Bolt, *J. Acoust. Soc.*, **19**, 79, 1947.
6. *Proc. Inst. Radio Eng.*, **32**, 72, 1944.
7. *J. Acoust. Soc.*, **13**, 234 and 244, 1942. *See also* Rettinger, *ibid.*, **19**, 343, 1947 ; Somerville and Ward, *Acustica*, **1**, 40, 1951.
8. Aigner, *Akad. Wiss. Wien, Ber.*, **123**, 1489, 1914.
9. *J. Acoust. Soc.*, **5**, 127, 1933.
10. Meyer and Just, *E.N.T.*, **5**, 293, 1928 ; *Zeits. tech. Phys.*, **10**, 309, 1929.
11. *Phys. Rev.*, **7**, 125, 1916. *See also* Davis and Littler, *Phil. Mag.*, **3**, 273, 1927.
12. *J. Acoust. Soc.*, **4**, 322, 1933. *See also* Kosten and Zwikker, *Physica*, **8**, 469, 1941.
13. *Proc. Phys. Soc.*, **58**, 358, 1946. *See also Engineering*, **165**, 97, 1948 ; Willms, *Akust. Zeits.*, **6**, 150, 1941 ; Rochester, *J. Acoust. Soc.*, **12**, 511, 1941.

14. *Phys. Rev.*, **7**, 125, 1916.
15. *J. Acoust. Soc.*, **7**, 190, 1936.
16. *Phil. Mag.*, **30**, 237, 1940.
17. *J. Inst. Eng. Scotland*, **82**, 225, 1938. *See also* HUGHES, *ibid.*, **85**, 55, 1942.
18. *J. Acoust. Soc.*, **8**, 118, 1936. *See also* SMITS and KOSTEN, *Acustcia*, **1**, 114, 1951.
19. *Elekt. Nach. Tech.*, **13**, 95, 1936.
20. *Proc. Phys. Soc.*, **48**, 690 and 914, 1936.
21. *Ibid.*, **48**, 919, 1936.

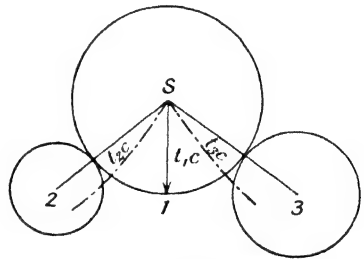
CHAPTER THIRTEEN

TERRESTRIAL SOUND AND SIGNALLING

Sound Ranging. This problem is that of determining the position of an explosive sound from the instants of reception of the sound at three or more listening stations.[1] The recording is done by some sort of microphone, but the currents induced in the microphone circuit are brought to a central station, where they each actuate a separate style writing on a common revolving drum (cf. Regnault's apparatus, p. 6) or deflect a beam of light acting on a moving film in a camera.[2] The three traces are drawn side by side, and from the distance separating the three " kicks " due to the arrival of the sound at the detectors, the position of the source relative to these can be found. Latterly the cathode ray oscillograph has been used as a recorder.

Thus let t_1, t_2, t_3, be the times for the sound to reach the detectors from the source. If t_1 is the shortest time, a circle of radius t_1c with the source as centre represents the distance reached by the wave at this instant (Fig. 110). Circles drawn with the other two stations as centre and having radii $(t_2 - t_1)c$, $(t_3 - t_1)c$ respectively will therefore touch the first circle since $t_2 - t_1$, $t_3 - t_1$, represent the remaining time required for the spherical wave to reach these two stations. Then having the positions of the three stations shown on a map, we can draw a circle of radius $(t_2 - t_1)c$ round the second, and $(t_3 - t_1)c$ round the third; the problem is then to find the centre of a circle which will touch these two and pass through the first station. Practically this is done by having a number of concentric circles drawn on tracing paper, and pushing the paper over the map until the circle which fits best is found.

FIG. 110.—Sound Ranging.

An alternative construction is derived from the consideration that as far as stations 1 and 2 are concerned the locus of S is a curve such

that the differences of distance from any point on the curve to the two stations is always $(t_1 - t_2)c$; the curve is therefore a hyperbola. Consideration of the 1st and 3rd stations shows that the locus of the source with regard to these is another hyperbola. The two hyperbolas intersect at S. Various instruments have been employed to construct rapidly these hyperbolas.

This sound-ranging device was employed by both sides to locate large guns during the First World War, but first apparently by the French under the direction of Esclangon.[3] In reality, sound ranging, so simple in theory, was not accomplished without a great deal of preliminary experiment. The greatest difficulty was that, owing to the powerful guns employed, the projectile left the muzzle with a speed exceeding sound. Accordingly three sounds were in general registered by each observation post: (1) the *onde de choc* forming the envelope

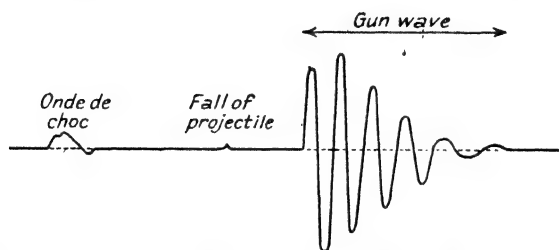



FIG. 111.—Response of Manometer to Firing a Gun (*Esclangon*).

of waves sent out by the projectile as it progressed at a speed exceeding that of sound (cf. p. 24 and Fig. 10*a*); (2) the "gun-wave" which left the muzzle with the projectile and travelled entirely as a sound wave to the post; (3) the fall or explosion of the projectile which travelled from another spot at a speed c and started some seconds later than (2). Obviously only (2) was of value in determining the *emplacement* of the gun, and unless the gun-waves could be picked out from the other sounds, the sound-ranging of such guns would be impossible.

Both *onde de choc* and gun-wave give rise to the sensation of detonation (cf. p. 275), being in general followed by a roll of sound due to successive reflections, but the evolution of pressure being slow in either case, definite tones are not heard. Esclangon noticed, however, that the effect of the gun-wave on a resisting surface was much more pronounced than that of the *onde de choc*, the former being able in fact to break windows on occasion. The gun-wave involves much

slower evolution and much greater amplitude of air motion, and can be detected by a manometer of large volume or instrument of large capacitance, whereas the *onde de choc* is too transient and alternates too rapidly to produce appreciable movements of the large mass though still not fast enough to give the sensation of pitch. As detectors a number of manometers consisting of large reservoirs of air whose motion was detected by membranes, manometric flames, or hot wires were constructed. The typical response in one of these is shown in Fig. 111. This shows how the sound required is easily sorted from the others by the large response of the manometer. Sound ranging was then accomplished from the traces of several of these manometers to the incidence of the gun-wave.

Sound Signals in the Atmosphere. For signals required to transmit over short distances, there is employed one or other of the devices described in Chapter V for producing a large amplitude of vibration in a diaphragm coupled with a horn. When, however, it is necessary to transmit a fog-signal audible for several miles round, from a lighthouse for example, greater power must be employed. The instrument most in favour is known as the "diaphone," and uses compressed air.[4] It is in principle an oscillating engine, employing a cylinder, piston, and valves, but fed with compressed air instead of steam, and governed so as to run at half the frequency of the tone to be emitted. This is the auxiliary apparatus and serves to open a valve between the reservoir and a horn, at each end of the piston's stroke. By this means a series of powerful puffs of air, with a very sharp "admission" and "cut-off," is delivered into the horn at its resonant frequency, producing a sound which can be heard for several miles and whose pitch is very nearly independent of the pressure of air in the reservoir. On hearing such a sound in conjunction with a simultaneous wireless signal, or light signal in clear weather, a vessel at sea can estimate its distance from the lighthouse by the time which elapses between the receipt of the practically instantaneous light signal and the sound signal.[5] As the propagation of sound in the atmosphere is attended with all the uncertainties discussed in the first chapter the diaphone is being superseded for such purposes by the submarine signals whose transmission through the sea is more certain.

Dahl and Denk [6] have made a study of the propagation of sound in a gaseous medium whose motion is of complex character. In some instances of the propagation of sound from fog-horns over the sea they noted marked variations in time of the intensity of sound recorded

by a condenser microphone at a given point in the sound field. Over a period of three seconds, during which the output of the transmitter remained quite constant, variations of intensity at the source amounted to 10 db. This is ascribed to the turbulence in the air. Rolling and whirling air bodies distributed irregularly through the structure act as scattering centres, particularly important for a sound having a wave-length of the same order as the diameter of the air pockets. They suggest that turbulence in the atmosphere might be studied by such acoustical means as the author has attempted to do (cf. p. 320).

These phenomena were first noted when a study was made for the Trinity House Brethren of sound propagation from fog signals under various weather conditions. Indeed, work on the audibility of fog signals goes back more than two centuries, when Derham believed that he had established that rain, fog and snow acted as sound absorbers. Tyndall [7] in 1874 showed the error of the still prevailing opinion; small objects like water droplets cannot cause much dissipation of energy in a sound beam of low frequency, but *irregular patches* of fog and rain by acting as reflectors and refractors make the air a turbid medium from the point of view of sounds of considerable wave-length. He recognized that pockets of high or low temperature in the atmosphere can do the same. From his experiments Tyndall concluded then that inhomogeneity of the distribution of humidity and temperature are the main enemies of sound propagation in the atmosphere.

Various authors have established that the measured decrease in sound intensity was more than ten times as large as that calculated by the classical formula based on friction and heat conduction alone. The final explanation of this discrepancy is due to the investigations of V. O. Knudsen; [8] he proved that the moisture content of the air causes a considerable additional absorption. The molecular absorption increases rapidly with increasing frequency. From the values at a low degree of air humidity it increases sharply up to a maximum, which at a relative humidity of 12–17 per cent. depends on the frequency. From there onwards the absorption falls uniformly with further increase in humidity. Fig. 112 shows the curve for molecular absorption in relation to relative humidity for various frequencies. Knudsen's data have been recalculated for a reduction figure of db./100 m. From the point of view of sound transmission, particularly in the case of higher frequencies, it would be a bad thing if we had to reckon in our latitudes with humidities of 15–20 per cent.,

because at 18 per cent. humidity, for example, the molecular absorption for 10 kc./sec. amounts to 28 db./100 m. However, low humidity values such as this occur only under special climatic conditions, e.g. in deserts. Knudsen was able to confirm the extremely high sound absorption in the atmosphere occurring in the American Salt Desert at Salt Lake City. In our latitudes the relative humidity hardly ever falls below 30 per cent. even in summer; at this humidity the damping is only about half the extreme value given above.

When a surface is warmed the thermal currents above it result

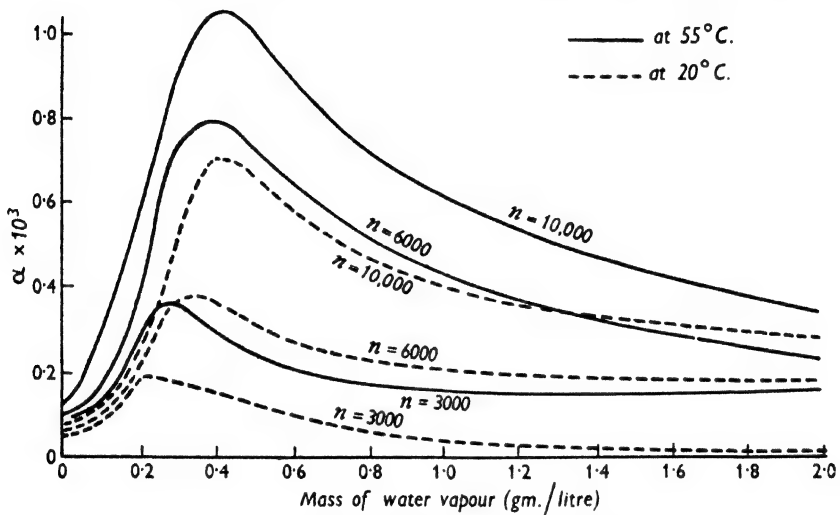



Fig. 112.—Absorption of sound in Air containing Water Vapour (*Knudsen*).

in eddies, but less regular in most instances. (It is these which cause the ghostly undulating appearance in the appearance of objects behind the thermal strata.) From the small dimensions of these eddies, it may be assumed that these will selectively affect the high-frequency sound-waves. The microphone records wide variations in the intensity received from high pitch sounds, whereas the effect of turbulence on the lower frequencies is to lower the intensity received, it is true, but not to make it unsteady.[9]

What might be called good sound conditions out of doors—calm weather overcast without much insolation—results in a minimum loss of about 3 db. per 100 ft. in the middle of the audible range. Bad conditions are gusty high winds or—near the ground—calm with strong

insolation—when the attenuation may amount to 200 db. in 100 ft. on the average, but fluctuating within $\pm$ 50 per cent. of this.

Peculiar conditions arise when the mean diameter of the turbulent eddies approximates to the wave-length of the sound, rather like those which occur when light penetrates a fog. There will be marked scattering of the beam to either side of the direct path as the author has shown using high-pitched sound and an artificial fog formed of light solid spheres held in suspension in a fluid medium.

Apart from these selective effects, turbulence is, of course, a notable dissipater of energy and is as likely to convert some of the particle motions in sound waves into heat as it is to act on their less frequent motions which we associate with gustiness.

Absorption of Ultrasonics in the Atmosphere. Effect of Fog and Smoke. Apart from the operation of the inverse square law, sounds in air are affected by absorption of another type to an extent which depends on the frequency. The tones of shortest wave-length are usually most affected, the mechanism of the absorption being similar to that produced by the scattering of light waves in a turbid fluid. Neklepajev [10] showed that sounds having frequencies of the order of 40 kc./sec. were strongly absorbed in clear air, while Altberg and Holtzmann [11] found that the absorption of an artificially produced fog increased five or six times as the wave-length of sound was reduced from 6·5 to 1·5 cm. This type of absorption becomes conspicuous as the wave-length of the sound approaches the average size of the particles in suspension in the medium. Under disturbed conditions one hears the lower components in the complex sounds emitted by church bells and aircraft more plainly than the higher tones; whereas the reverse is the case on a still night when the dissipative factors are less in evidence.[12] The production of ultrasonics in air has revived interest in this question, and experiments have been made on a small scale to compare the transmission through an artificial fog and through clear air. Laidler and Richardson [13] made smokes of measured concentration by burning stearic acid or magnesium in a chamber in which a quartz oscillator was working and measured the supersonic absorption. As a variation with size of the obstacle was expected from the theoretical investigation of Sewell,[14] some experiments were also made with a fog made of the spores of the puff-ball fungus, which consist of uniform spheres one two-hundredth of a millimetre in diameter. Care has to be taken to use very low power in the source since Andrade and Parker [15] and Brandt and Hiede-

mann [16] have noted a strong tendency of the smokes to coagulate under the influence of ultrasonic radiation.

In all the experiments a rapid increase of absorption with frequency was noted. This is borne out in practice. It is well known that on a misty day or on one accompanied by rain showers, sounds of high frequency are considerably attenuated, while those of lower frequency are little affected. The latter are in fact more likely to be disturbed in propagation on a gusty day. On such occasions, the distorting elements in the sound field are the eddies set up by turbulence, which can well be compared in size with the wave-length of audible sounds of quite low pitch, in spite of their intangibility, although the actual degradation of energy cannot be envisaged so precisely as with solid or liquid obstacles and high frequency sounds.

Attenuation of Sound under Water. If we assume plane waves the amplitude attenuation coefficient in fresh water is in theory 24×10^{-17} multiplied by the square of the frequency (or 2×10^{-15} in decibels). At 100 kc./sec., this corresponds to a fall of 1 db. in about 1 km. At the same frequency the actual attenuation is about 10 times as great. The difference is ascribed to the presence of magnesium sulphate, a weak solution of this substance in the laboratory showing a large absorption compared to distilled water.

In fact there will be in deep sea and lakes spherical radiation, so that the attenuation is given by

$$I = I_0/r^2 + I_0 e^{-\alpha r}$$

or, in decibels: $-20 \log r - \alpha r$.

In a shallow lake (or under the special canalized conditions described below), however, the energy apart from some loss in reflections from the free surface and bed is confined to a disc-shaped volume, and the first term in the above-equation would be replaced by that appropriate to cylindrical waves, viz. I_0/r. (This is almost the only practical case of cylindrical wave propagation in sound.)

Refraction of Sound under Water. As the equation on p. 30 shows, the velocity of sound in the sea near the surface is a function of salinity and temperature. It will also increase with depth due to the greater bulk elasticity of water under pressure.

The general trend of temperature downwards is a diminution, at least for the first kilometre, giving a curvature to the sound paths like that in the lower strata of the atmosphere (cf. p. 20). Under these

circumstances sounds sent out from a ship may be inaudible at a moderately distant under-water listening post (Fig. 113, lower track).

Sometimes, however, a layer of cold surface water may overlay a

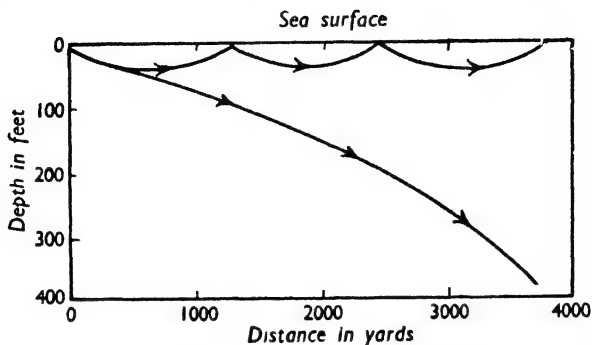



Fig. 113.—Paths of Sound Waves under the Sea.

warmer sub-stratum. In this case the sound waves are curved upwards (Fig. 113, upper track) and the sound " skips " along under the free surface giving very good audibility to hydrophones just submerged at a distance.

Sound Signals in the Sea. For signalling through the sea, the submarine bell has long been employed. Normally only the bell and its clapper are submerged, the apparatus floating so that the striking mechanism is above water. The latter is worked by compressed air and actuates the clapper at fixed intervals corresponding to the flashes of light given by a lighthouse. Other bells of less importance to navigation have no mechanism, the clapper hanging free and the bell striking irregularly under the action of the roll of the buoy to which it is attached on the sea. The sound from such a bell penetrates the sea to a considerable distance, and the submarine bell has stood the test of many years' experience, but when an unusually large range is required, more elaborate and expensive apparatus is needed. The density of water being so much greater than that of air, a considerable expenditure of energy is necessary to produce compressional waves of the needed amplitude, and instruments like the ordinary telephone transmitter are useless.

The commonest signal, using diaphragm vibrations, is the Fessenden oscillator.[17] Two types are in use, based on the electro-dynamic (cf. p. 129) and electro-magnetic principles respectively. The latter

type, as manufactured by the Signal-Ges., Kiel, is shown in section in Fig. 114. The diaphragm is formed of the base *B* of the iron casing of the instrument, which also supports two copper rods *R*, *R* fixed at their upper ends to the ends of two copper tubes *T*, *T* which support another cast-iron plate *P*, over and just clearing *B*. Alternating current led in from the top excites two coils with the requisite frequency, causing violent periodic attraction between *B* and *P*, and exciting longitudinal vibrations in *R*, *R* and *T*, *T*. The net effect is that the base is thrown into powerful transverse vibration, which sends out sound waves into the water.

Fig. 114.—Submarine Oscillator (*Du Bois Raymond, Hahnemann, and Hecht*).

Finding the Direction of a Source of Sound. We have seen in the eleventh chapter that the binaural faculty enables us to get a very good idea of the direction in which a source of sound lies. This faculty was used during the war of 1914–18 for locating hostile sounds in all three of the elements, with the aid of intensifying apparatus. The sounds whose direction was sought were usually those of hostile aircraft, and artificial ears consisting of two long conical trumpets were mounted with parallel axes at the ends of a strut several feet in length, which could be turned so as to point towards the source. Connecting tubes ran from the narrow ends of the trumpets to the listener's ears, and their position was adjusted until the intensity at the two ears was the same. In order to get the elevation as well as the bearing of the aeroplane, two pairs of trumpets, each pair having two listening tubes, and two listeners, one for each azimuth, are employed, each having control of the appropriate movements. Experience was necessary before the listeners could accurately follow the motion of the aeroplane, and avoid impeding each other's adjustments. Fig. 115 shows the apparatus devised by the French engineers, in which the four trumpets are arranged in a cross. The British instrument had the trumpets arranged in a T-pattern on two struts at right angles. For night

observation it was arranged to co-ordinate the movements of the listening apparatus with those of the searchlights.

The sensitivity of search on apparatus having trumpets 15 ft. long and 12 ft. apart was reckoned to be 0·1 deg. under favourable conditions. To get greater magnification and consequent sensitiveness of the apparatus, large spherical mirrors of concrete or similar material were constructed by the participating armies, and the listening and direction-finding carried out in the neighbourhood of their foci, with apparatus, of course, on a smaller scale. Generally the mirrors could be turned about an axis to assist in the adjustment.[18]

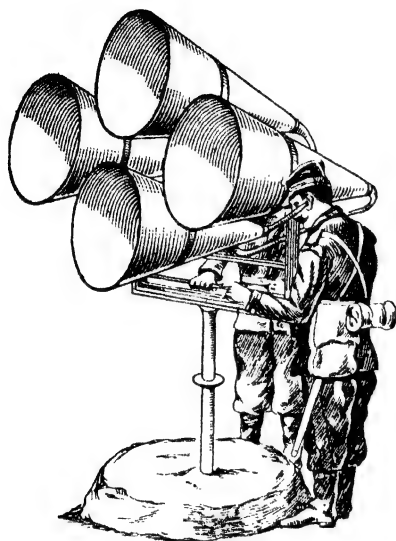



Fig. 115.—Direction Finder.

The listening apparatus led incidentally to a study of the sounds produced by an aeroplane.[19] Among periodic sounds which were discernible that of the exhaust and a number of tones from the airscrew due to lateral vibration of the blades stood out prominently, ranging from 100 to 400 vibrations per second. When the plane is double-engined, beats may sometimes be heard between the two engines; there are also the Æolian tones of the struts, but these are probably

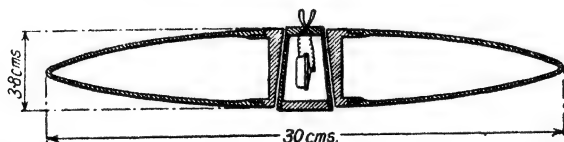



Fig. 116.—Light-Body Hydrophone ([*Bragg*] *Wood and Young*).

too weak to be detected at a distance. Beside these there are the "reflection tones" (cf. p. 58).

Direction-finding in the sea by means of a single microphone was found possible by the British, using a "light-body" hydrophone suggested by Bragg [20] (Fig. 116). This instrument consists of a large

diaphragm in a lens-shaped case, open to the water on both sides. The carbon microphone itself was placed in a central boss of the diaphragm, and responded to its movements. The principle on which it works is that when a body is in forced vibration under the action of another body the relative amplitudes are inversely as the respective masses. The diaphragm, conversely to that of the Fessenden oscillator, is a light body, less dense than water (a volume of air in a glass case, in the example figured), and so the amplitude of the diaphragm exceeds that of the water, securing enhanced response of the microphone without the bugbear of resonance. It was found that, with the case open at both sides, the response was least when the case and diaphragm were turned edge-on to the sound. With a diaphragm shielded on one side by a baffle, direction is found by turning the hydrophone to face the incident sound, when the response is a maximum.

Echo-Sounding. The depth of the sea, or, more strictly speaking, the distance from the surface to the nearest large solid mass, can be determined by timing a sound to the bottom and back. It may be noted in passing that the same principle of detection is now used under the name of "radar" by sending out a pulse of *electro-magnetic* radiation of short wave-length and obtaining its reflection from a distant object recorded on a cathode ray oscillograph. This is, in fact, the electro-magnetic analogue of the acoustic apparatus described on p. 204.

In the pattern of echo recorder used by the British and United States Navies,[21] the sound is produced by an oscillator excited at certain definite instants. The current is supplied by a contact spindle rotating at constant speed. Also on the spindle is another contact set in an ebonite disc which periodically connects the telephones to the microphone receiving the echo. No sound is heard in the telephones unless the receiving circuit is closed at the instant the echo is received. The contact brush can be displaced round the spindle until the echo is caught, by an amount depending on the depth of the water. The two types differ in detail, but this description illustrates in broad outlines the mechanism of both.

The energy reflected, when a pulse or a sound consisting of a few long-period waves is returned from a surface, will be small, unless the surface is of considerable extent, for only a small part of the emitted energy will fall upon it. For this reason the attempt to use the above apparatus, for detecting the presence of an iceberg or of a large ship in the neighbourhood of a vessel carrying an echo-sounding device,

fails. If, however, waves of very short length are emitted through an aperture, interference will destroy the energy diffracted out of the direct path, and a beam of sound will be produced, just as the exceedingly short waves constituting light are concentrated in a beam on passing through an orifice. To render this device effective, waves of frequency above the audible limit—ultrasonic waves—are necessary. It was for this purpose, i.e., sounding, that Langevin [22] first employed the piezo-electric quartz resonator (p. 250). An *ad hoc* aperture is not necessary, the two metal plates forming the electrodes of the crystal resonator (*EE*, Fig. 93) being of sufficient length to ensure that the longitudinal waves shall be directed in a beam limited by the planes

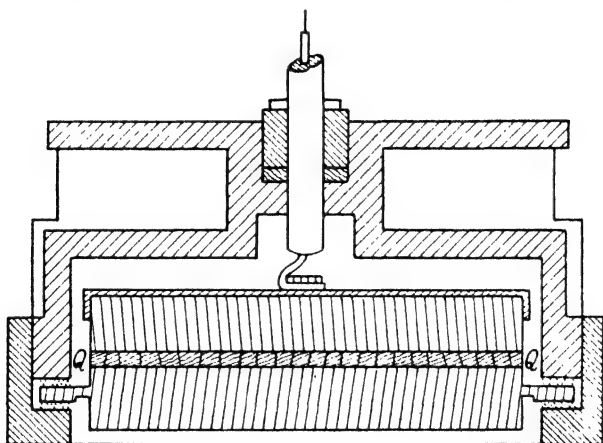



Fig. 117.—Piezo-electric source for Echo-sounding.

of the two electrodes. The technique of sounding by means of these ultrasonic waves has been developed by Boyle and collaborators in Canada.[23]

Langevin obtained a great improvement in the efficiency of the sender by making the electrodes of thick 3 cm. steel, rigidly holding a number of quartz slabs (*QQ*, Fig. 117) in a sort of sandwich, so that the mass of crystal and steel was forced to vibrate as a whole. This mass possessed much greater elasticity than the original arrangement, and with one steel face in contact with the water it was estimated that, using a peak potential of 2,500 volts between the metal plates, one kilowatt of ultrasonic power was obtained. Magneto-striction has also been used.

The Langevin–Florisson system for ultrasonic echo-sounding consists of such a piezo-electric sandwich in contact with the water, with its controlling electric circuit in which a short train of rapidly damped electric oscillations is excited by means of an electric spark. This wave-train is converted into a corresponding wave-train of about $\frac{1}{1000}$ second duration, i.e., short compared with the shortest echo-time encountered. The crystal acts as its own detector both of the emitted and returned ultrasonic waves, as the electric circuit also contains an amplifier which passes on the fluctuations in the circuit, due to departure and arrival of ultrasonic waves, to a sensitive string

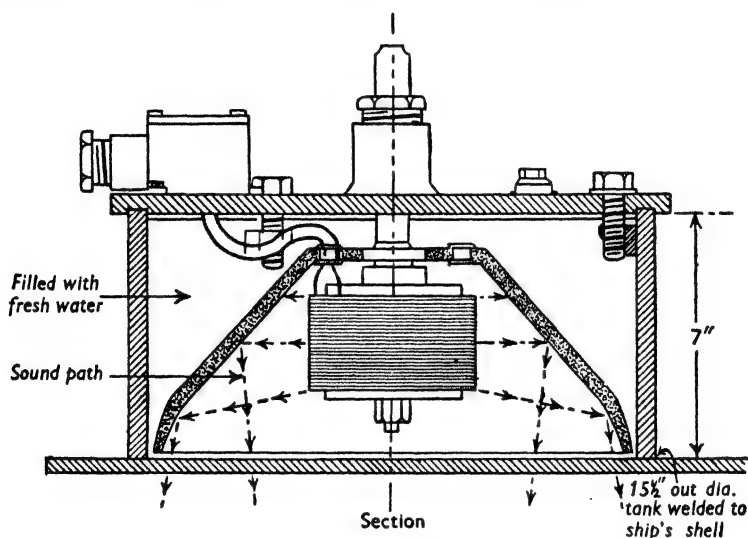



FIG. 118.—Kelvin–Hughes Magneto-striction Echo-Sounder.

galvanometer contained in an "analyser." This analyser is an instrument like Behm's recorder which sets in motion at constant speed a spot of light moving along a scale; and exhibiting "kinks" in its motion at the passage and return of the waves in the manner previously described. For greater accuracy the constant speed of the beam of light is controlled by a phonic motor, worked off a tuning-fork. A relative error of 1 per cent. may be introduced within the specified range (4 to 360 metres) on account of variations in velocity with depth; some of this error could, if required, be reduced by correction from tables.

In the echo-sounder made by the firm of Kelvin and Hughes the

source consists of a number of nickel stampings threaded on a rod wound with coils toroidally so that on excitation the system expands and contracts radially, sending ultrasonic waves through water against the inner wall of a metal horn from which they emerge axially in the form of a beam. The frequency is 15 or 30 kc./sec. and the effective aperture about 2 ft. (Fig. 118).

Distance is not the only information given by the echo-sounder; something about the nature of the bed may be deduced from an examination of the record of the echo. If the latter is sharp and distinct a correspondingly distinct and hard bed is indicated. If, however, the record is fuzzy, with the appearance of some of the radiation having penetrated before being reflected (like what one would expect in a light beam reflected from a mirror from which the silvering was beginning to peel), the inference is that the bed is oozy and indistinct. Indeed, interesting information about the nature of the bed of a lake has been gained by traversing lines across it in a vessel equipped with an echo-sounding device.

Echo Detection. Mortimer and Worthington [24] have studied the depth and nature of the bed of Lake Windermere with this technique. Besides an echo from the floor of the lake, others coming from depths up to 13 m. lower were found. By extracting cores of the mud with a special apparatus, it has now been confirmed that these reflections come from the clay bands found in the cores and due to seasonal variations in the sediments brought down in post-glacial epochs. Comparison between echo records and core analyses yields values for the velocity of sound in the mud which vary between 1,540 and 1,680 m./sec. These preliminary experiments indicate that echo-sounding equipment may be of considerable use in the study of sublacune geology.

A recent device for locating submarines and signalling their approximate position to aircraft is the "radio-sono buoy." The aircraft pilot drops a buoy, weighing about 12 lb., and having an aerial sticking out of the water. The buoy has a high-frequency hydrophone which relays the sounds which it receives, provided they exceed a minimum intensity, by radio waves to the aircraft. By dropping several of these buoys and noting their position, the pilot can assess the position of an enemy submarine from the relative signal strengths.

Another device, the homing torpedo, has two "ears" in the form of high-frequency hydrophones, one on each side. From the relative phase or signal strength of the sounds picked from the propeller of

an enemy vessel, relays turn the torpedo towards it just as a person will turn his head towards a source of sound he is locating.

Most of the ultrasonic noise of a propeller is due to cavitation, so that if a submarine wishes to escape detection by Asdic it must proceed below cavitation speed. There is also a considerable background of noise in the sea on account of surf, etc., which in rough weather may mask the sound of all but the fastest ships. Suggestions have been put forward for a mine exploding under the close action of the sound from a ship, but measurements by the author made during the war show that, unless they are made rather insensitive, such mines may explode under the action of ultrasonic background of a rough sea, at any rate near the shore.

Since the velocity of sound in water is 1,500 metres per sec. whereas that of electro-magnetic waves is 30 million metres per sec., it is evident that a model radar echo-map using ultrasonics may be constructed to one twenty-thousandth scale of the actual thing, that is, supposing the time scale is to remain unchanged. Thus to model the device in which an aircraft pilot flying at 5,000 ft. say, obtains by radar echoes the position and nature of the terrain beneath him, a quartz crystal of frequency about 15 megacycles/sec. just under water in a tank 3 in. deep may pulse towards the floor of the tank. A smooth bottom imitates, for radar, the specular reflection which electromagnetic waves suffer at the sea surface ; a sandy bottom, the diffuse reflection from a rough ground.

In the actual trainer, a radar emitter and receiver in the same building and controlled by the trainee monitored the ultrasonic circuits, while the quartz was carried slowly along the tank (at the scale speed) to simulate the steady flight of the aircraft at constant height.

Cracks in Metals. On a smaller scale, a similar idea may be applied to establish the existence of suspected discontinuities in a metal structure in the form of hidden cracks. Since it is not feasible to switch over the oscillator from emission to detection during the passage of the ultrasonic wave-train, a separate emitter and detector are clamped to one side of the specimen at a small angle to each other or, alternatively, on opposite sides of the specimen. When the radiation arrives at a boundary of any kind in the metal, most of it is reflected, so that the existence of a crack in the specimen in line of the beam is shown by a rise in the energy received by the neighbouring ultrasonic detector or a fall in that received on the opposite side.[25]

Sound Waves through the Earth's Surface. The Geophone. Sensitive sound detectors for waves passing through the earth have been designed and used in connection with mining, both military and civil. The geophone, invented by the French during the 1914 war, is actually a small seismograph, consisting of a very heavy lead weight surrounded by a light case to which it is connected circumferentially. On receipt of an impulse the lead weight remains comparatively motionless because it is suspended between the two discs forming the case, and because its mass is so great; so that the motion is taken up by the case. The relative motion of the lead and the case causes compressions and rarefactions in the listening tube at the back. In operation the front of the case is pressed against the earth, preferably rock, and the two free ends of the listening tube to the ear. If two are used relative direction can be determined by the binaural principle. The geophone is now being used in mine rescue work.

Echo-prospecting. Under this name we may class the methods now being developed to determine the depth of mineral strata below the surface of the earth, by reflection of sound waves in the earth from the surface of discontinuity between the softer earth and the harder strata. These methods have been exploited by Ambronn,[26] and

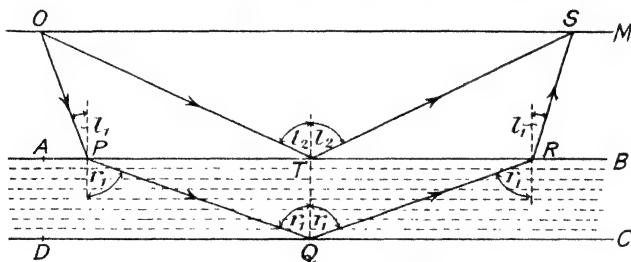



FIG. 119.—Echo-Prospecting.

the details are still secret, but the general principle is illustrated in Fig. 119.

This represents a layer of ore in which the speed of sound is 600 m. per second, beneath a soft layer in which it is only 300 m. per second, and which extends to the ground level. At O compressional waves are excited in the earth. On reaching the surface of separation AB they are partly reflected and partly refracted, but if they are incident on AB at the critical angle (p. 14), they are entirely reflected. The value of the critical angle in this case is 30 degrees for $\sin i = \frac{300}{600} = \frac{1}{2}$.

Of the rays which strike AB, some (incident at i less than 30°) will enter the ore, be partially or entirely reflected at CD (depending on the stratum below CD), refracted again at AB and reach the surface again, following a path such as $OPQRS$; the rest of the energy which they represent returning to the surface direct from P. The rays which strike AB at an angle greater than 30° will be entirely reflected. A certain one of these rays, OTS, incident at an angle i_2, will strike the surface at S, the same point as the ray OP which traversed the lower medium. If $OA = l_1$ and $AD = l_2$, the respective depths of the strata; the time taken for the path $OTS = \frac{2l_1}{V_1 \cos i_2}$; for the path $OPQRS$ it is $\frac{2l_1}{V_1 \cos i_1} + \frac{2l_2}{V_2 \cos r_1}$, where i_1 and r_1 are related by

$$\frac{\sin i_1}{\sin r_1} = \frac{V_1}{V_2}.$$

It is fairly obvious, under the conditions assumed, $V_2 = 2V_1$, that if S is near O the wave will traverse the path OTS more quickly than the path $OPQRS$, but if we take for S a point distant from O, the latter wave will have accomplished a greater portion of its journey in the ore, where its velocity is twice that of the wave following the shorter distance OTS in the soft earth. The refracted wave may therefore arrive before the totally reflected one.

If now, we set up a number of observation stations along the surface OM, and ignore the direct surface wave, the return wave will arrive in regularly increasing times as we recede from O, until suddenly there is a kink in the time curve along OM, representing the refracted wave which has penetrated the ore and is now gaining upon the totally reflected wave. The existence of this discontinuity in the times of transit is a sure sign of a harder stratum below. From these times and from the directions i_1 and i_2 from which the waves return to the surface—found by binaural geophones—the quantities l_1, l_2, V_1, V_2 can be calculated from the preceding equations, giving valuable information as to the nature of the stratum, $ABCD$.

The method should also be applicable when the upper medium is the sea or a lake, and the lower one its bed.

References

1. *See Nature* **104**, 278, 1919; Trowbridge, *J. Frank. Inst.*, **189**, 193, 1920.
2. Blondel, *Comptes Rendus*, **175**, 1371, 1922.
3. *Revue Scientifique*, 115 and 369, 1921.

4. KING, *Phil. Trans. Roy. Soc.*, **218**, 211, 1919.
5. JOLY, *Proc. Roy. Soc., A*, **92**, 101, 1914; *Phil. Mag.*, **36**, 1, 1918.
6. *Nature*, **139**, 550, 1937.
7. *Sound*, **1**, 284, 1883.
8. *Phys. Rev.*, **43**, 1051, 1933; *J. Acoust. Soc.*, **6**, 119, 1935.
9. RICHARDSON, *Proc. Roy. Soc., A*, **203**, 149, 1950. *See also* EAGLESON, *J. Acoust. Soc.*, **12**, 427, 1941; SIEG, *Elekt. Nach. Tech.*, **17**, 193, 1940; SCHILLING *et al.*, *J. Acoust. Soc.*, **19**, 222, 1947.
10. *Ann. Physik*, **35**, 175, 1911.
11. *Phys. Zeits.*, **26**, 149, 1925.
12. STEWART, *Phys. Rev.*, **14**, 376, 1919; MILNE, *Phil. Mag.*, **42**, 96, 1921.
13. *J. Acoust. Soc.*, **9**, 217, 1938. *See also* EPSTEIN, *C. I. T. Karman Vol.*, 162; HARTMANN and FOCKE, *Phys. Rev.*, **57**, 221, 1940.
14. *Phil. Trans. Roy. Soc.*, **210**, 269, 1910.
15. *Trans. Faraday. Soc.*, **32**, 1111 and 1115, 1936.
16. *Ibid.*, **32**, 1101, 1936.
17. *U.S. Pat.*, 1217, 1885.
18. TUCKER, *J. Roy. Aero. Soc.*, **28**, 504, 1924. *See also* FAGE, *Proc. Roy. Soc., A*, **107**, 451, 1925; PARIS, *Phil. Mag.*, **13**, 99, 1932, and **16**, 50, 1933.
19. WAETZMANN, *Zeits. tech. Physik*, **4**, 193, 1923.
20. *See* WOOD and YOUNG, *Proc. Roy. Soc., A*, **100**, 252 and 261, 1921. WOOD and BROWNE, *Proc. Phys. Soc.*, **25**, 183, 1923.
21. HAYES, *J. Frank. Inst.*, **197**, 323, 1924.
22. *See La Nature*, **49**, 4 and 125, 1921.
23. In *Trans. Roy. Soc. Canada*, BOYLE, LEHMANN and REID, **19**, 194, 1925; BOYLE and REID, **20**, 233, 1926; BOYLE and TAYLOR, **20**, 245, 1926; *also* BOYLE and LEHMANN, *Phys. Rev.*, **27**, 518, 1926; BOYLE and TAYLOR, *Trans. Roy. Soc. Canada*, **21**, 79, 1927; BOYLE and LEHMANN, **21**, 115, 1927.
24. *Nature*, **145**, 212, 1940.
25. MEYER and BUCHMANN, *Akust. Zeits.*, **3**, 132, 1938. *See also* BRADFIELD, *Proc. Phys. Soc.*, **63**, 305, 1950.
26. *Die Anwendung phys. Ausschlussmethoden in Berg-, Tief- u. Wasserbau*, 1921. *See also* BERGER, *Schalltechnik*, 87, 1926.

CHAPTER FOURTEEN

REPRODUCTION AND SYNTHESIS OF SOUND

Mechanical Recording. The gramophone has developed out of the sound analysers of Scott and Edison (cf. p. 201). All the early instruments employed a needle moving along a sinuous groove and directly attached to the centre of a diaphragm clamped to the throat of a small horn. This horn at first increased in flare and length, to the advantage of the reproduction, until æsthetic considerations stepped in and demanded that it should be concealed in some form of cabinet. In spite of the poor reproduction in the bass of such a short horn as—in spite of ingenious foldings—is necessary to fit the confines of the cabinet, this type has persisted.[1]

It is sometimes thought that, for perfect reproduction free from distortion, the trace of the groove should exactly correspond to the wave-pattern in the air outside the horn, that, e.g., a sine wave should be reproduced as a sine curve on the record. A little thought, however, will show that this is not essential. All that we are concerned with in reproduction is the correspondence between the final sound as given out by the instrument when the record is played and the original sound. It does not matter what form the trace takes as long as any change or distortion is compensated in the rest of the reproducing system, diaphragm, horn, etc. As a matter of fact both parts of the system are designed to have a response proportional to the intensity, so that for a range over the whole gamut at the same (physical) loudness, both record and reproduction should give constant energy, n^2a^2, which means that the amplitude, a, is to be inversely as the frequency n. Since $2\pi na$ is the mean velocity in S.H.M., this is sometimes known as the " constant velocity " system of recording. The natural frequency of the diaphragm, usually of mica, is made high in the musical scale by making it thin and clamping it tightly at the edges. Maxfield and Harrison[2] have used the principle of mechanical impedance in order to design a sound box—this being the name given to the diaphragm and needle assemblage—which shall transform the vibrations of the needle most efficiently into sound waves. The diaphragm and the stylus bar which acts as a transformer between it and the pick-up, possess both inertance (in virtue

of their respective mass) and capacitance or compliance (due to stiffness). The compliance due to the arm of the stylus near its junction with the diaphragm and that of the air chamber facing it must be in shunt upon that at the edge of the diaphragm where it is clamped, and the stylus where it is pivoted, for if the former were made zero, i.e., were perfectly rigid, they would act as a short circuit, in the sense that no oscillation would be transmitted into the horn. Although it is not possible to calculate these quantities exactly, it is evident that the mechanical impedance of the pick-up must be matched to that of the diaphragm, and this again to that of the horn. The latter point has already been dealt with. In order that the diaphragm may execute piston-like vibrations it must have a high natural frequency, so that it does not develop nodal lines, with consequent out-of-phase sections when responding to notes of high pitch, and must further be loaded by an air cavity ; this is the purpose of the air chamber in the throat of the horn. The impedance of this cavity, calculated as for the Helmholtz resonator (p. 223), is to equal that of the horn (p. 237) and that of the diaphragm treated as a piston (= its mass merely). As the diaphragm has, in fact, a certain amount of compliance the upper arm of the stylus bar must be made somewhat resilient to cope with this. This was noted as an empirical fact long before these theoretical ideas were worked out. As with any transmitting system the ideal aimed at is to have the impedance of the system a pure resistance over the range of frequencies to be transmitted. At those frequencies for which the system behaves as a whole as a pure reactance, it acts as a filter. Apart from this extreme case, any reactance in the impedance, i.e., any component of pressure in phase with the velocity, will do no useful work but may do considerable damage by its reaction upon the needle, causing the latter to resist being guided by the groove, and so producing wear. With a hard needle the record will suffer ; if a soft fibre needle is used the damage is less serious as it is the needle which is worn.

Electrical Pick-ups. The new vogue which the gramophone received about 1927 was due rather to the introduction of electrical methods of recording than to the improvement in the mechanical reproduction outlined above. In this system, which is like a telephone system with an indefinite time lag, the oscillations of the microphone diaphragm are converted into the corresponding electrical oscillations, amplified if necessary and returned as mechanical oscillations of the recording needle, usually by electro-magnets.[3] A similar

circuit terminating in a loud-speaker enables the sound to be picked up from the record and reconverted into sound waves in the air at will. This arrangement holds many advantages over the old method. The electrical amplifier enables the instruments and soloists to be placed at reasonable distances from the apparatus which is recording their efforts and to be grouped so that the balance of tone is not upset, whereas formerly the soloist had his head almost inside the horn and was literally hemmed in by a very small orchestra whose accompaniment he effectively swamped in the resulting record. Acoustic adjustment of the room becomes possible instead of the former excessive reverberation which was necessary to add to the depth of the recording groove. Finally electric filters, always more compact and easier to design than acoustic filters, can be incorporated in the circuit, to relieve distortion, tone down the high frequencies of the scratch, etc. Unfortunately, a filter which removes the scratch noise also takes out all the upper tones in the reproduction. To get over this difficulty, Pierce and Hunt [4] suggest that the electric record after being picked up should first be passed through an amplifier which exaggerates the extreme treble at the expense of the bass and, *ipso facto,* masks the scratch noise. When the wave-form is then passed on to the loud-speaker, it will have the proper proportions re-established by another amplifier which over-emphasizes the bass. In the resulting reproduction the high frequencies are retained, but the scratch noise—it is said—is mislaid.

Loud-speakers. We have already referred in a number of places to the principle which should underlie a loud-speaker. As far as the acoustical part of the apparatus is concerned theory would indicate the exponential horn (p. 237) as the most effective type. The piston type is, however, more often met with. This consists of a large, waxed paper diaphragm, one or two feet across, which is set back at its centre to form an obtuse angled cone. Sometimes two cones one behind the other may be found. The cone is actuated at its centre by the electrical mechanism. As the diaphragm is comparatively loose it has no marked resonance though such resonance as there is lies in the bass. Owing to the asymmetric loading the membrane is liable to produce distortion, particularly at higher frequencies. Formerly the centre of the diaphragm was vibrated by a steel stylus in an alternating magnetic field but, *ceteris paribus*, the moving-coil type of reproducer is found to produce less distortion.[5] This bears the same relation to the older type as the D'Arsonval to the tangent

galvanometer. It consists of a light coil to which the current is fed, and which is fixed to the diaphragm and oscillates in the magnetic field of a permanent magnet. Other distortion-less loud-speakers have been operated on the electrostatic system, i.e., they are overgrown condenser microphones; while for frequencies above 10,000 cycles/sec. at which the response of these falls off, one operating on the piezo-electric principle (cf. Chap. X) is now available.

As it is difficult with a single coil and horn to cover the musical gamut adequately, often two coils will be mounted at either end of a permanent bar magnet (one of the new high induction alloys). These coils are then connected severally to two horn-type diaphragms, the smaller high-frequency one about 3 in. diameter lying within the other, about 1 ft. diameter. If the electric signal is fed to both simultaneously, there is a danger of phase interference when the acoustic signals leave the trumpets. The designer therefore prefers to feed only the low-frequency signal to the large speaker and only the high frequency to the small one. The "cross-over" takes place at 3,000 c./sec. and is accomplished by electrical filters.

A typical loud-speaker has its moving coil mounted at the centre of a paper cone which is pleated into circular corrugations. At low frequencies the cone vibrates almost with the coil as a whole but at frequencies higher than the fundamental of the cone it tends to vibrate in segments with circular nodal lines. The corrugations help to make the central zone vibrate in piston form without disturbing contributions from the outer rim. In fact, the pleats near the rim stop vibrating instead of making out-of-phase contributions to the sound in advance of the speaker.

There is, nevertheless, a loss of radiating efficiency in any loudspeaker at low frequencies unless it is mounted flush with a large board to act as a baffle. This arises because the vibrations in the vicinity of the back and the front of the membrane are out of phase and their respective radiations to points in the vicinity would interfere adversely if allowed to mingle. The phase difference between such contributions depends on the distance from the source to the rim of the baffle, i.e., on the latter's diameter in relation to the wavelength of the sound; the larger the baffle, the greater improvement it affords for the low frequencies.

In the apparatus used by Davis [6] for obtaining rapidly the response curve of a loud-speaker, the voltage from the microphone is taken after amplification to one pair of electrodes of a cathode ray oscillo-

graph. To the other pair of plates at right angles to the first is led from the valve oscillator a D.C. voltage whose value is proportional to the log of the frequency. This is accomplished by passing the output through a frequency-weighting network and amplifier.

Thus the electron spot in the viewing screen receives a horizontal displacement proportional to the frequency and a vertical one proportional to the output of the microphone. As then the oscillator is taken through the gamut, the spot on the screen traces out the response curve (which can be photographed) of the loud-speaker as recorded by the distortion-free microphone. The frequency-weighting network, to which reference has been made, consists essentially of a resistance-capacity stage of amplification in which the coupling condenser is small. Consequently the amplification factor increases nearly as the frequency. In the rectifier which follows, the D.C. output is made nearly proportional to the log of the frequency.

Reciprocity Theorem. A theorem due originally to Rayleigh [7] has assumed importance in the calibration of a transducer.

If a pressure p in an acoustic generator (with a certain impedance Z) produces in a linear receiver (with the same impedance Z) a volume velocity $\dot{X}$, the same amplitude will be produced in the erstwhile generator if the pressure p be subsequently applied to the erstwhile receiver. It is only true if there are no dissipative effects in the transducers and obviously only if the constants of any inverse electro- or magneto-strictive effects in the transducers are equal to those pertaining to the direct effects.

This principle is applied in the "free-field" calibration of microphones. A transducer is first operated with a driving current i as a source in an anechoic chamber while a measurement of sound pressure p is made at a point P some distance l away. It is then used as a microphone with a very small spherical source whose impedance is known placed at P. If the volume velocity of the spherical source is $\dot{X}$ while the open-circuit voltage on the terminals of the transducer is e, the reciprocity principle states in effect that the transfer impedances in the two experiments are equal, i.e.,

$$\frac{p}{i} = \frac{e}{\dot{X}}$$

The sensitivity of the microphone is then expressed by the ratio of e to p_0, the free-field sound pressure in the absence of the microphone, but p_0 may also be expressed in terms of the formulæ for a wave having

the velocity amplitude $\dot{X}$ at a distance l from the source and that for the impedance of a spherical source of vanishingly small radius (from eqn. (88), p. 234). The actual measurements required in the calibration (apart from frequency) are those of e, i, and l, assuming the ρc of the medium to be known.

Sound Film. Two methods of recording sound on film are in commercial use. In one method light is reflected from a mirror which oscillates with the sound on the same principle as the phonodeik (p. 202), is passed through a slit and produces a record of *variable width* having a serrated edge. The other recorder is virtually a string oscillograph in which there are two strings or ribbons of duralumin 0·0005 × 0·006 in. forming a " light valve " through which the light passes to affect the film to a greater or less extent. The record on the developed film is of fixed width but *variable density.* The ribbons are tensioned to a natural frequency of 9,500 cycles per second, which is above the band at present recorded. There is a certain amount of " background noise " due to faults in the recording apparatus. It is obvious that this background becomes more obnoxious as the level of recorded sound is lowered. A steady current is accordingly passed through the ribbons so as to close the gap completely when no sound is being recorded. As the intensity level of the latter increases the biasing current is automatically reduced, releasing the ribbons, for the background noise will then become more completely masked by the wanted sound.[8]

In reproduction the film runs *continuously* through a gate in the projector, a short distance ahead of the picture gate itself. The light from an incandescent filament after crossing the sound track falls on a photoelectric cell, the current from which is amplified and fed to the loud-speakers.

We must also mention here the process sometimes known as the magnetophone, whereby speech or music may be recorded by magnetic means on a steel strip and reproduced later. This is now frequently used by the B.B.C. when a temporary record of a speech is required during broadcasting, to be radiated later in the day in another programme. Although this official adoption is recent, the process dates back to the early days of wireless telegraphy and was a device used for tape-recording by Poulson. A continuous steel band is carried round on pulleys in front of a coil through which an alternating current engendered by the speech or music passes. This magnetizes the steel in the direction of its motion in a corresponding fashion.

When reproduction is desired, the band is passed in front of another coil with Permalloy core and the induced current is fed into an amplifier. The record can be "washed off" for subsequent use by passing it near a sufficiently strong direct current or alternating current in this wise, that the first saturates the iron, the latter demagnetizes it by taking it through hysteresis cycles of successively smaller amplitudes as the band retreats from its influence.

Electric Musical Instruments.—These are of three main types: (1) Instruments which retain, in their construction, considerable portions of conventional instruments. Such are the pianofortes in which the vibrations of the wires, struck in the usual fashion, are picked up by electro-magnets near them and the resulting induced currents after amplification are passed to a loud-speaker. Such pianofortes have no soundboards.

(2) Instruments which claim to produce "music from the æther" and are actually self-excited valve oscillators giving notes whose quality and frequency are determined by the circuit details and, in particular, the capacity in the tuned portion. The capacity is varied by moving a sliding condenser or, more simply, by approaching the hand to give an infinite variation in frequency. Others have keyboards giving a step-by-step variation of the capacity of the effective condenser.

(3) Keyboard instruments in which the frequency of each note depends on periodic variations in capacity of condensers with revolving plates or on the number of studs on a revolving axle which pass an electromagnetic pick-up in one second. In some, for example, the Hammond organ, the designer has so shaped the stud, that, after a little judicious filtering of the electric current, a S.H.M. of known frequency is obtained from every pick-up. The quality, which is capable of being pre-set by the player, is made up of mixtures of the fundamental with harmonics, derived from S.H.M.'s picked up further along the rotor, where there is a greater number of studs per revolution. The resulting synthesized current is amplified and fed to the loud-speakers. Others purposely pick up a distorted wave, by using saw-edge teeth on the axle, and derive their desired smoothed current by suitable filters. In the last class, the rotating axle and stationary electro-magnets are sometimes replaced by a stroboscopic disc interrupting periodically the light which falls on a photo-electric cell, the current from which is the source of the sound.[9]

The quality of these instruments depends ultimately on the faith-

fulness of the reproduction in the loud-speaker, and it is this limitation which at the moment impedes their more general adoption.

Instruments for Æther Music. Under this ambitious heading —following the inventors—we may group those instruments which derive music directly from tuned circuits in oscillation and have no mechanical vibrators other than the diaphragm in the ultimate loud-speaker, if any. The prototype of these was the singing arc of Duddell (p. 249). By continuously variable or step-by-step changes in inductance it was possible to play tunes on the arc. Since then numerous instruments using valve oscillators (audions) have been patented, but those with keyboards and one oscillator to each note have proved too expensive both in first cost and upkeep of their numerous valves to gain a footing. There remain the solo instruments, of which the Theremin and the Trautonium have reached commercial production. Theremin's instrument is a beat-frequency oscillator in which the capacity controlling the frequency consists of a copper loop and a baton held in the hand (or sometimes the hand itself). The beat note when the hand is away from the loop is above the audible limit, but descends through the musical gamut as the hand approaches the loop in virtue of the increasing capacity, corresponding to the approach of an earthed conductor to an insulated plate in a familiar experiment of the elementary physics course. With this mode of control the music is confined to a series of single notes with a glissando between each which—together with the constant " feeling " for the note, like a horn-player burbling for the harmonics of his instrument—becomes rather distressing to the musical ear, in spite of the undoubted skill in performance of the inventor.

In the trautonium sponsored by the Telefunken Company of Germany, *portamento* playing is still possible, or one may proceed directly from note to note in proper musical style. A grid-glow tube is used as a variable-frequency generator. The grid potential which determines the pitch is controlled by the length of resistance wire cut off by the player pressing some point on the wire on to a mark on a metal plate behind the wire, like the frets on a banjo. Another resistance under the plate is varied by the pressure of the finger on the plate, and so alters the loudness of the sound. It is therefore possible to get as nice a variation in playing as a virtuoso may get from a violin by the mere graduation in the position and pressure of a single finger on the wire, including effects such as the vibrato and staccato playing.

Electrophonic Organs. The Telharmonium of Cahill antedated the valve amplifier and speaker. His apparatus, like those of the present day, produced electric currents of varying frequency and intensity from rotating elements. These currents were passed over the United States Telephone System and provided music for the household subscriber in a service very similar to that now available to subscribers in this country to the service operated by the British Broadcasting Corporation, in conjunction with the Post Office Telephone System. However, after the equipment had been built the experiment had to be abandoned owing to interference and inductive

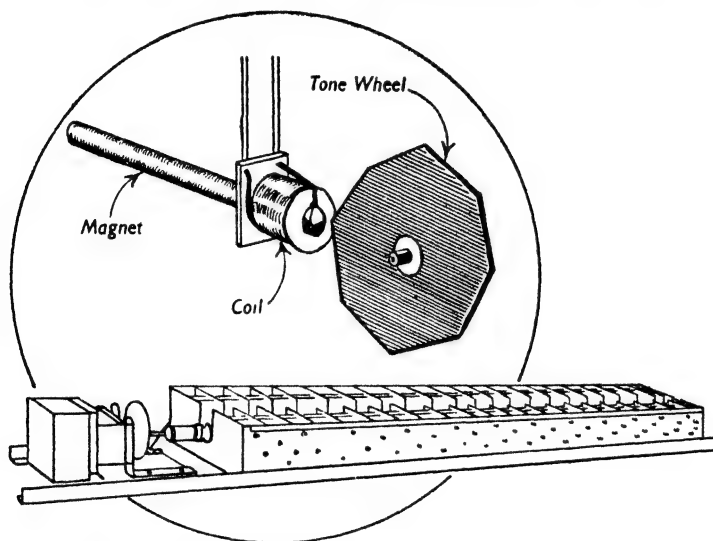



Fig. 120.—Principle of Electro-magnetic Tone Generator (Hammond Organ).

effects on other lines in the far from perfect telephone system of those days (1906), though his system of production contained all the essentials of an electrical organ. There are now a number of these instruments to choose from and at least three in Great Britain, of which the Hammond (an American invention) is probably the most widely diffused.

A synchronous motor drives a series of 91 tone generators through gears and pinions. One of these is shown in Fig. 120. The tone wheel is a polygonal plate, about the size of half a crown, rotating near a permanent magnet on which a coil is wound. As a high point on the wheel passes the magnet it induces a pulse of current in the coil.

The speeds of rotation and the number of corners on each wheel are so calculated that each disc produces one of the 91 partials used in building up the fundamentals and required timbre. The latter is decided in the harmonic controller wherein the various frequencies are super-imposed and flow as a single complex electrical wave to the pre-amplifier located in the console. There is of course an intensity control for each stop and a " swell pedal " which alters the overall intensity between the pre-amplifier and the loud-speakers.

The other type uses electrostatic production of tone. This is the principle employed in the Compton and the Midgley electrophonic instruments. Two electrodes are spaced a small distance apart in the

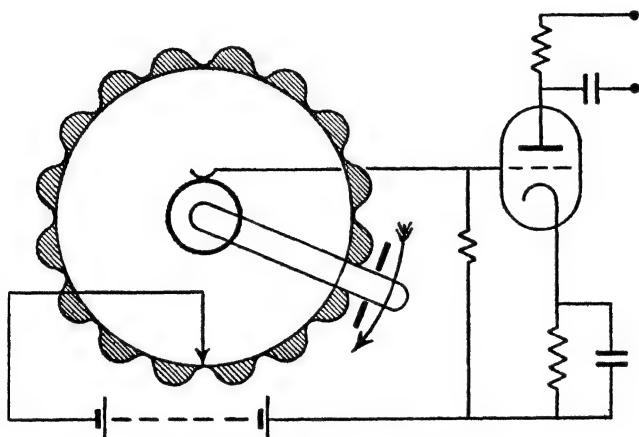



FIG. 121.—Principle of Electrostatic Tone Generator.

air, one moving relatively to the other. The two members constitute two plates of a condenser, usually at a fixed distance apart. An undulating variation in capacity between the members is caused by rotating one member, while the other on which sinusoidal grooves are inscribed is kept fixed (Fig. 121). As this system is liable to default due to buckling of one of the plates or lack of parallelism, Midgley keeps both condenser plates fixed, while varying the dielectric constant. To the surface of each of the stationary discs of insulating material are fixed eight concentric rings of conducting material cut into sine waves. The number of forms to each ring runs in powers of 2 from the centre to the outer ring, which has 256 undulations. A pair of these stationary discs face each other while between them another disc of bakelite rotates. The rotating disc is provided with

eight concentric rings of apertures, each of which covers a half-wave length of the corresponding sine wave. These condensers, of varying capacity, are connected in the grid circuits of valves so as to produce the necessary fundamentals and overtones for covering the musical gamut.

On most of these instruments the control of timbre is assured by stops as on the classical organ, in some of which the timbre is pre-selected to give close imitations of common organ stops or orchestral instruments. Other combinations lie at the choice of the player. Thus on the Hammond organ there is a small manual of six or seven coloured keys at each end, which determine the relative magnitude of the first six or seven harmonics of the fundamental of each note in the selective stops. Each of these control keys can be depressed through eight stages of intensity. Thus in the combination 6, 0, 7, 0, 3, 3, in which the numbers denote the relative intensities of the partials on the harmonic scale, the predominance of the odd harmonics point to a clarinet quality. A similar combination, with less of the fifth and sixth partials, would imitate a stopped flue pipe.

Although the three organs described normally give tones of fixed intensity, it is not difficult, though of course it adds to the elaboration, to introduce circuits which will damp the sound of each note from its inception and so mimic pianoforte, harp or bell tone, but of course the artificial wave-form does not vary during attenuation in the same way that it does in some of these instruments. It is not beyond the wit of man to devise circuits which will change the quality at the same time as they make the note die away. Some inventors indeed derive both constant and evanescent tones, as required, from the same damped source, e.g. struck reed. As soon as the sound has reached maximum intensity the pick-up is switched over to a circuit with a time constant which can be made to give long or short damping at will, or to maintain the tone without dissipation as long as the key is held down, in spite of the damping in the acoustic vibrator (this necessitates a negative resistance in the circuit). On the Midgley organ low-pass filters for preventing the high-frequency oscillations produced by the key contacts from reaching the amplifiers produce a lag in both the rise and fall of intensity when the key is depressed and released respectively. The former gives a gratuitous imitation of the slowness of speech of certain organ pipes, while the latter provides an artificial reverberation.

We have omitted from our classification those instruments which

lie on the border between musical producers and music reproducers or gramophones. There are, in fact, types in which the source is a sound film and light beam shining through the film on to a photo-electric cell, in exactly the same fashion as in the "talking film." The essential difference is that on the photoelectric organ the wave forms inscribed on endless film comprise a series all having the same fundamental wave-length but differing in complexity. The fundamental pitch is varied by the player altering the speed of rotation of the glass wheel on which the film is mounted, while the sideways shift of the beam of light alters the quality by bringing in a different trace. Such instruments become exceedingly complicated when adapted for playing more than one note at a time, and are scarcely likely to find favour save where talking film projector equipment is already installed. Even then it may be cheaper to purchase a library of sound films for the music required than to employ a photo-electric organ and organist.

We have spoken so far of the electric keyboard instrument as replacing the pianoforte and organ. This it can do quite well, since it is the aim of the makers of such instruments to secure uniformity of quality throughout the pianoforte and through each stop of pipes or reeds on the organ. When one considers how far it is feasible to replace an orchestral instrument with an electric tone producer, another problem arises. One must consider what are the characteristic "formants" of the instrument. A solo instrument of extensive range does not have the same wave-form throughout. That of the clarinet, for example, differs markedly in the upper register from the lower, while the timbre of the violin A played on the G string is not the same as that of the open A string. Each individual instrument has, in fact, characteristic resonances within its structure which reinforce certain partial tones in any note which is played and suppress others. No pre-set "mixtures" such as the Hammond organ has could give a satisfactory imitation of the clarinet or violoncello. To do this would necessitate the addition of elaborate filters having a fine structure of resonances to imitate the complex formants originating in the case and air cavities of these instruments.

References

1. *See* Williams, *J. Frank. Inst.*, **202**, 413, 1926; Thomas, *J. Sci. Instrum.*, **8**, 363, 1931.
2. *Bell. Syst. Tech. J.*, **5**, 493, 1926.
3. *See* Whitaker, *J. Sci. Instrum.*, **5**, 35, 1928; Kellogg, *J. Amer. I.E.E.*, **46**, 1041, 1927.

4. *J. Acoust. Soc.*, **10**, 14, 1938. *See also* BRAUNMUHL, *Akust. Zeits.*, **3**, 350, 1938.
5. *See* MCLACHLAN, *Proc. Roy. Soc.*, *A*, **122**, 604, 1929.
6. *Phil. Mag.*, **15**, 309, 1933.
7. *Sound*, **1**, 113, 1877; *see also* MACLEAN, *J. Acoust. Soc.*, **12**, 140, 1940.
8. *See* BRISTOL, *J. Frank. Inst.*, **205**, 179, 1928; SANDVICK, *ibid.*, **207**, 839, 1929; FREDERICK, *Rev. Sci. Instrum.*, **5**, 117, 1934.
9. *See* RICHARDSON, *Music. Assoc. Proc.*, **66**, 53, 1940.

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of subjects and authors cited